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Transmission Resonances Through a Fibonacci Array of Subwavelength Slits

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Abstract *The transmission properties of quasiperiodic arrays of subwavelength slits arranged forming a Fibonacci sequence is analyzed. By developing a theoretical framework in reciprocal space, the close link between the formation of surface electromagnetic (EM) modes, responsible for the transmission peaks, with resonant features appearing in the structure factor, is shown. This finding demonstrates that long-range order is the key ingredient to observe enhanced transmission through both periodic and quasiperiodic arrays of subwavelength slits.*

Keywords extraordinary transmission, surface EM mode, quasiperiodicity

1. Introduction

Since the discovery of the phenomenon of extraordinary optical transmission (EOT) through periodic two-dimensional (2D) arrays of holes by Ebbesen and co-workers (1998), many theoretical and experimental works have been devoted to the study of transmission resonances in metallic films drilled with periodic arrangements of subwavelength apertures. Apart from 2D hole arrays (Popov et al., 2000; Martín-Moreno et al., 2001; Sarrazin et al., 2003; Barnes et al., 2004; Klein Koerkamp et al., 2004; Bravo-Abad et al., 2006), resonant transmission has been reported and analyzed in several other structures, such as single apertures (Takakura, 2001; Bravo-Abad et al., 2004; Schouten et al., 2003; Lalanne et al., 2005; Gordon, 2006, 2007; Du & Luo, 2006), single apertures surrounded by corrugations (Lezec et al., 2002; Hibbins et al., 2002; Martín-Moreno et al., 2003; García-Vidal et al., 2003; Lockyear et al., 2004), one-dimensional (1D) arrays of slits (Schröter & Heitmann, 1998; Porto et al., 1999; Treacy, 1999; Went et al., 2000; Yang & Sambles, 2002; García-Vidal & Martín-Moreno, 2002; Skigin & Depine, 2005), and 1D chains of holes (Bravo-Abad et al., 2004).

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Very recently, EOT in quasiperiodic hole arrays has been reported in several experimental (Sun et al., 2006; Przybilla et al., 2006; Papanikolaou et al., 2006; Matsui et al., 2007) and theoretical (Bravo-Abad et al., 2007) studies. These works have demonstrated that the presence of long-range order in 2D hole arrays is the key ingredient to observe EOT phenomenon. However, to our knowledge, the appearance of transmission resonances in the 1D analogous structure, a quasiperiodic array of subwavelength slits, has not yet been explored.

In this work, we study the transmission properties of 1D quasiperiodic arrays of subwavelength slits. A complete theoretical analysis of the appearance of the EOT phenomenon is done by comparing transmission resonances in Fibonacci arrays of slits with those present in periodic arrangements. The picture that emerges from our theoretical study is that the same physical origin of the EOT phenomenon is common to both structures. Apart from the Fabry-Perot resonances, transmittance can be enhanced by the excitation of surface EM modes decorating the metallic interfaces. In our study, we have focused on the analysis of this last transmission mechanism, whose efficiency depends strongly on the degree of order present in the structure.

The article is organized as follows: in Section 2, we briefly describe the theoretical real-space formalism. This approach is applied in Section 3 to the study of the transmission properties of Fibonacci arrays of slits, where comparison with periodic arrays is discussed. In Section 4, we introduce the reciprocal space formalism that is more suitable for analyzing the transmissivity of structures in the absence of a well-defined regularity. Finally, in Section 5, the general conclusions of the work are summarized.

