Modulated phases in electron-hole bilayers & 2D dipolar Fermi gases

Francesca Maria Marchetti

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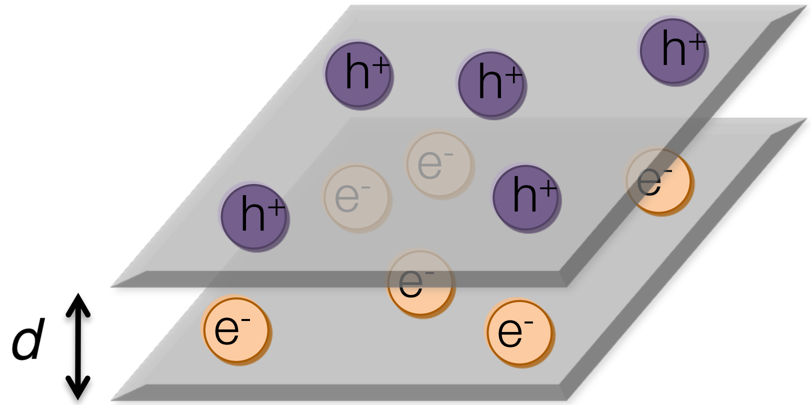
Outline

1. Imbalanced electron-hole bilayers

   ✧ spatially modulated pairing (FFLO)



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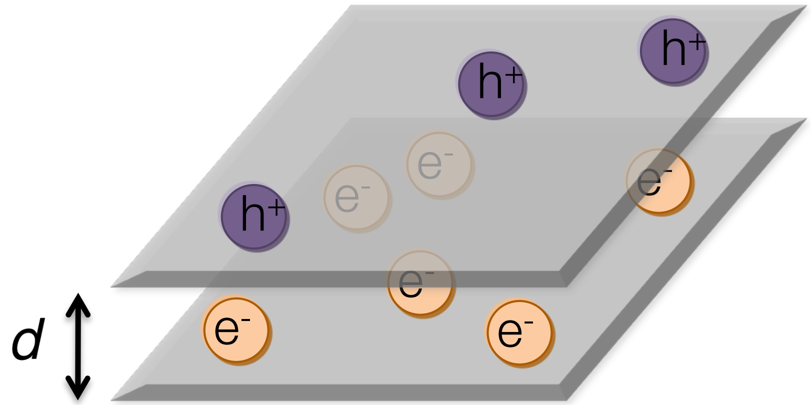
1. Imbalanced electron-hole bilayers

    ✧ spatially modulated pairing (FFLO)

    ✧ limit of extreme imbalance

    ✧ unusual bosonic limit of FFLO: supersolid



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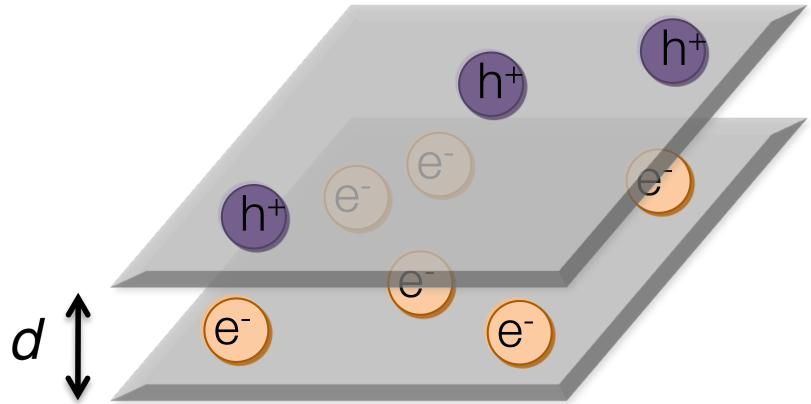
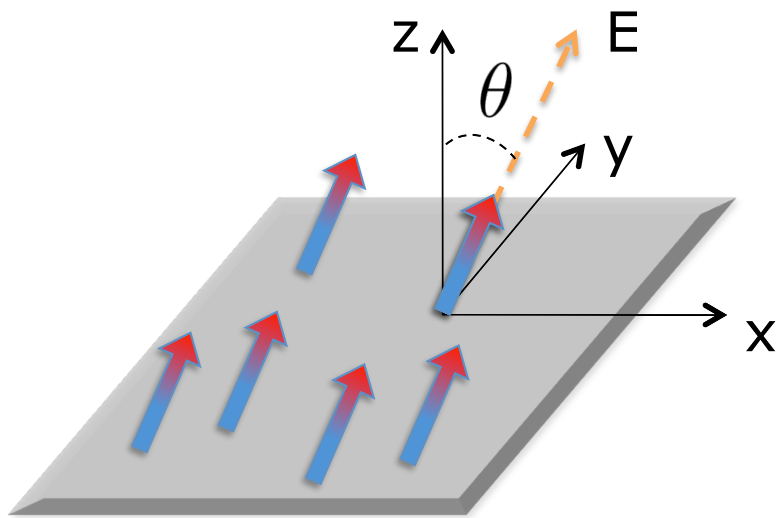
   - ✧ spatially modulated pairing (FFLO)
   - ✧ limit of extreme imbalance
   - ✧ unusual bosonic limit of FFLO: supersolid

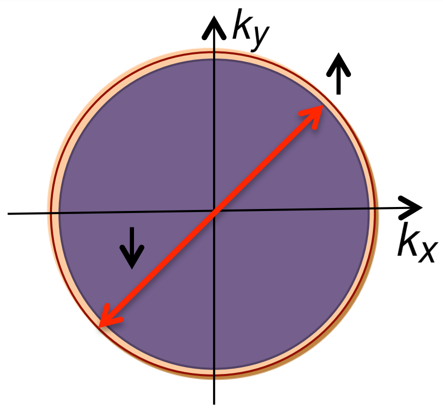


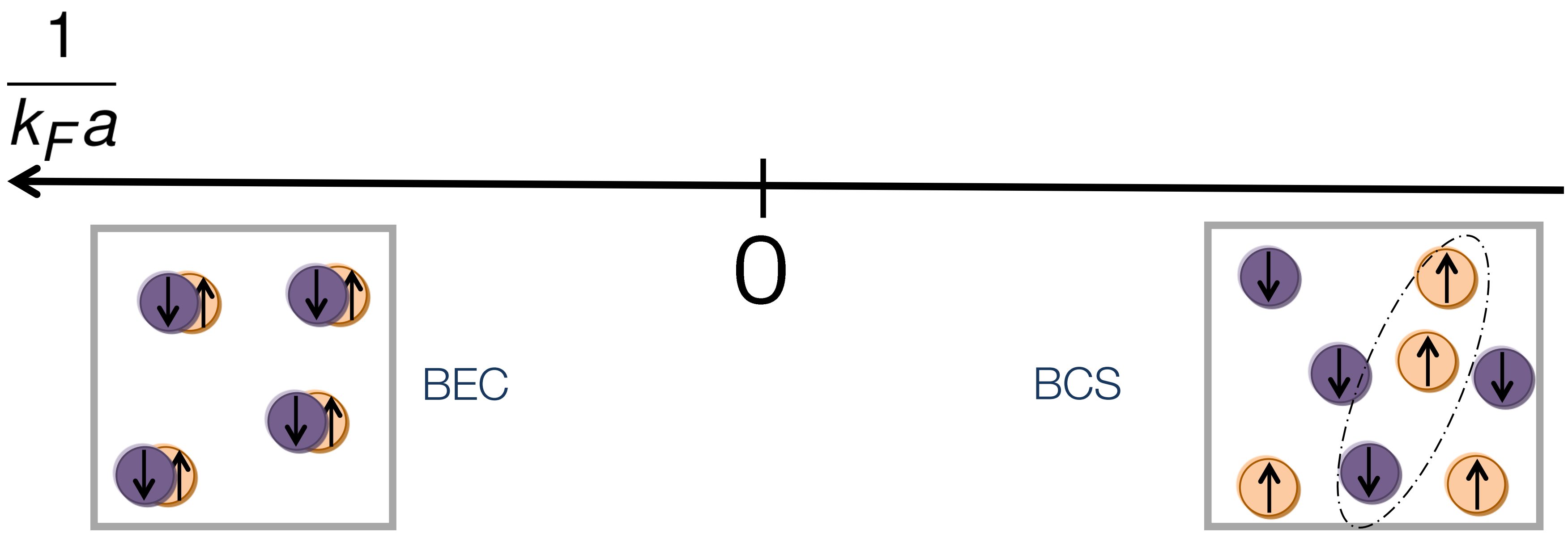
2. 2D dipolar Fermi gases
   - ✧ rich many-body physics (even 1 layer)
   - ✧ phase diagram (density modulation, superfluidity, collapse,…)
   - ✧ why beyond RPA



Background: Two component Fermi gases

- ✧ Tunable interactions
- ✧ BEC-BCS crossover





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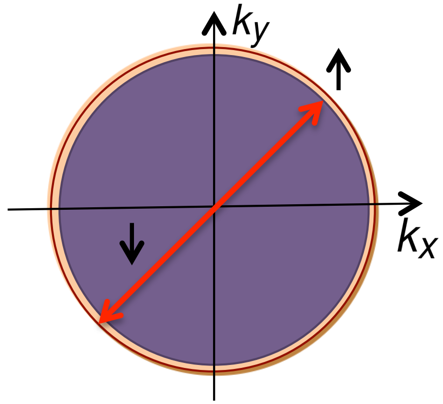
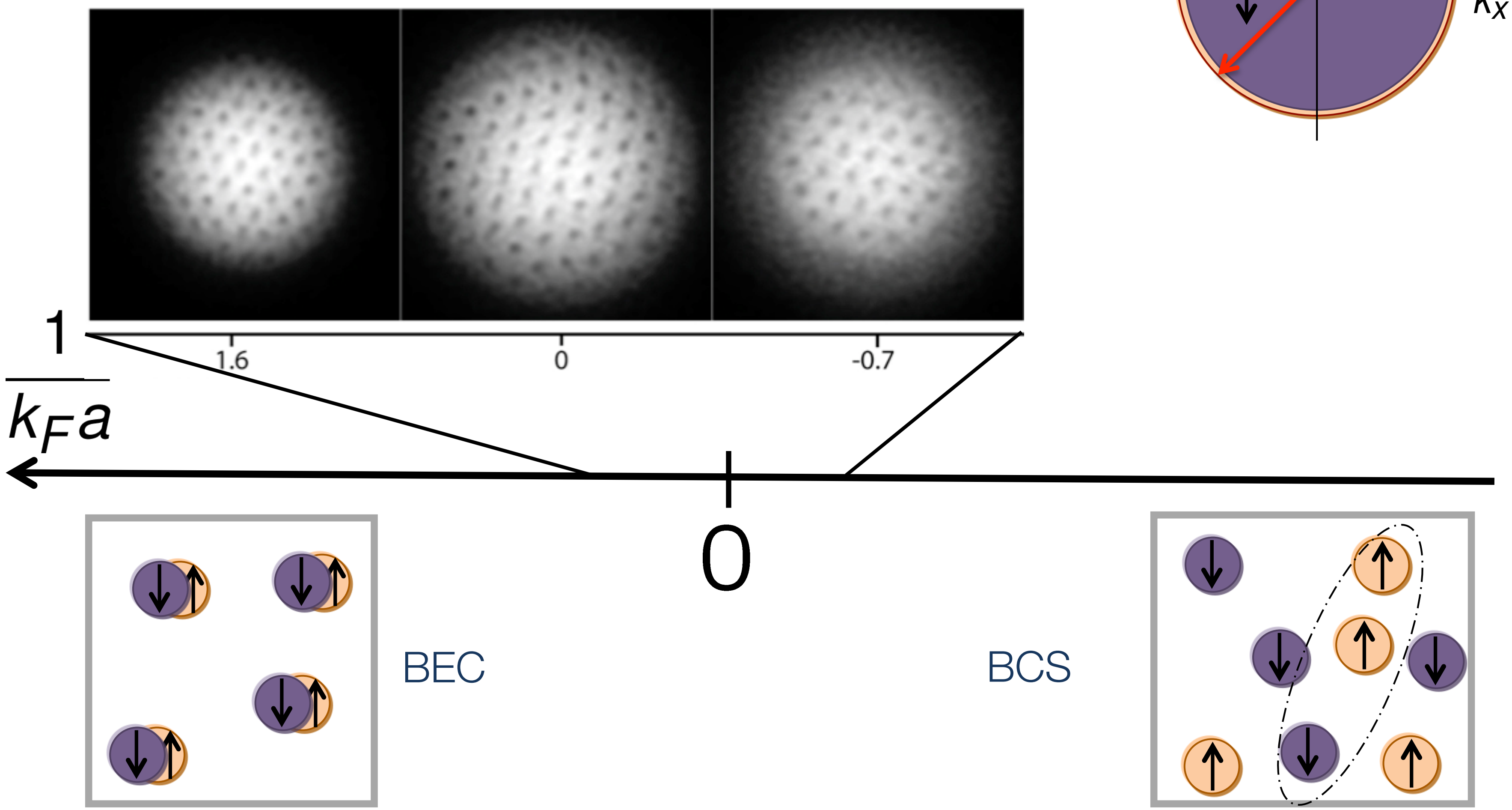
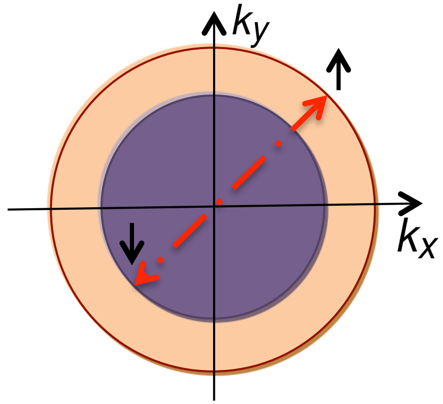


