Resonant Transmission of Light Through Finite Chains of Subwavelength Holes in a Metallic Film

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In this Letter we show that the extraordinary optical transmission phenomenon found before in 2D hole arrays is already present in a linear chain of subwavelength holes, which can be considered as the basic geometrical unit showing this property. In order to study this problem, we have developed a new theoretical framework, able to analyze the optical properties of finite collections of subwavelength apertures and/or dimples (of any shape and placed in arbitrary positions) drilled in a metallic film.

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After the discovery of extraordinary optical transmission (EOT) through 2D square arrays of subwavelength holes in an optically thick metallic film [1], several works have appeared in order to understand the basics of this remarkable phenomenon. From the theoretical side, the studies can be divided in those considering the simpler 1D analog of arrays of subwavelength slits [2–4] and the 2D arrays of holes [5–8]. Several of these works explained EOT in terms of the existence of surface electromagnetic (EM) resonances, something pointed out by the original experiments [1] and definitely corroborated by recent experiments [9]. However, a question that still remains open is what is the minimal system showing EOT, which is interesting both from the basic point of view and for possible future applications.

In this Letter we move a step forward in this direction and consider the optical transmission properties of finite chains of subwavelength holes, a basic structure with less symmetry than the original 2D array which, up to our knowledge, had not been considered before. Here we show that EOT is also present in these 1D finite systems. As an important by-product, we develop a formalism capable of treating the optical properties of even thousands of indentations (with any shape and placed arbitrarily) in metal films, something not possible with the present numerical methods, which are restricted to just a few of such indentations.

Let us first present the formalism, which is a nontrivial extension of the simpler one developed for sets of 1D indentations and that was successfully applied [10,11] for the understanding of enhanced transmission and beaming of light in *single* apertures flanked by periodic corrugations [12,13]. Here, we analyze the EM transmission through a planar metal film (with finite thickness h and infinite in the x-y plane) with a set of indentations at both input and output interfaces. These indentations may be either holes or dimples. Furthermore, each one of them may have any desired shape and may be placed in any position we wish. The only approximation in the formalism is that the metal is treated as a perfect conductor ($\epsilon = -\infty$). We have demonstrated in previous works that this model captures the basic ingredients of the enhanced transmission phenomena, being even of semiquantitative value in the optical regime for good conductors like silver or gold [14]. Additionally, results obtained within the perfect conductor approximation are scalable to different frequency regimes.

In our method, we assume a rectangular supercell, with lattice parameters L_x and L_y, along the x and y axes, respectively. This supercell may be real (if we are considering a bona fide periodic system) or artificial, if the number of indentations is finite. In this latter case, the limit $L_x, L_y \to \infty$ must be taken.

For an incident plane wave with parallel wave vector $\vec{k}_0$[15] and polarization σ_0, the EM field at $z = 0^-$ (at the metal interface in which radiation is impinging on) can be written, in terms of the reflection coefficients $r_{\vec{k}\sigma}$, as

$$\begin{aligned} |\vec{E}(0^-)\rangle &= |\vec{k}_0\sigma_0\rangle + \sum_{\vec{k}\sigma} r_{\vec{k}\sigma}|\vec{k}\sigma\rangle, \\ -\vec{u}_z \times |\vec{H}(0^-)\rangle &= Y_{\vec{k}_0\sigma_0}|\vec{k}_0\sigma_0\rangle - \sum_{\vec{k}\sigma} r_{\vec{k}\sigma} Y_{\vec{k}\sigma}|\vec{k}\sigma\rangle, \end{aligned} \quad (1)$$

where we have used the Dirac notation, and expressed the bivectors $\vec{E} = (E_x, E_y)^T$ and $\vec{H} = (H_x, H_y)^T$ (T standing for transposition) in terms of the EM vacuum eigenmodes, $|\vec{k}\sigma\rangle$. The expressions for these EM vacuum eigenmodes in real space are: $\langle\vec{r}|\vec{k}p\rangle = (k_x, k_y)^T \times \exp(\iota\vec{k}\,\vec{r})/\sqrt{L_x L_y |k|^2}$ and $\langle\vec{r}|\vec{k}s\rangle = (-k_y, k_x)^T \exp(\iota\vec{k}\,\vec{r})/\sqrt{L_x L_y |k|^2}$. The electric and magnetic fields in Eq. (1) are related through the admittances $Y_{\vec{k}s} = k_z/k_\omega$ and $Y_{\vec{k}p} = k_\omega/k_z$ (for s- and p-polarization, respectively), where $k_\omega = \omega/c$ (ω is the frequency and c the speed of light) and $|k|^2 + k_z^2 = k_\omega^2$. Notice that, according to Bloch's theorem, $\vec{k} = \vec{k}_0 + \vec{G}$, $\vec{G}$ being a (supercell) reciprocal lattice vector.