2. Real-Space Formalism

Our theoretical formalism is based on the modal expansion of the EM fields (Martín-Moreno et al., 2003; García-Vidal et al., 2003) in the different regions of the structure schematically depicted in Figure 1, a perfect conducting freestanding film of thickness h perforated by N slits. We will consider identical slits of width a which, in principle, can be disposed in any arrangement with x_α being the position of slit α (with $\alpha = 1, 2, \dots, N$). For light impinging at normal incidence on the film, this system presents translational symmetry along the direction parallel to the slits, and both light polarizations

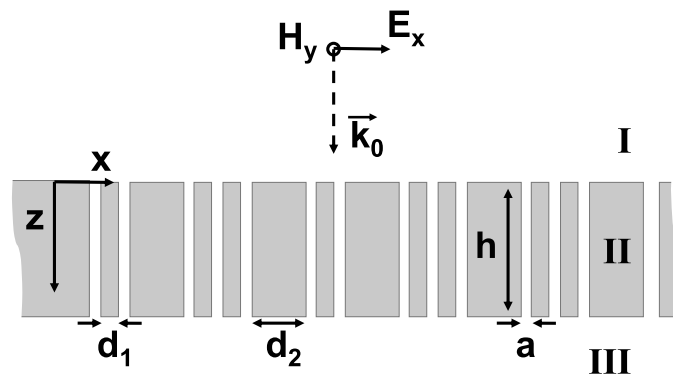


Figure 1. Schematic cross section of the structure analyzed, a perfect conducting film of thickness h drilled with an array of slits of width a . Slits are disposed following a Fibonacci sequence generated from two basic distances d_1 and d_2 (see text for definition).

(s and p) are decoupled. Taking advantage of this fact, we can restrict our analysis to the case of p -polarized illumination (magnetic field parallel to symmetry axis), which gives rise to transmission resonances in periodically perforated, freestanding films.

If the structure is illuminated by a plane wave of wavenumber k_0 ($k_0 = \omega/c = 2\pi/\lambda$) from the top ($z = 0$), the relevant EM fields in region I (see Figure 1) can be expanded as a continuum of refraction modes of the form:

$$\begin{aligned} E_x^I(x, z) &= e^{ik_0z} + \int_{-\infty}^{\infty} dk r(k) e^{ikx} e^{-ik_zz}, \\ H_y^I(x, z) &= e^{ik_0z} + \int_{-\infty}^{\infty} dk Y(k) r(k) e^{ikx} e^{-ik_zz}, \end{aligned} \quad (1)$$

where $k_z = \sqrt{k_0^2 - k^2}$ gives the normal component of the wavevector, $Y(k) = k_0/k_z$, the mode admittance, and $r(k)$ is the reflection coefficient for the mode with parallel wave vector k .

The use of perfect conducting boundary conditions yields nonzero EM fields in region II ($0 \leq z < h$) only inside the slits. Since we are considering the case in which the apertures' size is much smaller than the wavelength of the incoming wave ($\lambda \gg a$), EM fields inside each slit can be approximated by only considering its fundamental propagating waveguide mode:

$$\begin{aligned} E_x^{II}(x, z) &= \sum_{\alpha} (A_{\alpha} e^{ik_0z} + B_{\alpha} e^{-ik_0z}) \phi_{\alpha}(x), \\ H_y^{II}(x, z) &= \sum_{\alpha} (A_{\alpha} e^{ik_0z} - B_{\alpha} e^{-ik_0z}) \phi_{\alpha}(x), \end{aligned} \quad (2)$$

where $\phi_{\alpha}(x) = \theta(a/2 - |x - x_{\alpha}|) / \sqrt{a}$ is the normalized wave function of the fundamental waveguide mode at slit α , and $\theta(x)$ is the Heaviside function.

In region III ($z \geq h$), the transmitted EM fields can be also written in terms of diffraction modes:

$$\begin{aligned} E_x^{III}(x, z) &= \int_{-\infty}^{\infty} dk t(k) e^{ikx} e^{ik_zz}, \\ H_y^{III}(x, z) &= \int_{-\infty}^{\infty} dk Y(k) t(k) e^{ikx} e^{ik_zz}, \end{aligned} \quad (3)$$

where $t(k)$ is the transmission coefficient associated with parallel wavevector component k .