[Zwierlein et al. Nature (2005)]

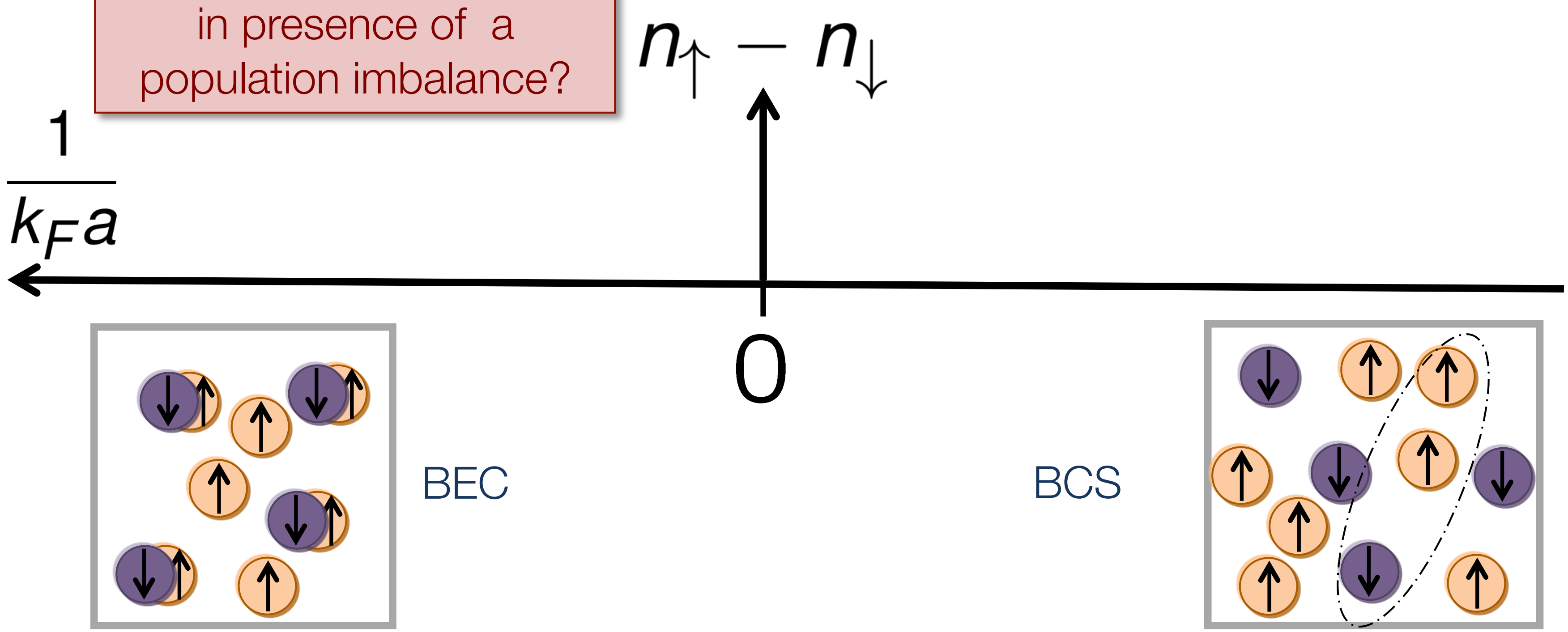


Background: Imbalanced Fermi gases

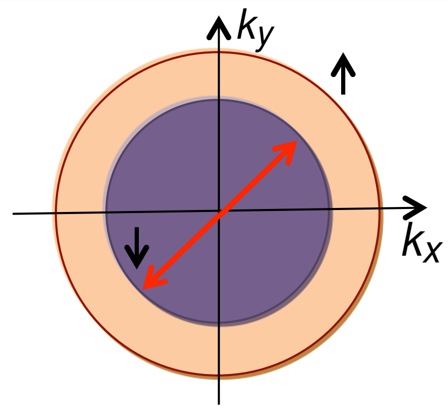
- Tunable interactions
- BEC-BCS crossover
- Density imbalance frustrates pairing

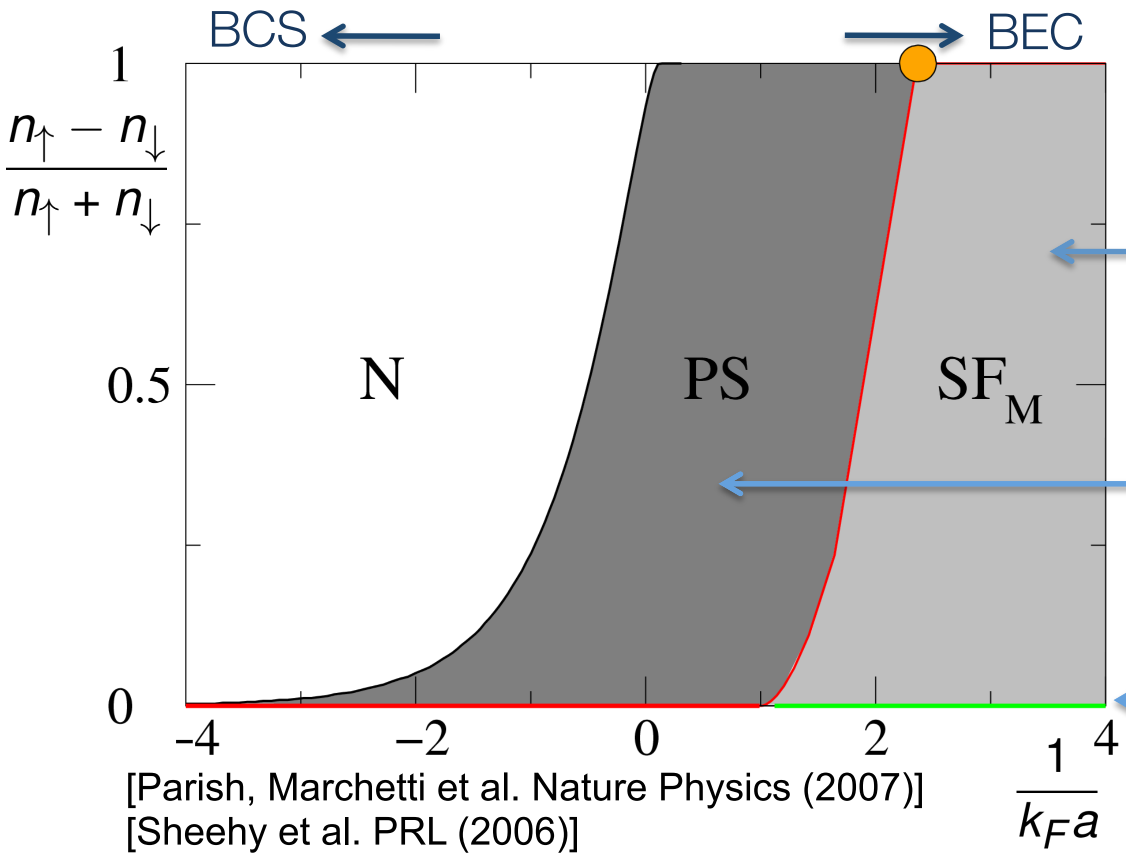


Can superfluidity persist in presence of a population imbalance?



Background: T=0 phase diagram

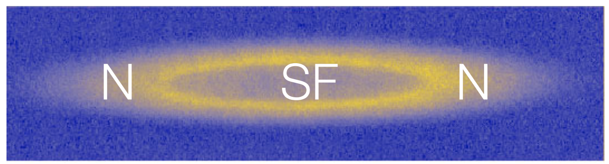




[Parish, Marchetti et al. Nature Physics (2007)]
[Sheehy et al. PRL (2006)]

Polarised SF or breached pairing (space homogeneous)

Macrooscpic phase separation between SF and N phases



[Shin *et al*. PRL (2006)]