In the region of transmission, the electric field at $z = h^+$ can be expressed as a function of the transmission amplitudes $t_{\vec{k}\sigma}$ as $|\vec{E}(h^+)\rangle = \sum_{\vec{k}\sigma} t_{\vec{k}\sigma} |\vec{k}\sigma\rangle$, from where the magnetic field can be readily calculated.

The EM fields inside the indentations can be written, in terms of the expansion coefficients A_α, B_α, as:

$$|\vec{E}(z)\rangle = \sum_\alpha |\alpha\rangle [A_\alpha e^{\iota q_{z\alpha} z} + B_\alpha e^{-\iota q_{z\alpha} z}],$$
$$-\vec{u}_z \times |\vec{H}(z)\rangle = \sum_\alpha |\alpha\rangle Y_\alpha [A_\alpha e^{\iota q_{z\alpha} z} - B_\alpha e^{-\iota q_{z\alpha} z}]. \quad (2)$$

In the previous equations, α runs over all "objects," which we define as any EM eigenmode considered in the expansion. An object is, therefore, characterized by the indentation it belongs to, by its polarization and by the indexes related to the mode spatial dependence. All that is required to be known are the electric field bivectors $|\alpha\rangle$ [16] and the propagation constants $q_{z\alpha}$ associated to the objects, as the admittance $Y_\alpha = q_{z\alpha}/k_\omega$ for TM modes, while for TE modes $Y_\alpha = k_\omega/q_{z\alpha}$. For indentations with such simple cross sections as rectangular or circular, the required expressions for $|\alpha\rangle$ and $q_{z\alpha}$ can be found analytically [17]; otherwise, they can be numerically computed [18]. By matching the EM fields appropriately on all interfaces, we end up with a set of linear equations for the expansion coefficients. We find it convenient to define the quantities $E_\alpha \equiv A_\alpha + B_\alpha$ and $E'_\alpha \equiv -(A_\alpha e^{iq_{z\alpha}h} + B_\alpha e^{-iq_{z\alpha}h})$, which are the modal amplitudes of the electric field at the input and output interfaces of the indentations, respectively. The set $\{E_\alpha, E'_\alpha\}$ must satisfy:

$$(G_{\alpha\alpha} - \epsilon_\alpha)E_\alpha + \sum_{\beta\neq\alpha} G_{\alpha\beta}E_\beta - G^V_\alpha E'_\alpha = I_\alpha,$$
$$(G_{\gamma\gamma} - \epsilon_\gamma)E'_\gamma + \sum_{\nu\neq\gamma} G_{\gamma\nu}E'_\nu - G^V_\gamma E_\gamma = 0. \quad (3)$$

The different terms in these "tight-binding" like equations have a simple interpretation: $I_\alpha \equiv 2i\langle\vec{k}_0\sigma_0|\alpha\rangle$ takes into account the direct initial illumination over object α. ϵ_α is related to the bouncing back and forth of the EM fields inside object α and is $\epsilon_\alpha = -iY_\alpha(1 + \Phi_\alpha)/(1 - \Phi_\alpha)$ where, for holes, $\Phi_\alpha = \exp(2iq_{z\alpha}h)$ and the same expression applies for dimples, but replacing h by W_α, the depth of the dimple. The main difference between holes and dimples is the presence of G^V_α, which reflects the coupling between the two sides of the indentation. For a hole, $G^V_\alpha = -2iY_\alpha\sqrt{\Phi_\alpha}/(1 - \Phi_\alpha)$, while for a dimple, $G^V_\alpha = 0$.