The modal expansion amplitudes in each region $\{r(k), A_{\alpha}, B_{\alpha}, t(k)\}$ are calculated by imposing continuity conditions for the parallel components of the EM fields at the two interfaces ($z = 0$ and $z = h$). The x -component of the electric field must be continuous everywhere at both interfaces, whereas the y -component of the magnetic field must be continuous only at the slits' openings. We project the two resulting x -dependent continuity equations over vacuum plane waves (for E_x) and slits waveguide modes (for H_y). Defining a new set of variables, $E_{\alpha} = A_{\alpha} + B_{\alpha}$, $E'_{\alpha} = -(A_{\alpha} e^{ik_0h} + B_{\alpha} e^{-ik_0h})$, which correspond to the modal amplitudes of E_x field at the entrance and exit of slit α ,

respectively, we end up with a system of $2N$ linear equations for the set of unknowns $[E_\alpha, E'_\alpha]$:

$$\begin{aligned} (G_{\alpha\alpha} - \varepsilon)E_\alpha + \sum_{\beta \neq \alpha} G_{\alpha\beta}E_\beta - G^V E'_\alpha &= I, \\ (G_{\alpha\alpha} - \varepsilon)E'_\alpha + \sum_{\beta \neq \alpha} G_{\alpha\beta}E'_\beta - G^V E_\alpha &= 0. \end{aligned} \quad (4)$$

All terms in Eqs. (4) have a simple physical interpretation. The inhomogeneous term $I = 2i\sqrt{a}$ gives the slits direct illumination, $\varepsilon = \cot(k_0h)$ describes the bouncing of light inside the slits, and $G^V = \sec(k_0h)$ describes the coupling of EM fields through them. Finally, the term $G_{\alpha\beta}$ carries light from slit β to slit α through the continuum of diffraction modes. It reads:

$$G_{\alpha\beta} = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dx' \phi_\alpha(x) G(x, x') \phi_\beta(x'), \quad (5)$$

where $\phi_\alpha(x)$ has been defined previously, and $G(x, x') = (i\pi/\lambda)H_0^{(1)}(k_0|x - x'|)$ is the Green's function associated with Helmholtz's equation in two dimensions (Arfken & Weber, 2001), with $H_0^{(1)}(x)$ being the zero-order Hankel function of the first kind.

By solving the set of equations in Eqs. (4), the modal amplitudes E_α and E'_α are obtained; hence, the EM fields in all space can be constructed. Within our formalism, the total transmittance of the structure can be written as a sum over slits contributions as

$$T = \sum_{\alpha} \text{Im}((G^V E'_\alpha)^* E_\alpha). \quad (6)$$

3. Transmission Properties of a Fibonacci Array of Slits

We apply the formalism introduced in the previous section to the analysis of the transmission properties of slits disposed following a Fibonacci sequence. This sequence is probably the earliest and best known deterministic aperiodic system. In its simplest version, it can be generated from two basic elements, $\{a, b\}$, by iteratively applying the substitution rules,

$$a \rightarrow ab, \quad b \rightarrow a. \quad (7)$$

Thus, we can construct Fibonacci chains of increasing numbers of elements, having

$$a \Rightarrow ab \Rightarrow aba \Rightarrow abaab \Rightarrow abaababa \Rightarrow abaababaabaab \Rightarrow \dots \quad (8)$$

Each sequence S_j in Eq. (8) can be obtained from the two preceding ones by applying the recursion relation $S_j = S_{j-1} \cup S_{j-2}$, where \cup means composition. Sequences of a and b elements obtained through this iterating process do not have a well-defined regularity, but they present several interesting properties due to their quasiperiodic character.

In a Fibonacci array of slits, the distance between consecutive slits follows a Fibonacci sequence of two basic lengths, d_1 and d_2 . The slits' positions in such an array are given by (Galdi et al., 2005)

$$x_\alpha = d_1 \text{int}\left(\frac{\alpha}{\tau}\right) + d_2 \left[\alpha - 1 - \text{int}\left(\frac{\alpha}{\tau}\right)\right], \quad (9)$$

where $\tau = (1 + \sqrt{5})/2$ is the golden ratio, and $\text{int}(\cdot)$ denotes the integer part function.