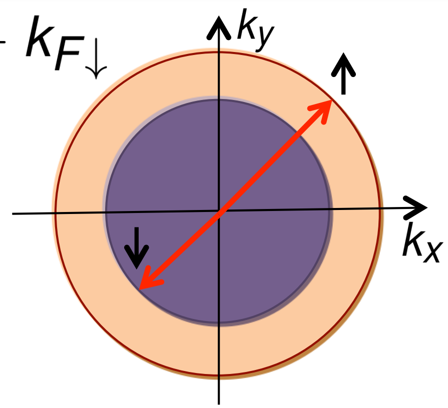
Fully paired SF phase

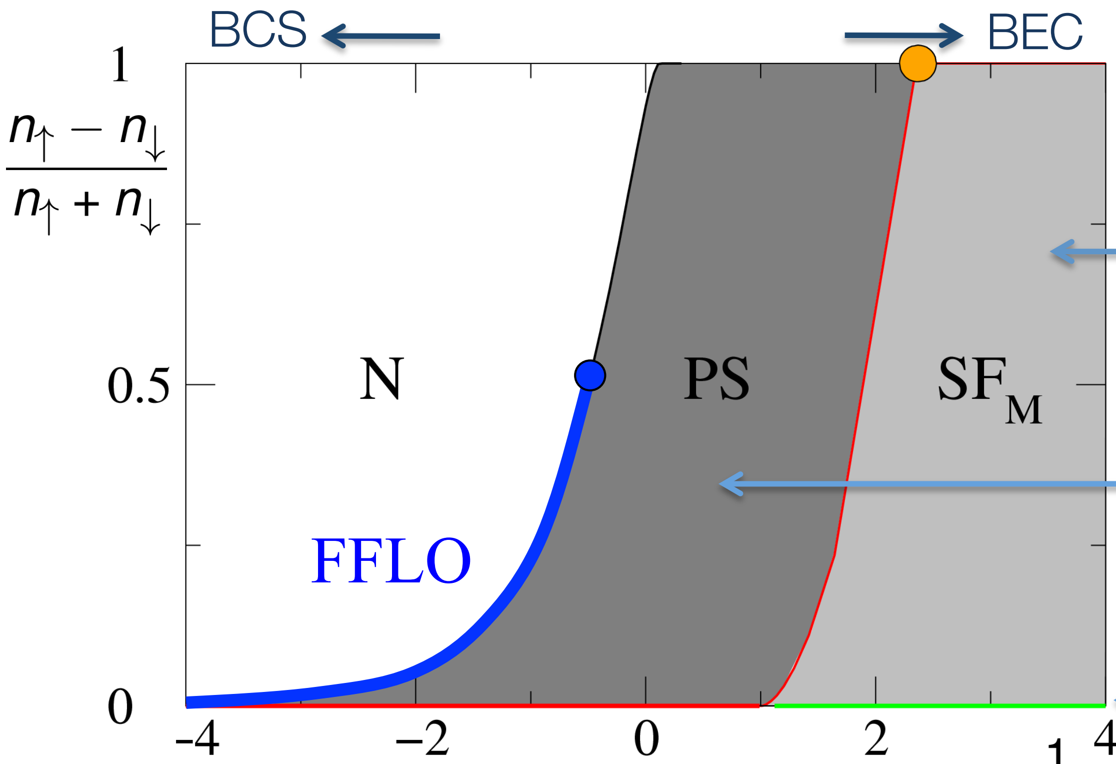
Background: T=0 phase diagram

FFLO phase $\Delta(\mathbf{r}) = \sum_{\mathbf{Q}} \Delta_{\mathbf{Q}} e^{i\mathbf{r}\cdot\mathbf{Q}}$

SUPERSOLID PHASE

$\mathbf{Q} \sim k_{F\uparrow} - k_{F\downarrow}$





$\frac{1}{k_F a}$

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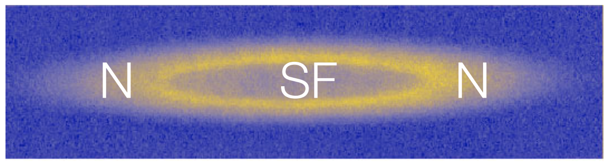


[Shin *et al*. PRL (2006)]

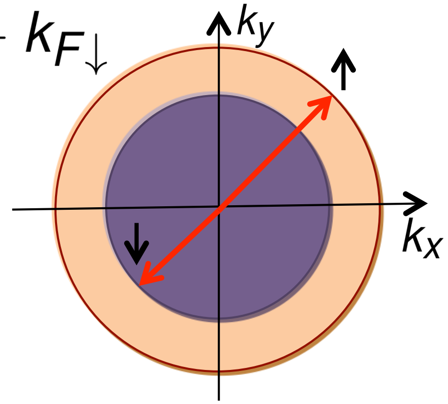
Fully paired SF phase

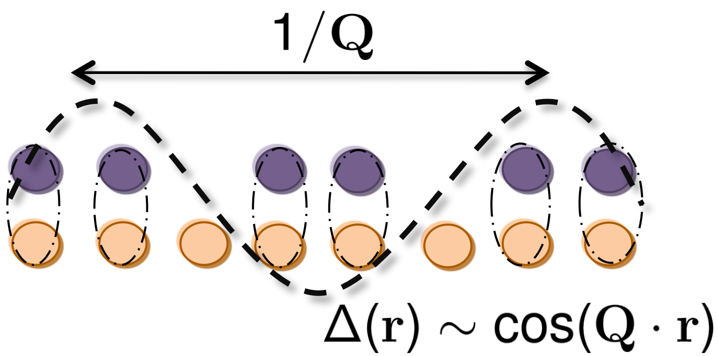
FFLO unlikely in 3D Fermi gases

FFLO phase $\Delta(\mathbf{r}) = \sum_{\mathbf{Q}} \Delta_{\mathbf{Q}} e^{i\mathbf{r}\cdot\mathbf{Q}}$

SUPERSOLID PHASE

$\mathbf{Q} \sim k_{F\uparrow} - k_{F\downarrow}$



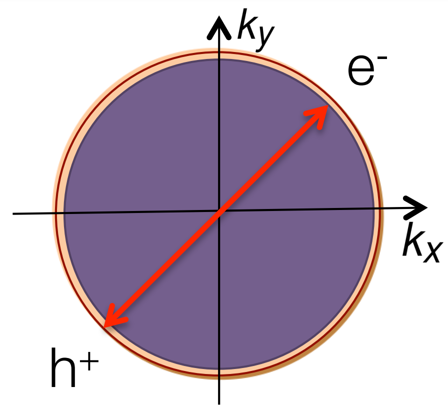


- ✧ Displacement of the Fermi surface:
  - …allows Fermi surface nesting
  - …allows the system to polarise
- ✧ However
  - …nesting is partial
  - …it costs kinetic energy

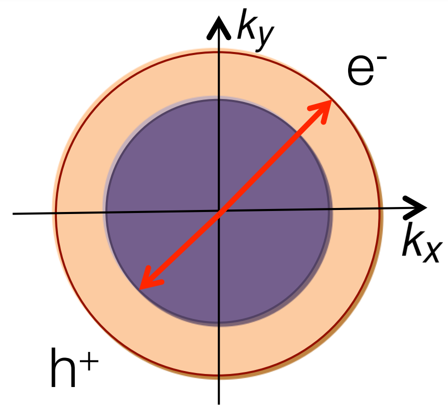
✧ Phase separation dominates over FFLO

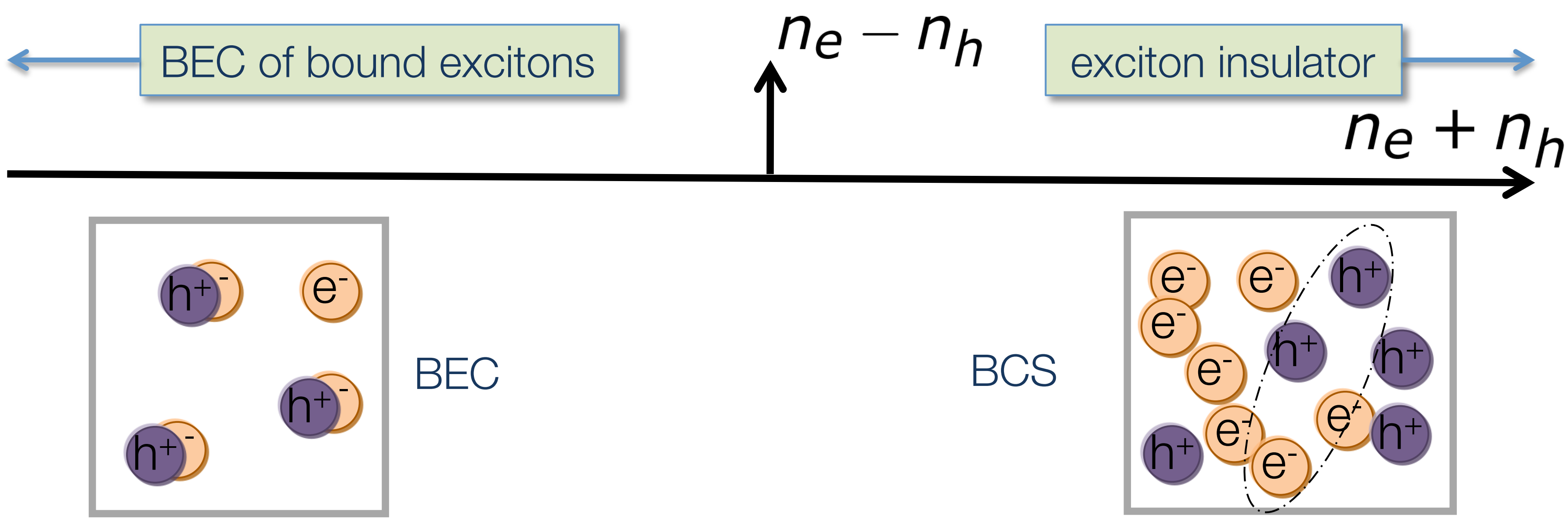
✧ Experiments (Rice & MIT): FFLO elusive

Electron-hole systems



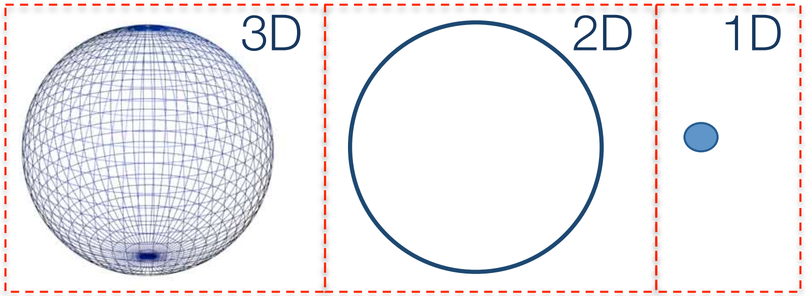
Electron-hole systems

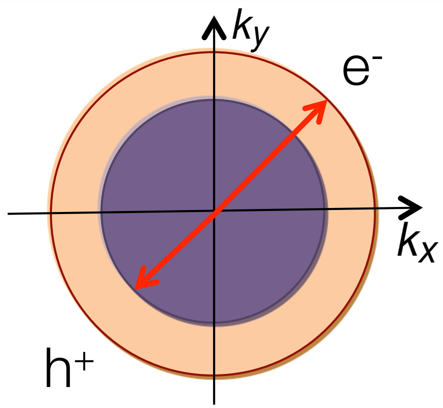




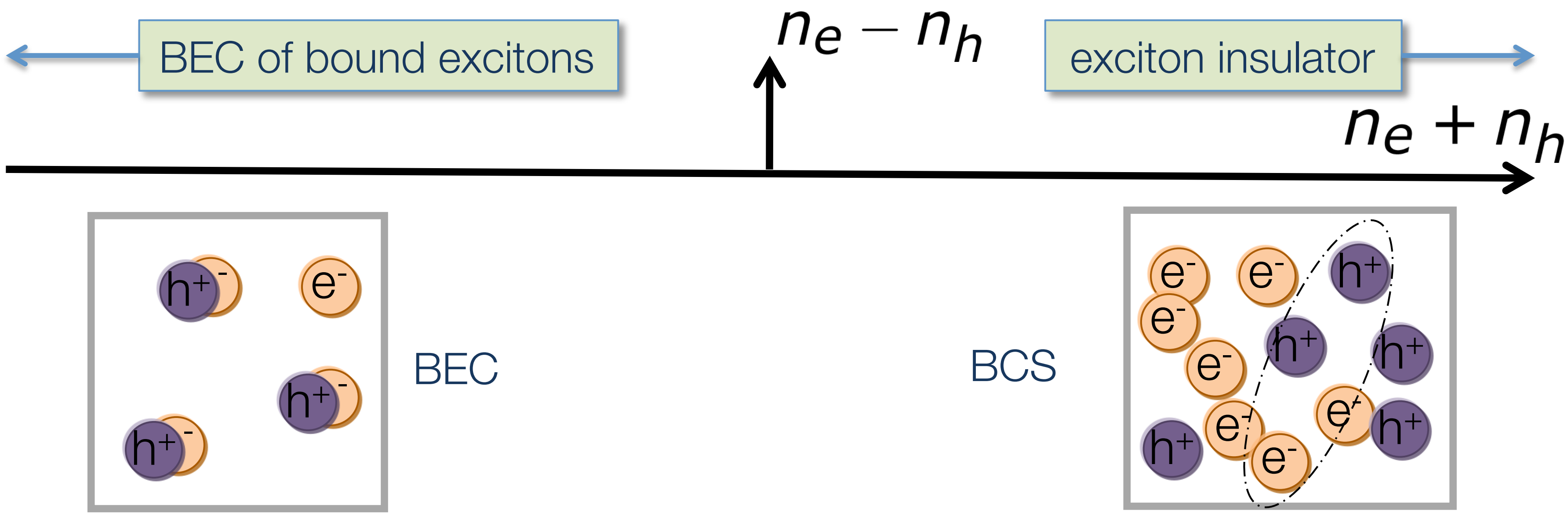
Electron-hole bilayers: FFLO more ‘likely’

