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$$k_z = i \sqrt{\pi^2/a^2 - \varepsilon_{\rm h} \mu_{\rm h} k_0^2} \qquad (2)$$

## Mimicking Surface Plasmons with Structured Surfaces

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Metals such as silver support surface plasmons: electromagnetic surface excitations localized near the surface that originate from the free electrons of the metal. Surface modes are also observed on highly conducting surfaces perforated by holes. We establish a close connection between the two, showing that electromagnetic waves in both materials are governed by an effective permittivity of the same plasma form. The size and spacing of holes can readily be controlled on all relevant length scales, which allows the creation of designer surface plasmons with almost arbitrary dispersion in frequency and in space, opening new vistas in surface plasmon optics.

The interaction of light with a metal surface is dominated by the free electrons that behave like a plasma with a dielectric function,  $\varepsilon = 1 - \omega_p^2/\omega^2$ , which is negative below the plasma frequency,  $\omega_{p}$ . As a consequence, metals support collective oscillations of the electrons bound to the surface (1). These are the surface plasmons, responsible for a host of phenomena unique to metals. Examples are the surfaceenhanced Raman scattering experiments (2), in which the Raman signal is enhanced by the surface plasmon resonances (sometimes by as much as a factor of  $10^6$ ) and by the strong absorption of light in silver colloids (the black parts of photographic negatives). Previous work has investigated waveguiding in surface plasmon polariton band gap structures (3), plasmon propagation in metal stripes (4), and two-dimensional optics with surface plasmon polaritons (5). A useful summary of work appears in a recent review (6). More recently, experiments have been performed on the transmission of light through subwavelength holes in metal films (7). It has been established (8)that resonant excitation of surface plasmons creates huge electric fields at the surface that force light through the holes, giving very high transmission coefficients.

Other materials with quite different dielectric functions from the plasma form can also support electromagnetic surface modes. A perfect conductor is an exception, but even such a material can be induced to support surface modes by drilling an array of holes in the surface. We show that these structured surfaces have many more properties in common with the electron plasma and that they can be described by an effective dielectric function of the plasma form, provided that the structure is on a scale much smaller than the wavelength of probing radiation. Reducing our description to a dielectric function of the plasma form has a conceptual elegance that unites all of these phenomena under the same umbrella. An effective response generated by the structure of the medium is commonly seen at microwave frequencies (9-11). Thus, it appears that surface structure may spoof surface plasmons, which provide an accurate paradigm for structured surfaces. Such a description allows us to unify a broad range of analogous phenomena. To illustrate these ideas, we begin with a simple model. Let us suppose that we have a surface that is a perfect conductor pierced by an array of holes. For simplicity, we shall assume a square cross section  $a \times a$  for the holes and a square array of side d (Fig. 1). We suppose that the holes and their spacing are much smaller that the wavelength of radiation,  $a < d << \lambda_0$ . An incident wave excites several waveguide modes in the holes but the fundamental mode will dominate because it is the least strongly decaying. Both electric and magnetic fields are zero inside the conductor but in the holes the electric field has the form

$$\mathbf{E} = E_0[0,1,0]\sin(\pi x/a)\exp(ik_z z - i\omega t),$$
  
 
$$0 < x < a, 0 < y < a \qquad (1$$

where  $E_0$  is a constant, x, y, and z are cartesian coordinates,  $i = \sqrt{-1}$ ,  $\omega$  is the frequency, t is time,  $k_0$  is the free space wave vector, and where  $\varepsilon_{\rm h}$  and  $\mu_{\rm h}$  are the permittivity and permeability, respectively, of any material that may be filling the holes. We take the *z* axis to be the surface normal. The magnetic fields along the *x* and *z* axes  $H_x$  and  $H_z$ , follow from this equation.