The structure factor of an arrangement of slits is defined as the discrete Fourier transform of the slits' positions

$$S(k) = \sum_{\alpha} e^{ikx_{\alpha}}. \quad (10)$$

In Figure 2(a), the modulus of $S(k)$ for a Fibonacci array of 200 slits is depicted. The two basic lengths defining the array are $d_1 = 0.68\Lambda$ and $d_2 = 1.55\Lambda$, where Λ is the mean distance between slits. Taking advantage of the scalable character that perfect conducting boundary conditions give to the system, in what follows, we will take Λ as unit length. In Figure 2(b), the structure factor for a periodic array of 200 slits of period Λ is plotted. It vanishes for all k s that are far from the multiples of $2\pi/\Lambda$ and is narrowly peaked at each multiple of $2\pi/\Lambda$. At these k s, all summands in Eq. (10) are in phase, and $|S|_{\max} = N = 200$. The structure factor for the Fibonacci array also presents several peaked features. The formation of these maxima relies on the appearance of a certain limited degree of coherence among summands in Eq. (10) that makes $|S|_{\max} < 200$ (for $k \neq 0$). These peaks are fingerprints of the presence of long-range order in the structure.

In Figure 3, transmission spectra of a perfect conducting film of thickness $h = 0.68\Lambda$ perforated with the Fibonacci (solid line) and periodic (dashed line) arrays of 200 slits of width $a = 0.17\Lambda$ are shown. Transmittances are normalized to the transmissivity of 200 independent slits. Single slit transmittance (T_0), taken as reference for normalization, is plotted in the inset of the figure. Within the wavelength range considered in our analysis, T_0 shows two maxima corresponding to the two first-slit waveguide resonances ($\lambda = 0.89\Lambda$ and $\lambda = 1.87\Lambda$) (Porto *et al.*, 1999; Takakura, 2001; Bravo-Abad *et al.*, 2004). The normalized Fibonacci and periodic array spectra display these maxima too,

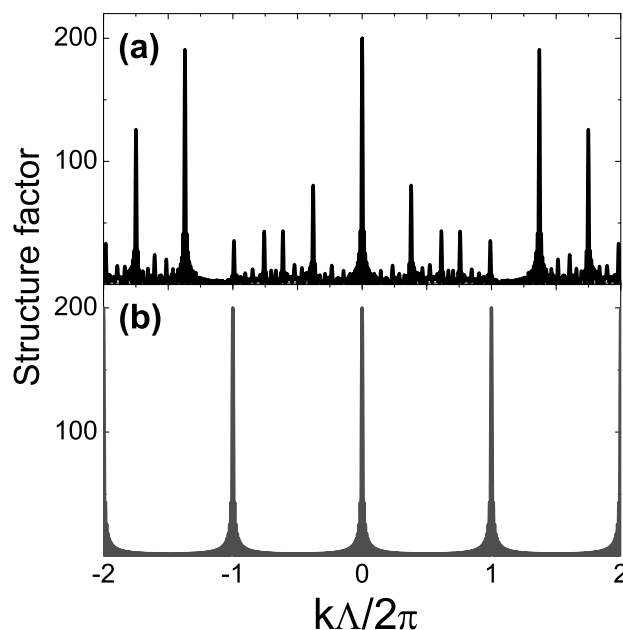


Figure 2. Absolute value of the structure factor corresponding to: (a) a Fibonacci array of 200 slits with $d_1 = 0.68\Lambda$ and $d_2 = 1.55\Lambda$ (where Λ is the mean distance between slits) and (b) a periodic array of 200 slits of period Λ .