The term $G_{\alpha\beta} = i\sum_{\vec{k}\sigma} Y_{\vec{k}\sigma}\langle\alpha|\vec{k}\sigma\rangle\langle\vec{k}\sigma|\beta\rangle$ controls the EM coupling between indentations. It takes into account that each point in the object β emits EM radiation, which is "collected" by the object α. If the system is periodic, $G_{\alpha\beta}$ can be calculated through the previous discrete sum by including enough diffraction waves. If the considered supercell is fictitious, the limit $L_x, L_y \to \infty$ transforms the previous sum into an integral over diffraction modes. It is then convenient to calculate $G_{\alpha\beta}$ through $G_{\alpha\beta} = \langle\alpha|\hat{G}|\beta\rangle$. The integral defining the dyadic $\hat{G}(\vec{r}_\parallel, \vec{r}'_\parallel) = \langle\vec{r}_\parallel|\hat{G}|\vec{r}'_\parallel\rangle$ can be evaluated, obtaining

$$\hat{G}_{ij}(\vec{r}_\parallel, \vec{r}'_\parallel) = g(d)\delta_{ij} + (2\delta_{ij} - 1)\frac{\partial^2 g(d)}{\partial d_i \partial d_j}, \quad (4)$$

where i, j can be either x or y, δ_{ij} is Kronecker's delta, $\vec{d} \equiv k_\omega(\vec{r}_\parallel - \vec{r}'_\parallel)$, and $g(d) = k_\omega \exp(\iota d)/(2\pi d)$ is proportional to the scalar free-space Green function associated to the Helmholtz equation in 3D.

$\hat{G}$ turns out to be the in-plane components of the EM Green function dyadic associated to an *homogeneous* medium in three dimensions [17]. As a technical note, the calculation of $G_{\alpha\beta}$ for objects in the *same* indentation from $\langle\alpha|\hat{G}|\beta\rangle$ suffers from the problems associated to the divergence of $\hat{G}$ at $|\vec{r}_\parallel - \vec{r}'_\parallel| \to 0$ [19–21], and we evaluate it directly from its integral over diffraction modes.

Therefore, our method reduces the calculation of EM fields into finding the EM field distribution right at the indentation openings, which is extremely efficient when the openings cover a small fraction of the metal surface. By projecting these fields into indentation eigenmodes, convergence (as a function of number of eigenmodes needed) is reached very quickly, especially in the subwavelength regime. Notice that Eq. (3) can also be used for an infinite periodic array of indentations by imposing Bloch's theorem on the set $\{E_\alpha, E'_\alpha\}$, something we will make use of when analyzing the case of an infinite linear chain (see below).

Once the self-consistent $\{E_\alpha, E'_\alpha\}$ are found, it is straightforward to find all expansion coefficients, and from them the EM fields in all space. For the total transmittance through subwavelength holes, we find $T = \sum_\alpha \mathrm{Im}[(G^V_\alpha)^* E^*_\alpha E'_\alpha]/Y_{\vec{k}_0\sigma_0}$.

We have tested our formalism against published results for two extreme systems: a single circular hole [22] and a square array of square holes [7]. In both cases we recover the known results, showing that our method is free from numerical instabilities and that, in finite systems, there are no spurious effects related to the $L_x, L_y \to \infty$ limit.

In the rest of this Letter we apply this formalism to the study of linear chains of subwavelength circular holes (see Fig. 1). For proof of principle purposes, we choose holes with radius $a/d = 0.25$ perforated in a metallic film of thickness $h/a = 1$, d being the first-neighbor distance between holes. These are typical geometrical values in experiments in 2D hole arrays (2DHA). Figure 2(a) shows the evolution of the transmittance versus wavelength as a function of the number of holes, N [23]. The incident plane wave impinges normally, and is polarized with the E field pointing along the direction of the chain. For the other polarization, the boost in transmittance is negligible. In Fig. 2(a), the total transmission is normalized to the hole area and then divided by the corresponding N. The most interesting feature of these spectra is that, as N is increased, a transmittance peak

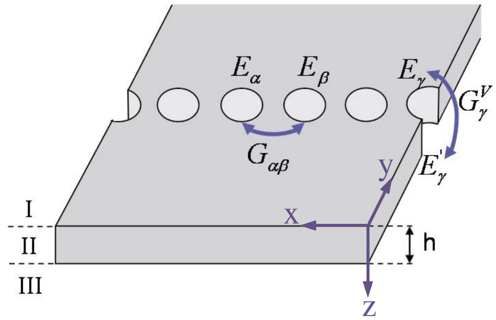

FIG. 1 (color online). A chain of circular holes in a metal film with thickness h, with a schematic representation of the terms appearing in the theoretical formalism presented in the text.