Externally incident radiation is insensitive to the details of the holes, which it cannot resolve, and sees only an average response that we shall describe by  $\varepsilon_z$ ,  $\varepsilon_x = \varepsilon_y$  and  $\mu_z$ ,  $\mu_x = \mu_y$  for an anisotropic effective homogeneous medium. We can easily determine  $\varepsilon_z$  and  $\mu_z$  by observing that the dispersion of the waveguide mode is unaffected by  $k_x$  or  $k_y$  in either of the two possible polarizations and therefore

$$\varepsilon_z = \mu_z = \infty$$
 (3)

Suppose that the effective homogeneous fields in the medium are

$$\mathbf{E}' = E'_{0}[0,1,0] \exp(ik_{x}x + ik_{z}z - i\omega t)$$
(4)

We require that  $k_z$  is the same as in the waveguide and  $k_x$  is defined by the incident direction. If this effective field is to match to the incident and reflected fields external to the surface, Eq. 1 and Eq. 4 must give the same average fields at the surface. Hence, after matching to incident and reflected waves,

$$\overline{E}_{y} = E_{0} \frac{a}{d^{2}} \int_{0}^{a} \sin(\pi x/a) dx = E_{0} \frac{2a^{2}}{\pi d^{2}} = E'_{0}$$
(5)

We argue that the instantaneous flow of energy across the surface,  $(\mathbf{E} \times \mathbf{H})_z$ , must be the



**Fig. 1.** Our model system:  $a \times a$  square holes arranged on a  $d \times d$  lattice are cut into the surface of a perfect conductor. Our theory predicts localized surface plasmon modes induced by the structure.

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same inside and outside the surface, both for the real and effective media,

$$(\mathbf{E} \times \mathbf{H})_{z} = \frac{-k_{z}E_{0}^{2}}{\omega\mu_{h}\mu_{0}} \frac{a}{d^{2}} \int_{0}^{a} \sin^{2}(\pi x/a)dx = \frac{-k_{z}E_{0}^{2}}{\omega\mu_{h}\mu_{0}} \frac{a^{2}}{2d^{2}} = (\mathbf{E}' \times \mathbf{H}')_{z} = \frac{-k_{z}E'_{0}^{2}}{\omega\mu_{0}\mu_{x}}$$

Finer details of the fields around the holes are matched by strongly evanescent fields in the vacuum. Because these fields do not escape from the surface and contain little energy, we neglect them. Substituting for  $E'_{0}^{2}$ , we deduce

(6)

$$\mu_x = \mu_y = \frac{2d^2\mu_h}{a^2} \left[\frac{2a^2}{\pi d^2}\right]^2 = \frac{8a^2\mu_h}{\pi^2 d^2} \quad (7)$$

We also know that

$$k_z = k_0 \sqrt{\varepsilon_y \mu_x} = i \sqrt{\pi^2/a^2 - \varepsilon_h \mu_h k_0^2} \qquad (8)$$

and hence if  $c_0$  is the velocity of light in a vacuo

$$\varepsilon_{\rm y} = \varepsilon_{\rm x} = \frac{1}{\mu_{\rm x}} \left( \varepsilon_{\rm h} \mu_{\rm h} - \frac{\pi^2}{a^2 k_0^2} \right) = \frac{\pi^2 d^2 \varepsilon_{\rm h}}{8a^2} \left( 1 - \frac{\pi^2 c_0^2}{a^2 \omega^2 \varepsilon_{\rm h} \mu_{\rm h}} \right)$$
(9)

which is the canonical plasmon form with a plasma frequency of

$$\omega_{\rm pl} = \frac{\pi c_0}{1 \sqrt{\varepsilon_{\rm h} \mu_{\rm h}}} \tag{10}$$

This is exactly the cut-off frequency of the wave guide. It is easy to see that this is a general result: Any array of closely spaced holes in a perfect conductor will have this form of effective response, with the plasma frequency given by the cut-off frequency of the waveguide mode.