1. Enhanced Fermi surface nesting



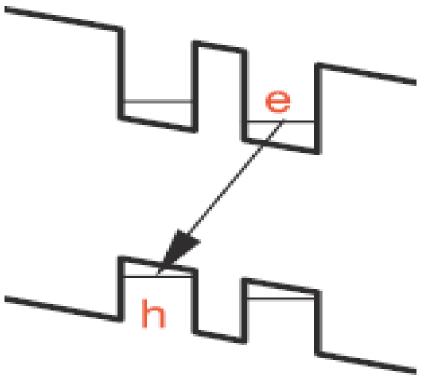


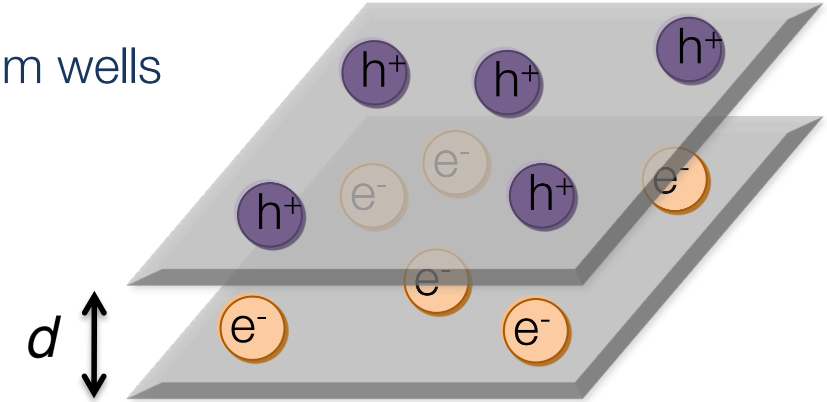
2. No phase separation on macro-scales



Electron-hole bilayers: Experimental realisations

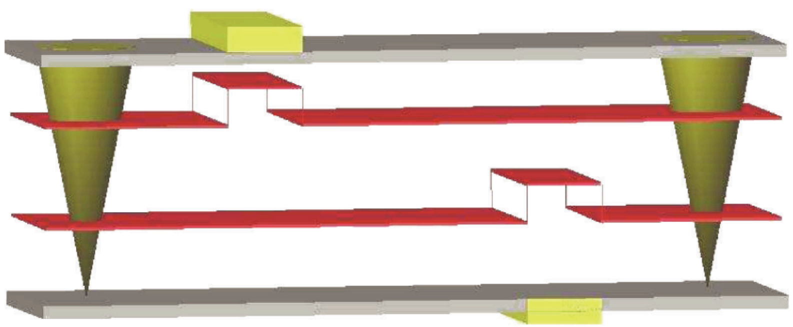
1. Optically pumped coupled quantum wells





[Butov et al. Nature (2002)]
[Snoke et al. Nature (2002)]
[…and many more…]

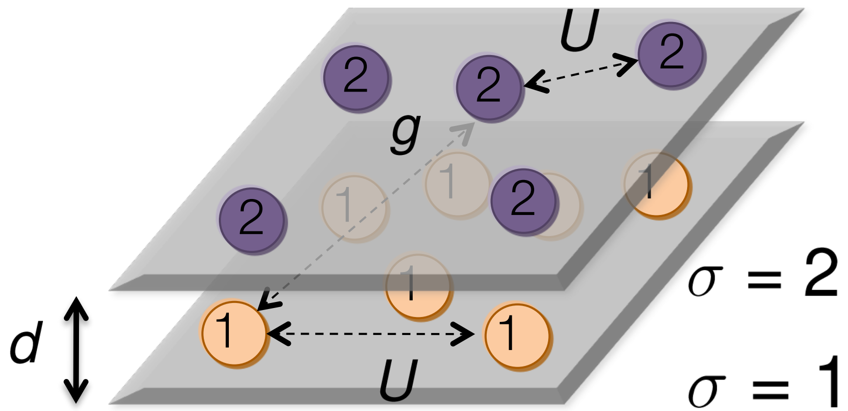
2. Individually contacted doped layers



[Croxall et al. PRL (2008)]
[Seamons et al. PRL (2009)]

⇒ Equilibrium phase diagram

Electron-hole bilayers: Hamiltonian



$\epsilon_{\mathbf{k}\sigma}$

$$H = \sum \frac{k^2}{2m_\sigma} c_\sigma^\dagger c_\sigma + \frac{1}{\Omega} \sum g_q c_1^\dagger c_2^\dagger c_2 c_1 + \frac{1}{2\Omega} \sum U_q c_\sigma^\dagger c_\sigma^\dagger c_\sigma c_\sigma$$

$U_q = \dfrac{2\pi e^2}{\varepsilon q}$ bare intra-layer Coulomb interaction

$g_q = -U_q e^{-qd}$ bare inter-layer

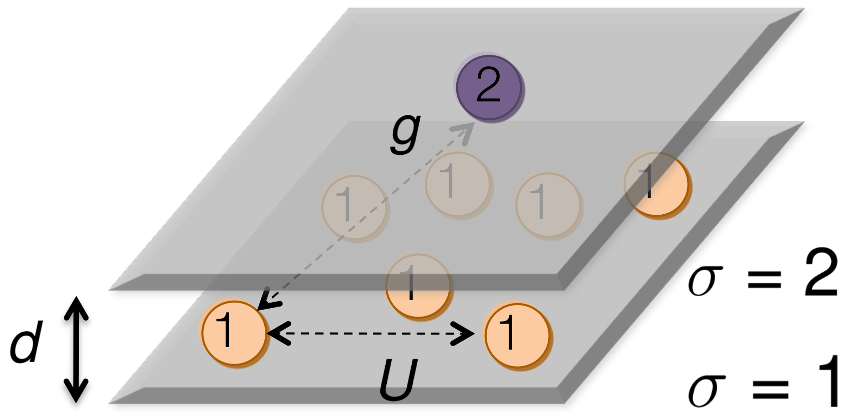
$\Rightarrow$ No spin (spin polarised)

Limit of large imbalance: Variational ground state

✧ single particle in the 2nd layer + Fermi see in the 1st layer

relative k   CoM Q

$$|\Psi(\mathbf{Q})\rangle = \sum_{\mathbf{k}>k_F} \varphi_{\mathbf{kQ}}\, c^\dagger_{\mathbf{Q}-\mathbf{k},2}\, c^\dagger_{\mathbf{k},1} |FS\rangle$$



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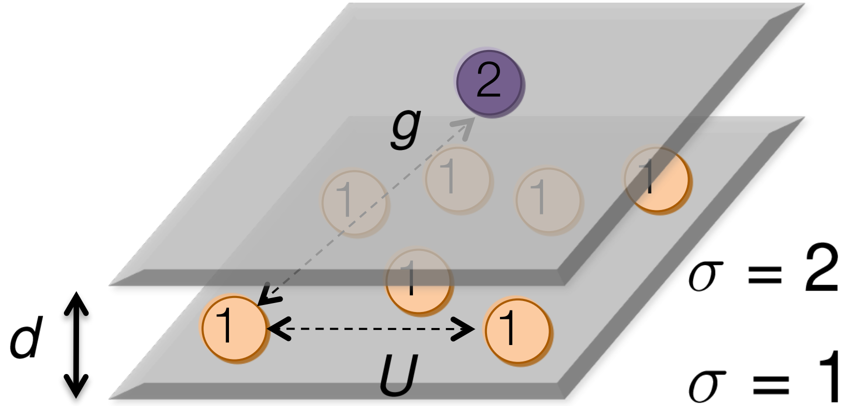
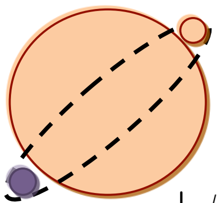
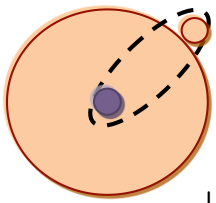




$|\psi(\mathbf{Q}=0)\rangle$

(SF=superfluid)



$|\psi(\mathbf{Q}\neq 0)\rangle$

(FFLO)

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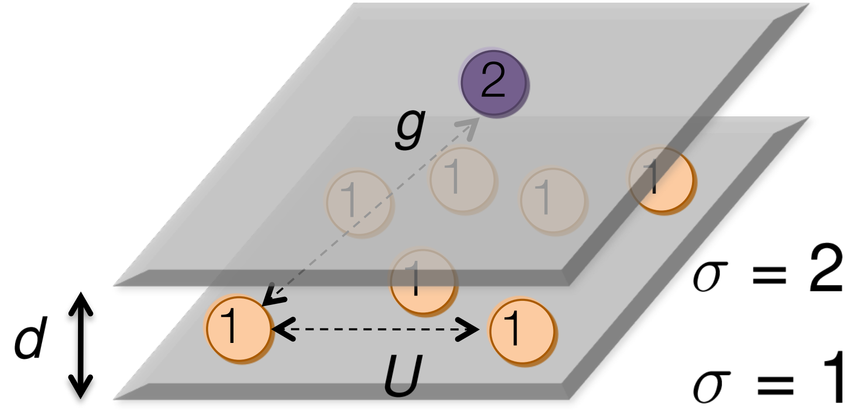

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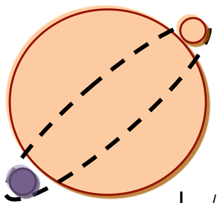
$$\varphi_{\mathbf{kQ}} = \delta_{k,k_F}\delta_{Q,k_F}$$

$$|\Psi_0\rangle = c^\dagger_{0,2} c^\dagger_{k_F\hat{\mathbf{k}},1} |FS\rangle$$