emerges at λ close to d, showing that enhanced transmission is also present in linear chains of subwavelength holes. The transmittance peak value, $T_{\max}$, grows almost linearly with N (for small N) eventually reaching saturation [see top inbox of Fig. 2(a)]. In order to gain physical insight into the origin of this transmission resonance, it is interesting to analyze a simplified model for the infinitely long chain. In this model, only one mode per hole is considered: the least evanescent mode with the electric field pointing mainly along the chain axis. Therefore, for all wavelengths, each hole behaves as a small dipole. In this geometry, Bloch's theorem implies $E_\alpha = E\exp(\iota k_x \alpha d)$, and $E'_\alpha = E'\exp(\iota k_x \alpha d)$, which renders the system of Eqs. (3) easily solvable. For the case we are considering (normal incidence, $k_x = 0$), this procedure yields

$$[(G_S - \epsilon)^2 - G_V^2]E = I(G_S - \epsilon), \tag{5}$$

where $G_S = G_{\alpha\alpha} + \sum_{\beta \neq \alpha} G_{\alpha\beta}$, $G_V = G_\alpha^V$, and $\epsilon = \epsilon_\alpha$. Figure 2(b) shows that the spectral location of the transmittance peaks for the infinite chain coincide with the cuts between $|G_S - \epsilon|$ and $|G_V|$, implying that the origin of EOT relies on a resonant denominator. Therefore, EOT is associated, as in the case of 2DHA, to the excitation of coupled surface EM resonances, which radiate into vacuum as they propagate along the surface. This is yet another instance of surface EM modes (in this case a leaky mode) appearing in a perfect conductor due to the presence of an array of indentations [24]. The results for finite chains [top inbox of Fig. 2(a)] show that this EM resonance is characterized by a typical length, L_D (for the case considered $L_D \approx 80d$) . When the size of the finite chain is smaller than L_D, the resonance is not fully developed and the transmittance is smaller than the one obtained for an infinite chain. However, for large enough N, the system can effectively be considered as infinite and its associated transmittance peak approaches the asymptotic value (with a $1/N$ contribution coming from holes located at a distance $\approx L_D/2$ from the chain ends). We have found that L_D is completely governed by the geometrical parameter a/d, increasing as a/d decreases.

It is also interesting to compare the peak value calculated for an infinite linear array [$T_{\max}^{1D} \approx 4$; see Fig. 2(a)] with the one obtained for an infinite 2D hole array with the same geometrical parameters [$T_{\max}^{2D} \approx 5$; see bottom inbox of Fig. 2(a)]. That is, when going from a linear chain to a 2D hole array, the transmittance per hole is only increased by 25%. This means that the basic unit of the extraordinary transmission phenomenon observed in 2DHA is the linear chain of holes, and that the 2D hole array can be considered as an array of weakly coupled 1D arrays. We have checked that this conclusion remains valid even for non-normal incidence, provided the in-plane component of the incident E-field points along the direction of the chain.

In order to study how the EOT in 2DHA develops from the EOT in linear chains, we have calculated the transmittance of a collection of finite linear chains separated by a distance d. Figure 3 renders the normalized-to-area transmittance versus wavelength for different stripes formed by several (ranging from 1 to ∞) chains of 41 holes. As shown in the figure, the strongest effect appears when going from 41×1 to 41×3, suggesting that the EM coupling between chains of subwavelength holes is very short-ranged. The short-range nature of the interchain interaction is more clearly seen in the inset to Fig. 3, which renders the transmission peak value, $T_{\max}$, through