This effective medium implies a bound surface state when there is a divergence in the reflection coefficient of the surface for large values of  $k_{\parallel}$  where  $k'_{z} = i \sqrt{k_{\parallel}^{2} - k_{0}^{2}}$  is imag-



**Fig. 2.** Dispersion relation for spoof surface plasmons on a structured surface. The asymptotes of the light line at low frequencies and the plasma frequency at large values of  $k_{\parallel}$  are shown.

inary. For the polarization where the magnetic field is parallel to the surface

$$r = \frac{k'_z - \varepsilon_{\parallel}^{-1}k_z}{k'_z + \varepsilon_{\parallel}^{-1}k_z} = \infty \qquad (11)$$

which implies a dispersion relationship typical of a surface plasmon polariton

$$k_{\parallel}^{2}c_{0}^{2} = \omega^{2} + \frac{1}{\omega_{\rm pl}^{2} - \omega^{2}} \frac{64a^{4}\omega^{4}}{\pi^{4}d^{4}} \quad (12)$$

Figure 2 shows a sketch of the dispersion. Notably, at low frequencies the surface mode approaches the light line asymptotically, and the fields associated with the mode expand into the vacuum. At large  $k_{\parallel}$ the frequency of the mode approaches  $\omega_{p}$ , in contrast to an isotropic plasma where the asymptote is  $\omega_p/\sqrt{2}$ . Although a flat perfectly conducting surface supports no bound states, the presence of holes, however small, produces a surface plasmon polariton-like bound surface state. Indeed, almost any disturbance of the flat surface will bind a state. Bound states are found in the following circumstances: a surface with an array of holes of finite depth, however shallow; a surface with an array of grooves of finite depth; and a surface covered by a layer of dielectric, however thin.

It is also possible to produce hybrid surface plasmons by cutting holes in metals such as silver that already have a surface plasmon. The holes will increase the penetration of fields into the metal and lower the frequency of existing surface plasmons. In this case, it is hard to distinguish between the "real" and the spoof surface plasmons as they merge one into the other.

Our theory as presented is valid when the spacing between the holes is much smaller than the wavelength, so that incident radiation cannot resolve the individual holes. However, the theory can be progressively extended to larger hole spacings by including diffracted waves, more of which appear as the spacing gets bigger. We can do this by including Fourier components of  $\varepsilon$  and  $\mu$  so that the effective medium also gives rise to diffracted beams. The main effect of diffraction is to couple the spoof plasmons to incident radiation.

Our result resolves a longstanding debate over transmission through subwavelength holes. As mentioned above, when the anomalously high transmission was first observed (Eq. 7), it was attributed to resonant excitation of surface plasmons. Subsequent calculations (Eq. 8) showed that subwavelength holes in a perfect conductor also gave rise to similar anomalous transmission even though the free surface of an unperforated conductor has no surface modes, leading to discussion as to the true origin of the effect. Now we can see that there are not two separate mechanisms: The holes will spoof surface plasmons which play the same resonant role as do the real ones on silver.

The ability to engineer a surface plasmon at almost any frequency (metals are nearly perfect conductors from zero frequency up to the threshold of the terahertz regime) where none existed before and to modify at will the frequency of surface plasmons on metals such as silver opens opportunities to control and direct radiation at surfaces over a wide spectral range. Unlike other schemes for creating metamaterials with a plasmalike response (9-11), the present scheme is readily implemented, given that hole drilling techniques are available over a range of length scales down to suboptical wavelengths. Invoking a picture of the surface plasmon as a wave in two dimensions, we now have the ability to control the refractive index perceived by this wave. We stress that this is not the refractive index of the underlying material but is given by the extent to which the surface plasmon wave vector deviates from the light line,  $n_{\rm sp} = k_{\parallel} / \omega_{\rm sp}$ . We have the option to design a lens to focus surface plasmons. Alternatively, we could contour the refractive index,  $n_{sp}(x, y)$ , and, adopting a particle picture of the surface plasmon, imagine it rolling like a ball over this landscape. More prosaic applications such as creating channels to act as waveguides are also possible.

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