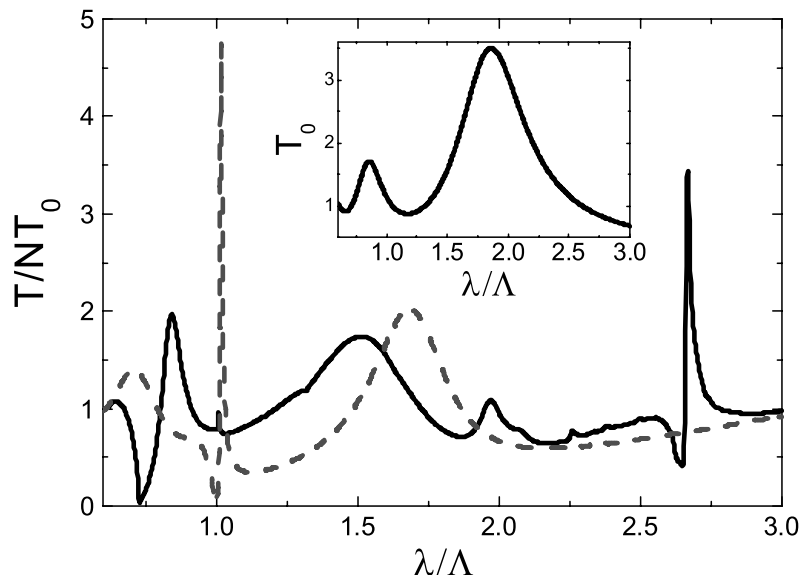


Figure 3. Transmission spectra of the Fibonacci (solid line) and periodic (dashed line) arrays of 200 slits considered in the text. The slit width is $a = 0.17\Lambda$ and the film thickness is $h = 0.68\Lambda$. Transmittances are normalized to the transmissivity of 200 independent slits. Inset: Transmission spectrum of a single slit of the same dimensions.

but shifted to shorter wavelengths. Although the localized character of slit waveguide resonances makes the associated transmission maxima almost independent of the number of slits, the shift in resonant wavelength leads to low maxima $T_{\max} \leq 2$ in both normalized spectra (see Figure 3).

Periodic and Fibonacci array transmittances display higher and narrower peaks that are not present in the single-slit spectrum. In the last few years, transmission properties of periodic arrays of slits have been thoroughly studied (Schröter & Heitmann, 1998; Porto et al., 1999; Treacy, 1999; Went et al., 2000; Yang & Sambles, 2002; García-Vidal & Martín-Moreno, 2002; Skigin & Depine, 2005). It is well known that the physical origin of the sharp dip in transmission—the Wood's anomaly (Wood, 1902)—and the associated narrow peak that periodic structures show at wavelengths close to the array period (see Figure 3) rely on the excitation of surface EM modes at the system interfaces. The aim of our work is to elucidate whether the origin of the very similar features found in the Fibonacci spectrum is the same.

In the coupling process between incident light and surface EM modes supported by an infinite periodic array of slits, Bragg momentum matching conditions must be satisfied. This fact relates the position of the resonant transmission peak with the corresponding structure factor for the infinite array, which has the form

$$S_{\infty}(k) = \sum_{\alpha=1}^{\infty} e^{ik\alpha\Lambda}. \quad (11)$$

This function defines a set of resonant parallel wavevectors that turn to be equal to the reciprocal lattice vectors ($b_j = 2\pi j/\Lambda$). Wavevectors $b_{\pm 1} = \pm 2\pi/\Lambda$ give the lowest incident energy and momentum needed for the coupling, opening a very efficient trans-

mission channel and leading to a peak in the transmission spectrum close to periodicity. For a finite periodic array, discrete resonant components of k parallel to surface are not well defined, but the close correspondence between locations of the first structure factor peak at $\lambda = \Lambda$ (see Figure 2(b)) and the surface EM resonance transmission peak at $\lambda = 1.02\Lambda$ (see Figure 3) remains.