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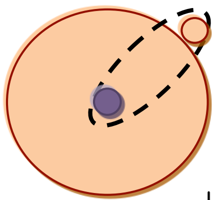
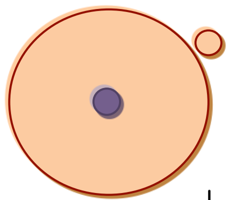


$|\psi(\mathbf{Q}\neq 0)\rangle$

(FFLO)

Interpolate by screening the interactions (within RPA) = dress with density fluctuations, including particle-hole excitations



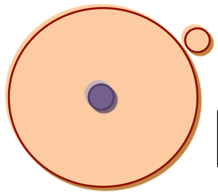
$|\psi_0\rangle$

(N=normal)

Good description of both low and high density limit

Phase diagram

1. Eigenvalue equation $\langle \Psi(\mathbf{Q}) | H | \Psi(\mathbf{Q}) \rangle$
2. Unbinding transition $\langle \Psi_0 | H | \Psi_0 \rangle$



$|\psi_0\rangle$

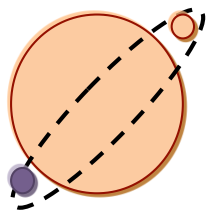
N=normal state

✧ Read also as mean-field (linearised) gap equation

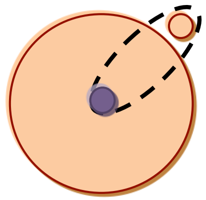
⇒ Current theories
 ⇒… some neglect screening
 ⇒… some neglect finite Q FFLO

[Yamashita et al. J Phys Soc Japan (2010)]
[Pieri et al. PRB (2007)]
[Subasi et al. PRB (2010)]

SF=superfluid



$|\psi(\mathbf{Q}=0)\rangle$



FFLO

$|\psi(\mathbf{Q}\neq 0)\rangle$

✧ Parameters:

exciton Bohr radius $a_0 = \dfrac{\varepsilon}{me^2}$

⇒ mass ratio $\alpha = \dfrac{m_2}{m_1}$
⇒ interlayer distance d/a_0
⇒ dimensionless density $r_s = \dfrac{2}{k_F a_0}$

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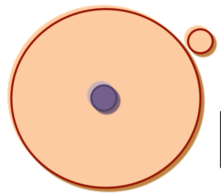

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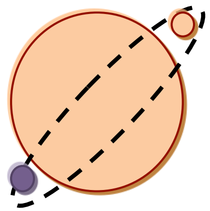
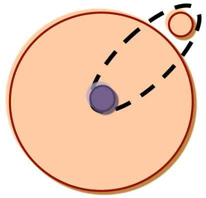


$|\psi(\mathbf{Q} = 0)\rangle$



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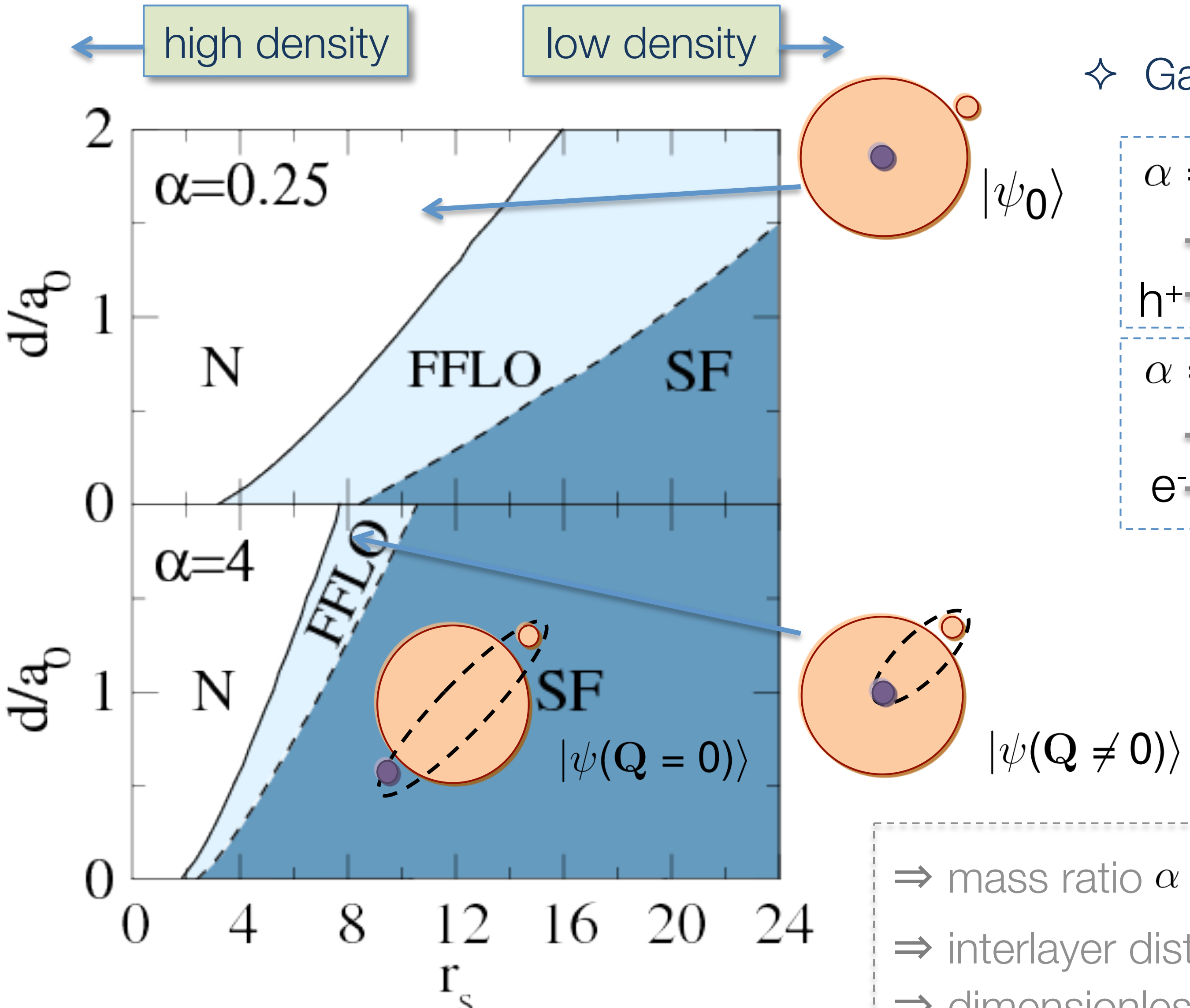
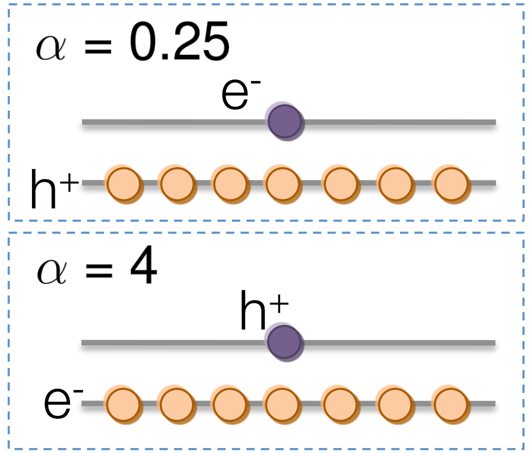
Phase diagram of fully imbalanced EH bilayer



[Parish, Marchetti&Littlewood EPL (2011)]

✧ GaAs:



FFLO region enhanced if minority particle lighter

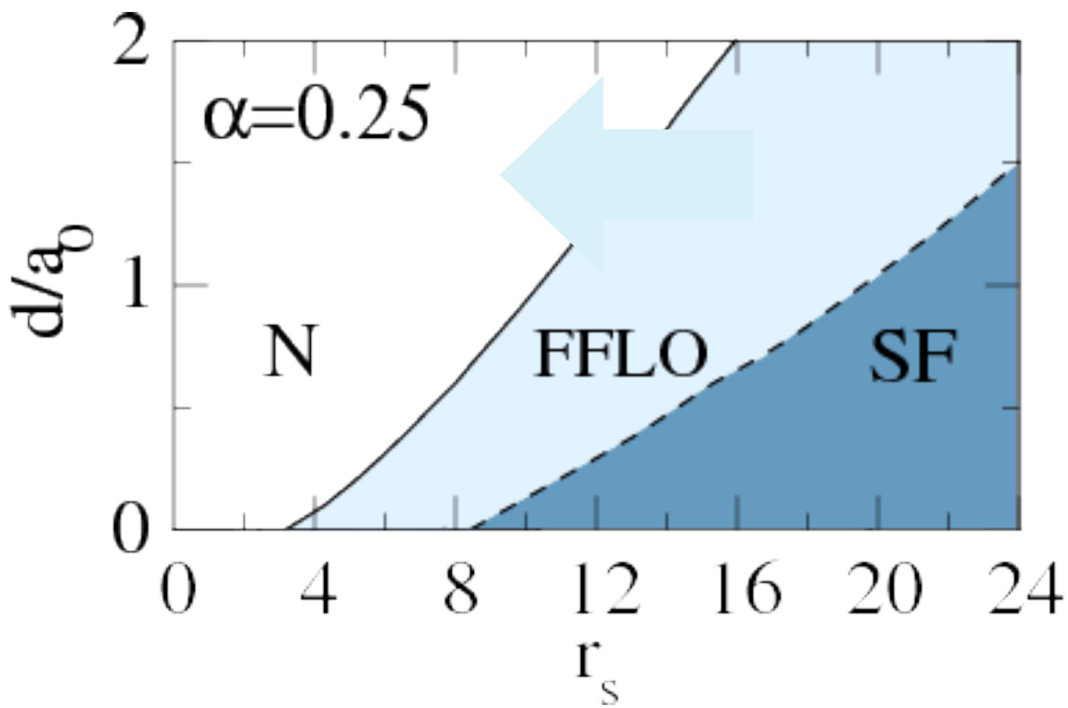
⇒ mass ratio $\alpha = \dfrac{m_2}{m_1}$

⇒ interlayer distance d/a_0

⇒ dimensionless density $r_s = \dfrac{2}{k_F a_0}$

Effect of screening on the LOFF phase

✧ Unscreened case



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⇒ Numerically

⇒ Analytically

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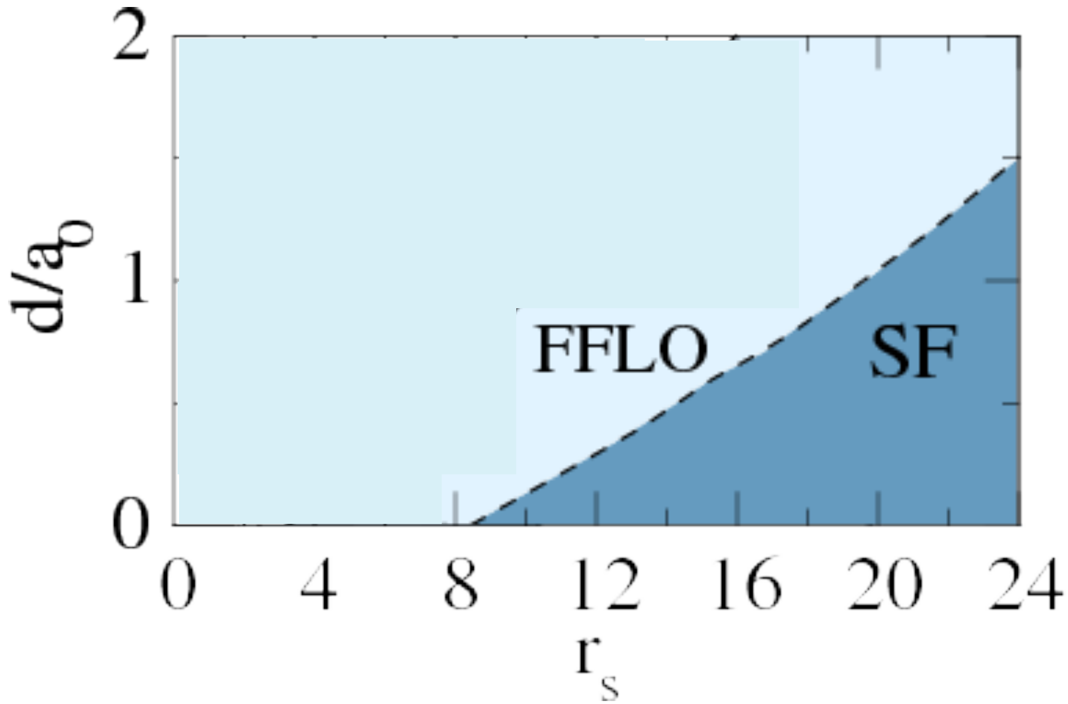