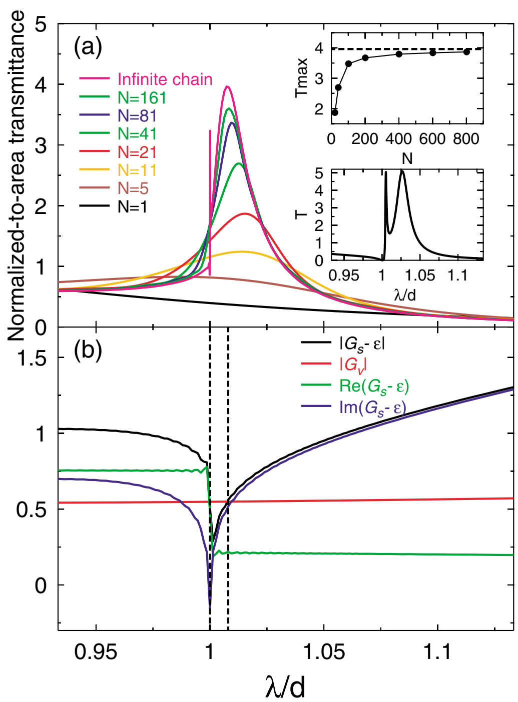

FIG. 2 (color). (a) Normalized-to-area transmittance (see text) versus λ/d for a linear array of N holes with $a/d = 0.25$ and metal thickness $h = a$. Top inbox: transmittance peak value, as a function of N, dashed line showing the value obtained for an infinite chain. Bottom inbox: normalized-to-area transmittance versus λ/d calculated for an infinite 2D hole array with the geometrical parameters defining the 1D arrays. (b) $G_S - \epsilon$ and G_V (see text for the definition of these magnitudes) as a function of λ/d.

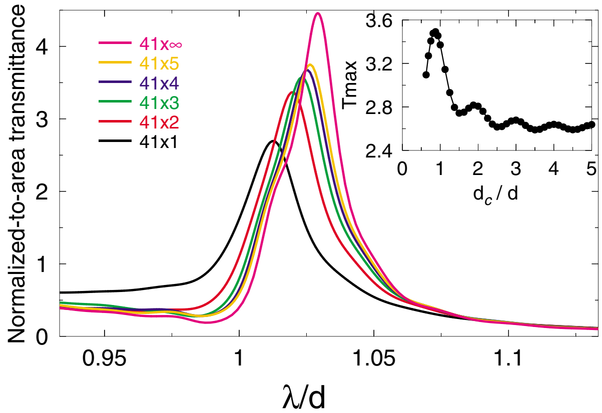

FIG. 3 (color). Normalized-to-area transmittance versus λ/d for several stripes formed by M linear chains of 41 holes with $a/d = 0.25$ and $h/a = 1$. M varies from 1 to 5. Also, the limiting case of $M = \infty$ is presented. Inset: transmittance peak value for the case of two linear chains (each one of them with the same parameters as before), as a function of their distance, d_c.

two finite linear chains as a function of the distance between them, d_c. As this inset shows, for this set of geometrical parameters, the chains are already practically uncoupled when $d_c = 3d$, the maximum coupling being for $d_c \approx d$.

When linear chains are added up to the structure, the transmittance peak shifts to longer wavelengths and its maximum value increases. The increase in transmittance is due to the fact that each chain takes advantage from the reillumination coming from other chains. The peak redshift is related to the corresponding decrease of frequency of the stripe surface EM mode, due to a reduction of its lateral confinement. More precisely, in this case, the stripe surface mode that couples to the incident plane wave with $\vec{k} = 0$ is essentially the sum, with equal phases, of the single chain leaky modes. In infinite 2DHA, two EOT peaks originate from the resonant coupling (via the holes) of the surface EM modes, with a narrow transmission peak appearing close to $\lambda \approx d$ (see bottom inbox of Fig. 2). Notice that in stripes of chains of finite length, only one peak is clearly resolved, the second expected peak appearing as a shoulder in the transmission curve (see Fig. 3). This suggests that finite-size effects may prevent the development of the narrowest peaks. Interestingly, the addition of linear chains also provokes the birth of a minimum in the transmittance spectrum, appearing at a wavelength slightly smaller than d. Eventually, in 2DHA, this minimum leads to a "Wood's anomaly," appearing just when a propagating Bragg diffracted wave becomes evanescent. In this case the reciprocal lattice vectors involved are $(\pm 1, 0)2\pi/d$, and Wood's anomaly appears due to a divergence in the EM density of states corresponding to those wave vectors. Notice that, in a linear chain, as the diffracted field contains a continuum of k_y components, the divergence in the density of states is smeared out; correspondingly, no Wood's anomalies appear in this case.

To summarize, we have found that EOT phenomena are already present in a single finite chain of subwavelength holes in a metallic film. For a chain, the transmittance per hole is comparable to that found in 2D arrays; therefore, the single chain can be considered as the basic entity of EOT and then 2D arrays can be seen as a collection of weakly EM coupled chains. As a by-product, we have developed a new theoretical framework that is able to treat the optical properties of even thousands of indentations (holes or dimples) placed arbitrarily in a metallic film.

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