The connection between the spectral location of transmission peaks with resonant features of the structure factor seems to hold also for the Fibonacci array. Its structure factor (see Figure 2(a)) has a first maximum at $k\Lambda/2\pi = 0.375$, which corresponds to $\lambda = 2.66\Lambda$. The highest peak in its transmission spectrum occurs at $\lambda = 2.68\Lambda$ (see Figure 3), whereas Wood's anomaly also has its own quasiperiodic counterpart in a narrow dip in transmittance just at structure factor maximum location ($\lambda = 2.66\Lambda$). All these similarities seem to point out that surface EM modes also play a crucial role in the transmission properties of the Fibonacci structure. The fact that both transmission features (dip and peak) are less pronounced for the Fibonacci structure can be interpreted as a consequence of the less efficient coupling process between the incident light and the surface EM mode due to the absence of a well-defined regularity in the system.

It is worth studying the evolution of the transmission peaks with the number of slits present in the quasiperiodic arrangement; this is done in Figure 4. The gradual increase of the transmission peak height is compatible with our hypothesis that the origin of the transmission resonance stems from the excitation of a surface EM mode. As the number of slits is increased, the structure factor becomes more and more peaked around the resonant wavevectors, resulting in a more efficient excitation of surface EM modes and, consequently, in a higher transmittance. A similar behavior is found for periodic arrays. In the inset of Figure 4, peak heights for Fibonacci (squares) and periodic (dots) arrangements with the same mean distances between slits are plotted. We can observe

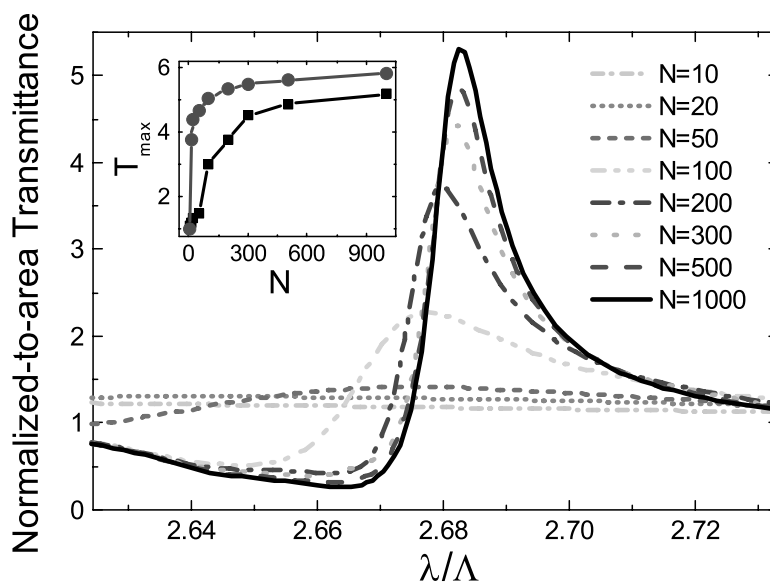


Figure 4. Evolution of the Fibonacci array transmission peak as the number of slits is increased. The transmittance is normalized to the EM flux impinging on the array area. The geometrical parameters considered are the same as in Figure 3. Inset: Evolution of the transmission peak height as a function of the number of slits in the Fibonacci (squares) and periodic (dots) arrays.

that transmittance increase is very similar in both cases, but with transmittance always lower for the Fibonacci structure than for the periodic array.

4. Reciprocal Space Formalism

From the analysis performed in the previous section, it seems clear that the structure factor plays a fundamental role in the transmission properties of slit arrangements that present long-range order (periodic and quasiperiodic arrays). In order to include this magnitude explicitly in our theoretical framework, we transform our real-space formalism into a reciprocal space one. As a result, we obtain a set of k -space, integral equations equivalent to Eqs. (4), governing the EM field's behavior. The unknowns of the new system of equations are the Fourier amplitudes of the electric field at film surfaces with parallel wavevector component k ($E(k) = \sum_{\alpha} E_{\alpha} e^{ikx_{\alpha}}$ and $E'(k) = \sum_{\alpha} E'_{\alpha} e^{ikx_{\alpha}}$). The set of linear equations in reciprocal space can be written as

$$\begin{aligned} -\varepsilon E(q) + \int_{-\infty}^{\infty} dk G(k) S(q-k) E(k) - G^V E'(q) &= IS(q), \\ -\varepsilon E'(q) + \int_{-\infty}^{\infty} dk G(k) S(q-k) E'(k) - G^V E(q) &= 0. \end{aligned} \quad (12)$$