[Parish, Marchetti&Littlewood EPL (2011)]

⇒ Numerically

⇒ Analytically

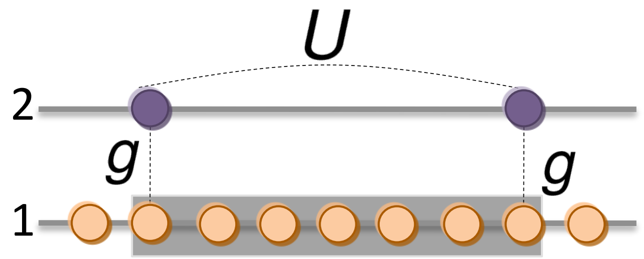
Which kind of FFLO phase?

Dilute gas of minority particles: Interactions

✧ Normal phase:

effective interaction between two unbound minority particles (RPA)

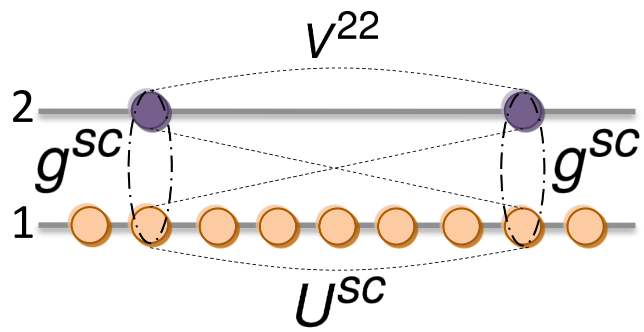
⇒ repulsive and dipolar $V^{22}(r) \to \frac{1}{r^3}$:
the dilute gas is a Fermi liquid



✧ Excitonic phase:

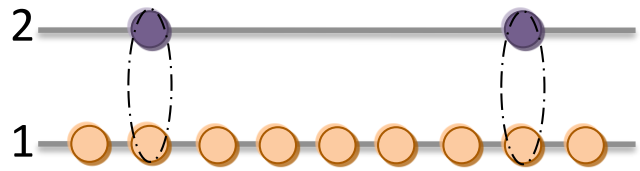
well separated excitons (dipoles)

⇒ also repulsive and dipolar $V^{ex}(r) \to \frac{1}{r^3}$:
no phase separation
(biexciton formation suppressed for
single spin species & $\frac{d}{a_0} \gtrsim 0.25$)



Phenomenology of ‘bosonic’ FFLO

- Landau theory for $|\psi|^2$ = exciton density



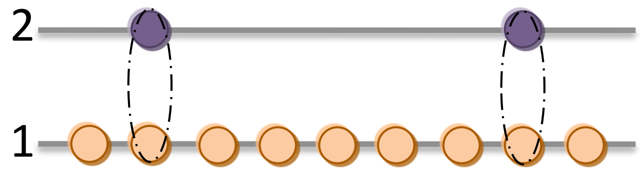
$$F[\psi] = \int d\mathbf{r} \left[-\mu|\psi|^2 \qquad\qquad + \frac{\lambda}{2}|\psi|^4 \right]$$

chemical potential

minimum energy at $|\mathbf{q}| = Q_{min}$

Phenomenology of ‘bosonic’ FFLO

✧ Landau theory for $|\psi|^2$ = exciton density
(weak crystalisation theory)



$$F[\psi] = \int d\mathbf{r} \left[-\mu|\psi|^2 + \gamma|(\nabla^2 + Q_{min}^2)\psi|^2 + \frac{\lambda}{2}|\psi|^4 \right]$$

chemical potential

minimum energy at $|\mathbf{q}| = Q_{min}$

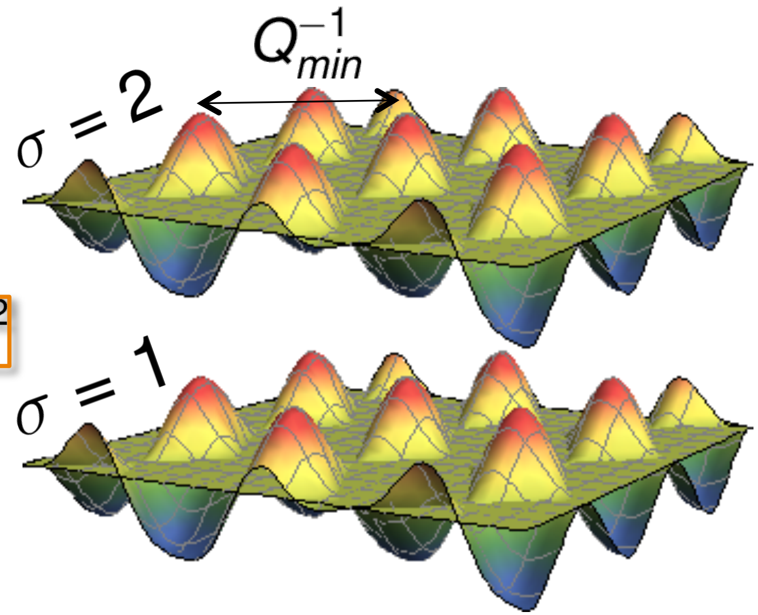
✧ Complex order parameter

$$\psi(\mathbf{r}) = \sum_n a_n e^{i\mathbf{q}_n \cdot \mathbf{r}} \Big|_{|\mathbf{q}_n| = Q_{min}}$$

✧ Minimal energy solution

$$|\psi(\mathbf{r})|^2 \propto |\cos(\mathbf{q}_1 \cdot \mathbf{r}) - i \cos(\mathbf{q}_2 \cdot \mathbf{r} + \varphi)|^2$$

⇒ supersolid: exciton condesate
with 2D spatial modulation
(diagonal and off-diagonal order)



Observing FFLO

- FFLO enjoys a sizeable region of existence away from the Wigner crystal
- Trions only for $d/a_0 << 1$

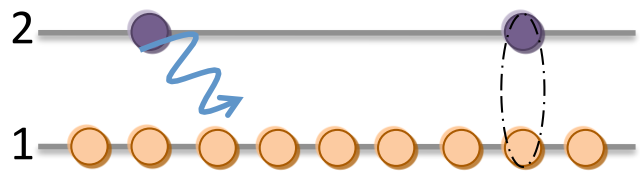
- Finite temperature
  Exciton binding energy in GaAs
  = upper bound for the FFLO critical temperature

$$\frac{E_B}{k_B} = 5\ \mathrm{K}$$

$$T_{BKT} \propto \frac{E_0}{k_B r_s^2} \frac{n_2}{n_1} \sim 100\ \mathrm{mK}$$

GaAs
$$\alpha = 0.25$$
$$a_0 = 7\ \mathrm{nm}$$
$$E_0 = 17\ \mathrm{meV}$$
$$d/a_0 = 0.5$$
$$r_s = 10$$
$$n_2/n_1 \sim 0.2$$

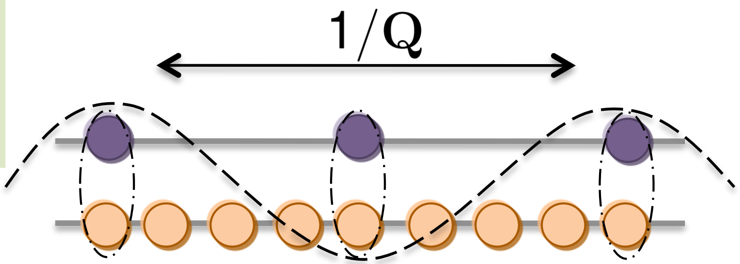
1. Light scattering off the spatial modulations
2. Photon angular emission



(electron hole recombining): finite momentum pairing ($\pm\mathbf{q}_1, \pm\mathbf{q}_2$)

Conclusions part 1

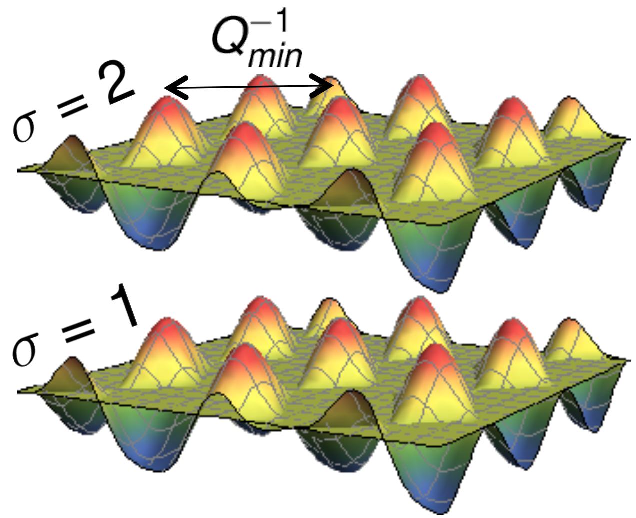
✧ Electron-hole bilayers:
promising for observing exotic pairing phenomena



✧ Evidence of FFLO phase at large imbalance:
finite-Q exciton in presence of a fermi sea

✧ Dilute gas of finite-Q excitons:
condensation with 2D spatial modulation (a supersolid)

⇒ bosonic limit of FFLO



✧ Prospects for experimental observation

2D dipolar Fermi gases

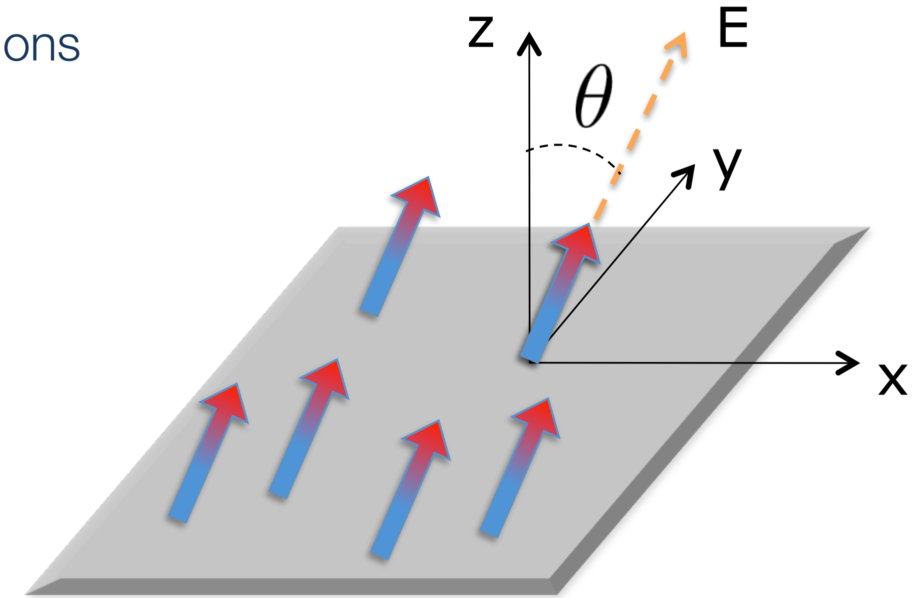
✧ Quantum gas of ultracold polar fermions