It is important to notice that, like their real-space counterparts, these equations only hold in the subwavelength regime ($\lambda \gg a$) and for normal illumination. All terms in Eqs. (12) can be understood in a very similar way to those of Eqs. (4). As ε and G^V describe EM fields traveling inside the slits, they remain the same as in the real-space equations. The illumination term now contains the direct coupling I , but weighted by the structure factor $S(q)$. Finally, the integral term $\int_{-\infty}^{\infty} dk G(k) S(q-k) E(k)$ describes all the scattering processes which couple $E(q)$ to the continuum $E(k)$, being the crystal momentum needed for the coupling provided by the structure through $S(q-k)$. The amplitude of each scattering process is measured by

$$G(k) = \frac{i}{2\pi} Y(k) |\Omega(k)|^2, \quad (13)$$

where $Y(k) = k_0 / \sqrt{k_0^2 - k^2}$ is the admittance of a plane wave with parallel wavevector k , and $\Omega(k) = (2/k\sqrt{a}) \sin(ka/2)$ is the overlap between this plane wave and the waveguide mode $\phi(x) = \theta(a/2 - x) / \sqrt{a}$.

In order to illustrate the strength of this reciprocal space approach, we first apply it to the case of an infinite periodic array of slits. In this case, $S_{\infty}(k)$, given by Eq. (11), selects parallel wavevectors equal to the reciprocal lattice vectors b_j in the integral term of Eqs. (12), removing the rest. Moreover, Bloch's theorem implies $E(k) = E(k + b_j)$ for all j , and Eqs. (12) transform into two simple equations for each k . For $k = 0$ these equations have the form

$$\begin{aligned} (\Sigma_0 - \varepsilon) E(0) - G^V E'(0) &= IS(0), \\ (\Sigma_0 - \varepsilon) E'(0) - G^V E(0) &= 0, \end{aligned} \quad (14)$$

where $\Sigma_0 = (2\pi/\Lambda) \sum_j G(b_j)$, and $G(b_j)$ is evaluated from Eq. (13).

As zero-order diffracted beam ($k = 0$) governs infinite array transmittance for $\lambda \geq \Lambda$, Eqs. (14) describe the mathematical foundation of the transmission resonances shown by the structure. In Figure 5, $|\Sigma_0 - \varepsilon|$ for an infinite periodic array with the same geometrical parameters as considered before is plotted (dotted line). As $G(b_{\pm 1})$ has a singularity when $k_0 = b_{\pm 1}$, Σ_0 diverges when $\lambda = \Lambda$. Therefore, $E(0)$ and $E'(0)$ vanish at this wavelength, leading to a zero in transmittance—the so-called Wood's anomaly in infinite periodic slit arrays. However, due to the rapid oscillations of Σ_0 close to this divergence, at a wavelength slightly longer than the period $|\Sigma_0 - \varepsilon| = G^V$. It can be easily demonstrated that this condition leads to an enhancement of $E(0)$ and $E'(0)$ obtained from Eqs. (14), which finally produces a narrow peak in the transmission spectrum. This resonance can be interpreted as the excitation of surface EM modes at the two film interfaces, as it is associated with an enhancement of the E -field amplitudes. As we are considering perfect conducting films, which do not support surface plasmon polaritons, these modes correspond to the so-called *spoof* surface plasmons supported by periodically corrugated perfect conductors (Pendry et al., 2004; García-Vidal et al., 2005).