⇒ ^{40}K-^{87}Rb tightly bound heteronuclear molecules

⇒ quantum degeneracy

⇒ 2D confinement

[Ni et al. Science (2008), Nature (2010)]
[Ospelkaus et al. Science (2010)]
[de Miranda et al. Nature Physics (2011)]



✧ Dipole-dipole interaction: long range and anisotropic

✧ Rich many-body physics (even for single component and single layer)

⇒ $\theta = 0$ isotropic & repulsive

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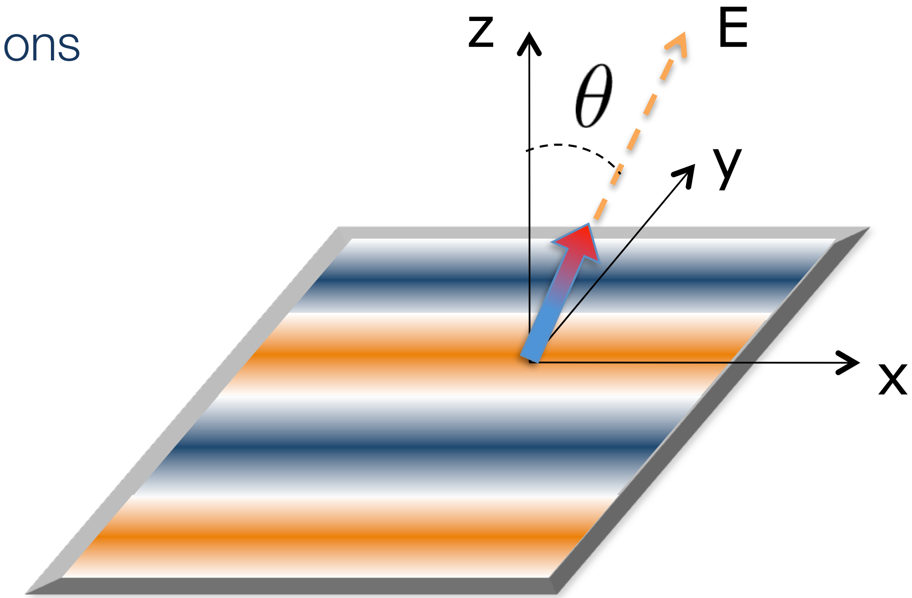

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spontaneous density modulation & crystalline phase

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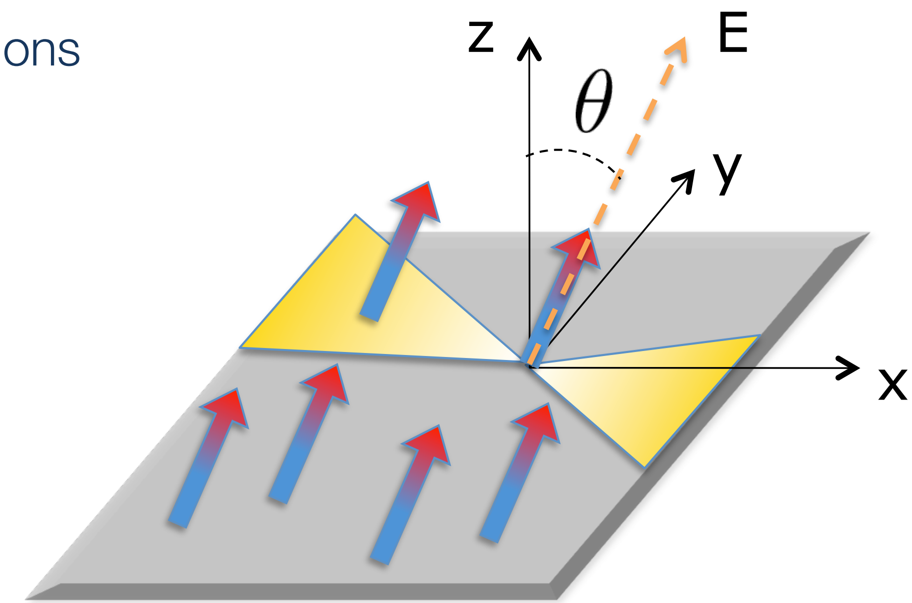

✧ Dipole-dipole interaction: long range and anisotropic

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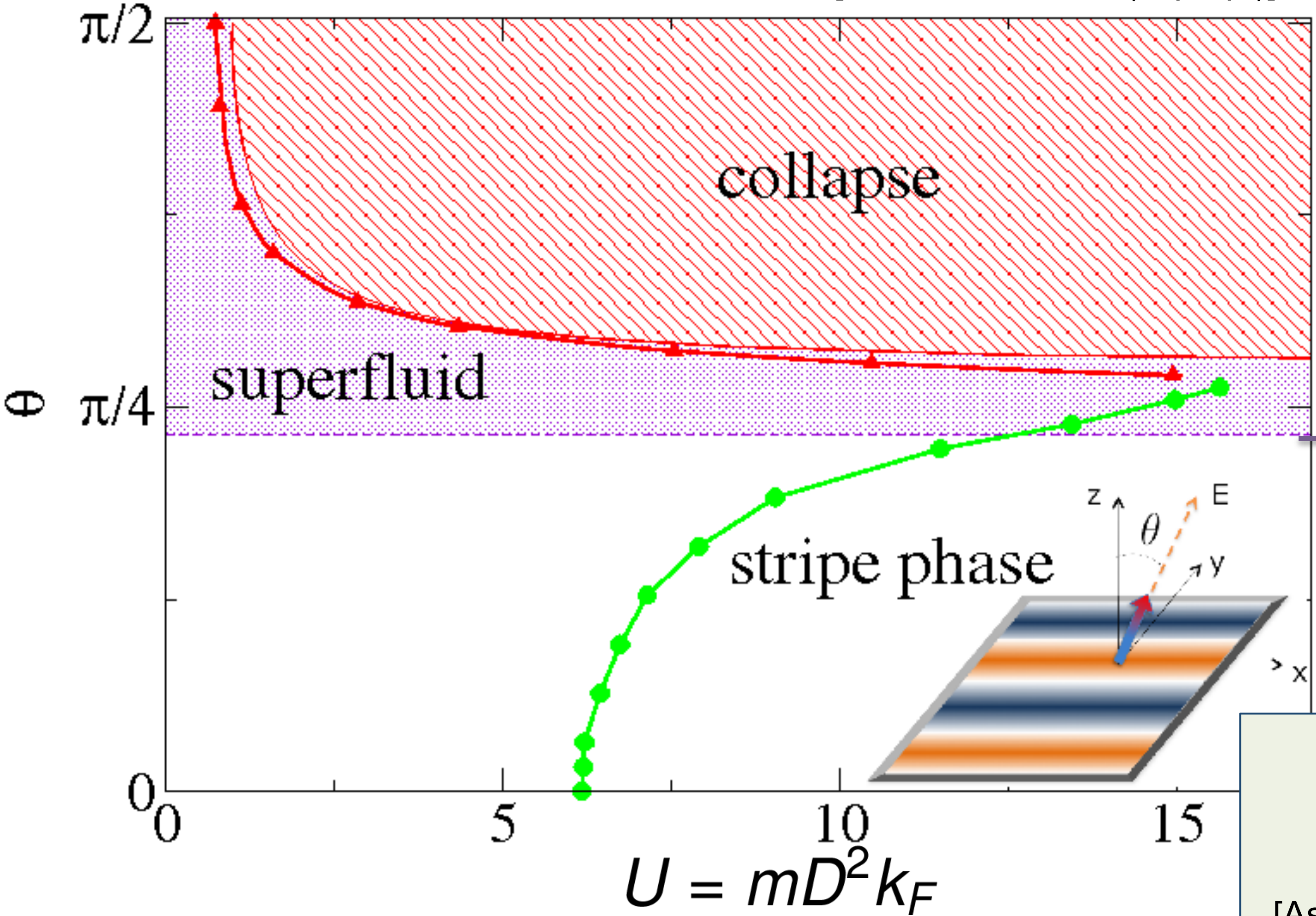
⇒ $\theta > \mathrm{asin}(1/\sqrt{3})$ attractive sliver

superfluidity & collapse

Phase diagram

$$V_\theta(q,\phi) = 2\pi D^2 \left[v_0 - q \left(\cos^2\theta - \sin^2\theta \cos^2\phi \right) \right]$$

[Marchetti&Parish (in prep.)]



[Bruun&Taylor PRL (2008)]

Wigner cristal

$U \gtrsim 60$

[Astrakhrchik PRL (2007)]

Why not RPA (e.g., for θ=0)

$$V_0(q) = 2\pi D^2 \left[v_0 - q\right]$$

- Hartree-Fock ground state energy per particle ($U = mD^2 k_F \to 0$):

$$\varepsilon(n) = \frac{4\pi n}{m}\left(\frac{1}{4} + \frac{32U}{45\pi}\right)$$