The arguments introduced for the infinite structure can be extended to the finite periodic and Fibonacci arrays. Now, Bloch's theorem cannot be applied to electric-field amplitudes, and Eqs. (12) must be solved for a continuum of parallel wavevectors q . However, considering only the $k = 0$ Fourier amplitude results in a very good approximation for large enough arrays. Under this approximation, equations governing the electric field at film interfaces can be written in the form of Eqs. (14), where Σ_0 is now numerically evaluated with the expression

$$\Sigma_0 = \frac{1}{E^{(l)}(0)} \int_{-\infty}^{\infty} dk G(k) S(-k) E^{(l)}(0), \quad (15)$$

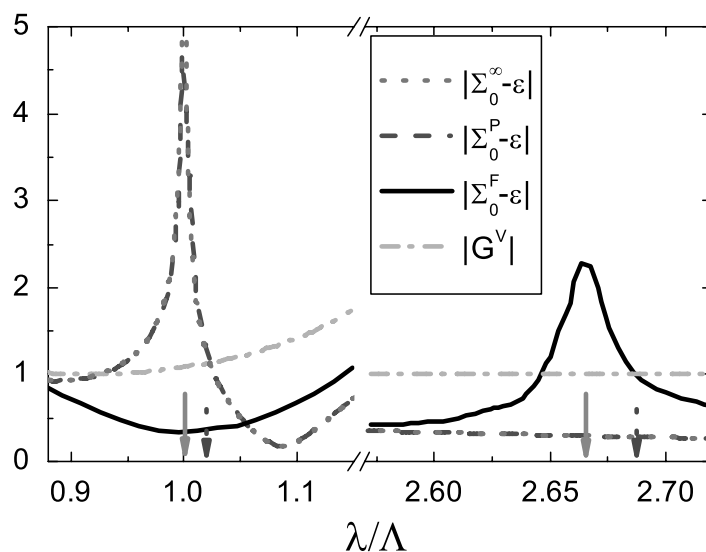


Figure 5. Relevant terms in Eqs. (14) for the infinitely periodic (dotted line), finitely periodic (dashed line), and Fibonacci (solid line) structures considered in the text. G^V (dashed dotted line) is the same for the three slit arrangements. Dotted arrows indicate the peak and solid arrows indicate the dip position in the periodic and Fibonacci transmission spectra (see Figure 3).

where $E(k)$ and $E'(k)$ are numerically obtained after solving the system of linear equations in real space. It is important to notice that we have checked that this function acquires the same values calculated from the modal amplitudes at input and output film surfaces.

In Figure 5, $|\Sigma_0 - \varepsilon|$ associated with the periodic (dashed line) and Fibonacci (solid line) arrays considered previously is represented. We have focused on the wavelength ranges in which the structures show surface EM resonance peaks. Although Σ_0 does not diverge for any of them, both arrangements develop a Wood's anomaly (solid arrows in Figure 5) at wavelengths for which $|\Sigma_0 - \varepsilon|$ has a maximum. Moreover, for both structures, the resonant condition $|\Sigma_0 - \varepsilon| = G^V$ still exactly gives the locations of the transmission peaks (dotted arrows in Figure 5), which now coincide with an electric-field enhancement close to the system interfaces. As for the infinite array, this field enhancement can be assigned to the excitation of *spoof* plasmon surface modes at the perfect conducting film sides.

5. Conclusions

In this article, we have presented a complete theoretical analysis of the transmission resonances appearing in quasiperiodic 1D arrays of subwavelength slits. By applying a formalism able to deal with structures that do not have a well-defined periodicity, we have identified the two mechanisms that yield enhanced transmission in the system: slit waveguide and surface EM resonances. By comparing the transmission properties of Fibonacci and periodic arrays, we have linked the formation of surface EM modes with the structure factor of the slit arrangement. The location of the resonant features in the structure factor marks the spectral locations of the transmission peaks, whereas the efficiency of the transmission resonance is controlled basically by the height of the peaks in the structure factor.

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