- Compressibility sum rule

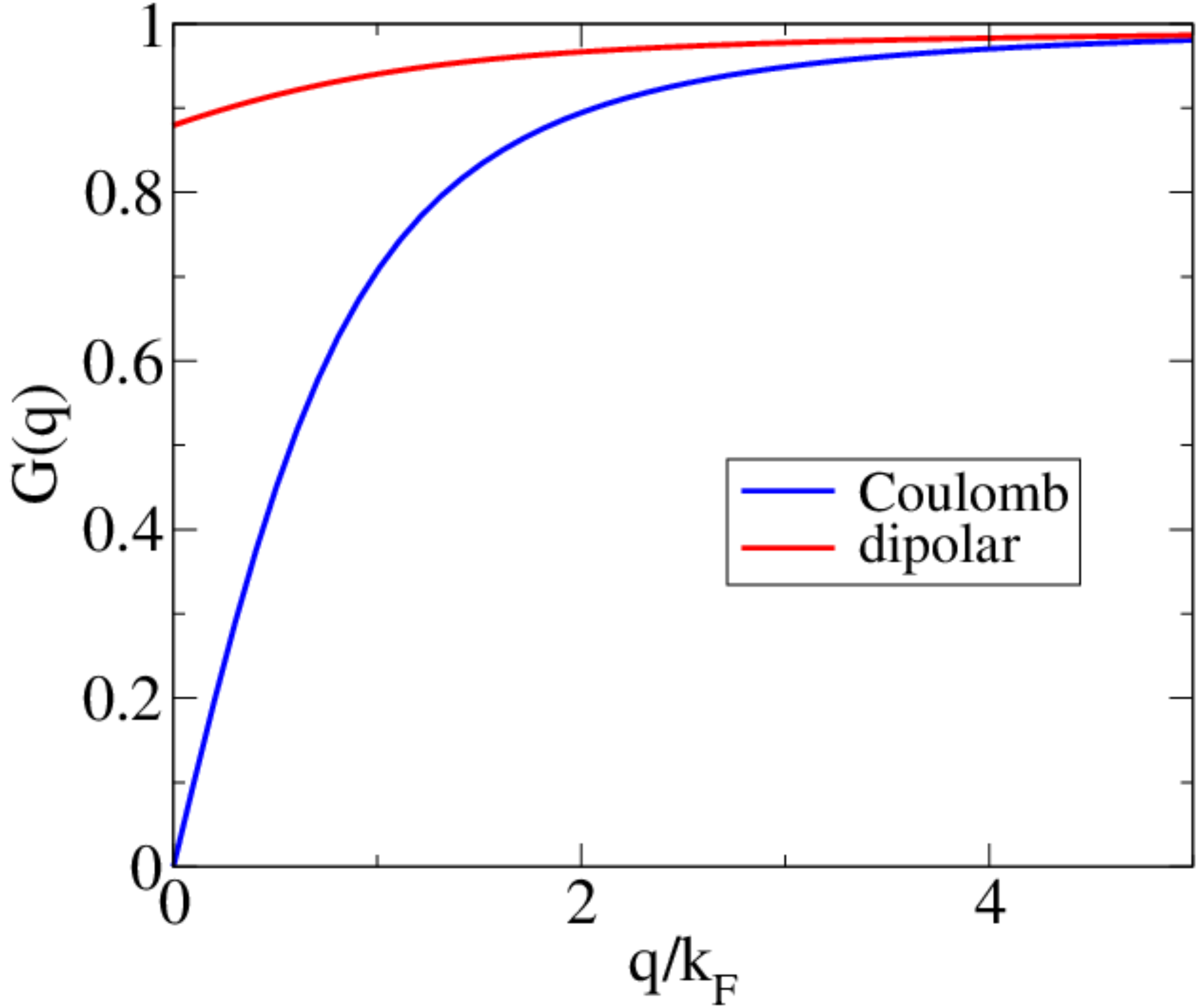
$$-m\frac{\partial^2\left[n\varepsilon(n)\right]}{\partial n^2} = \lim_{q\to 0}\chi^{-1}(q,\omega=0)$$

$$= \lim_{q\to 0}\frac{1}{\Pi(q,\omega=0)} - V_0(q)\left[1 - G(q)\right]$$

RPA local field factor

- The local field factor allows to include exchange correlations neglected by RPA ($G(q) = 0$)

Why not RPA



Phase diagram

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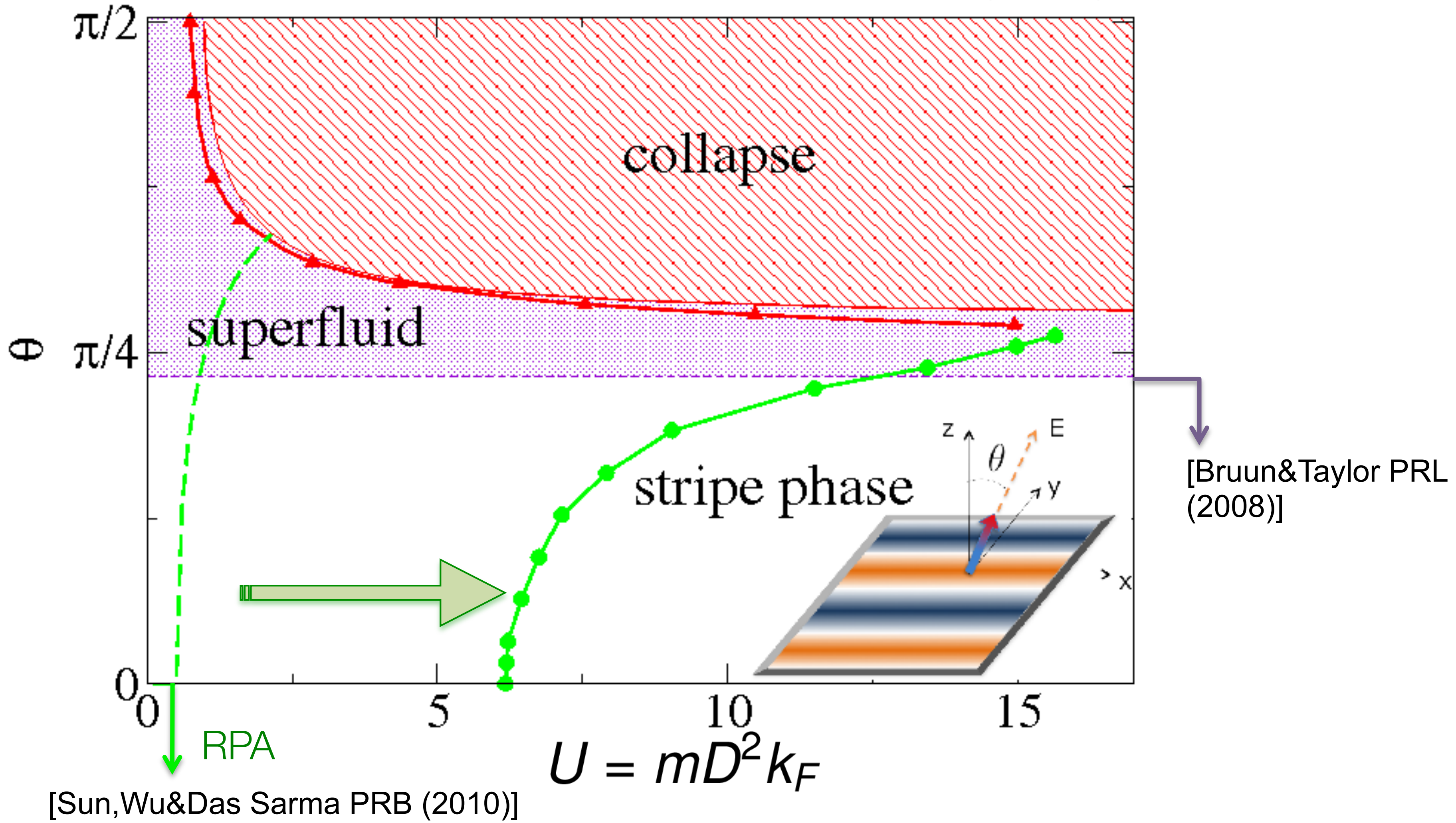
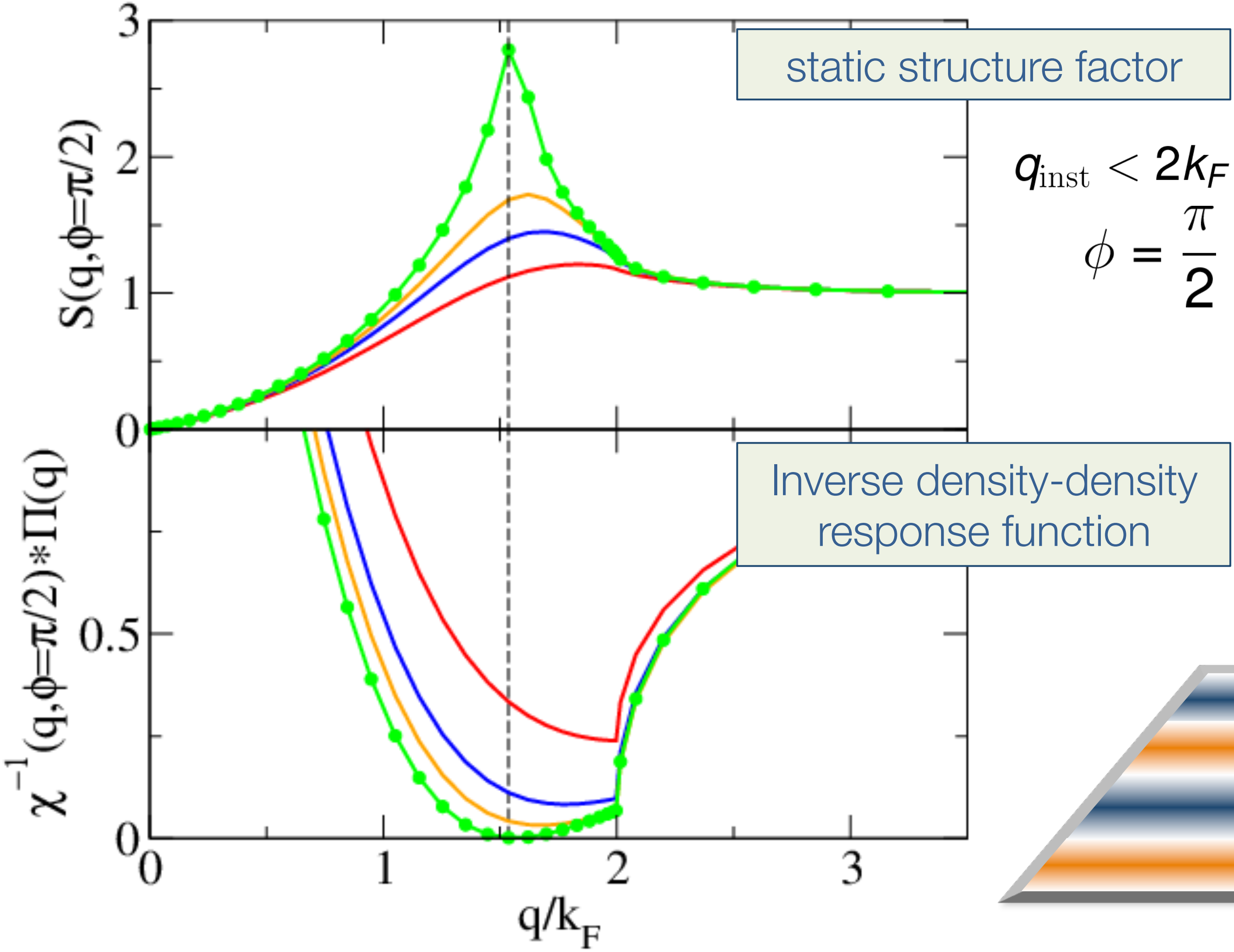


Phase diagram

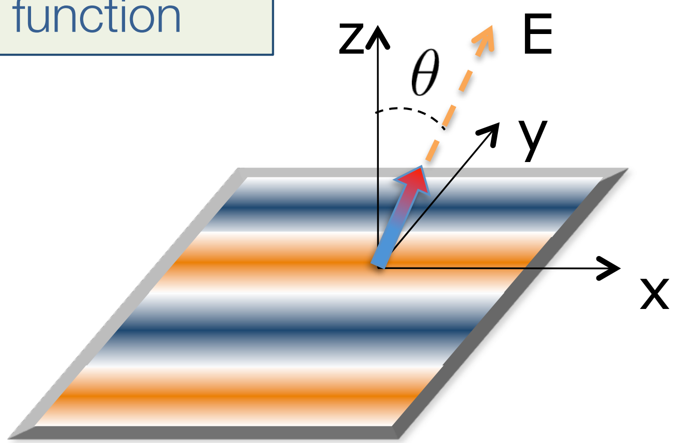
✧ Approaching the transition to the stripe phase

[Marchetti&Parish (in prep.)]



$$q_{\rm inst} < 2k_F$$

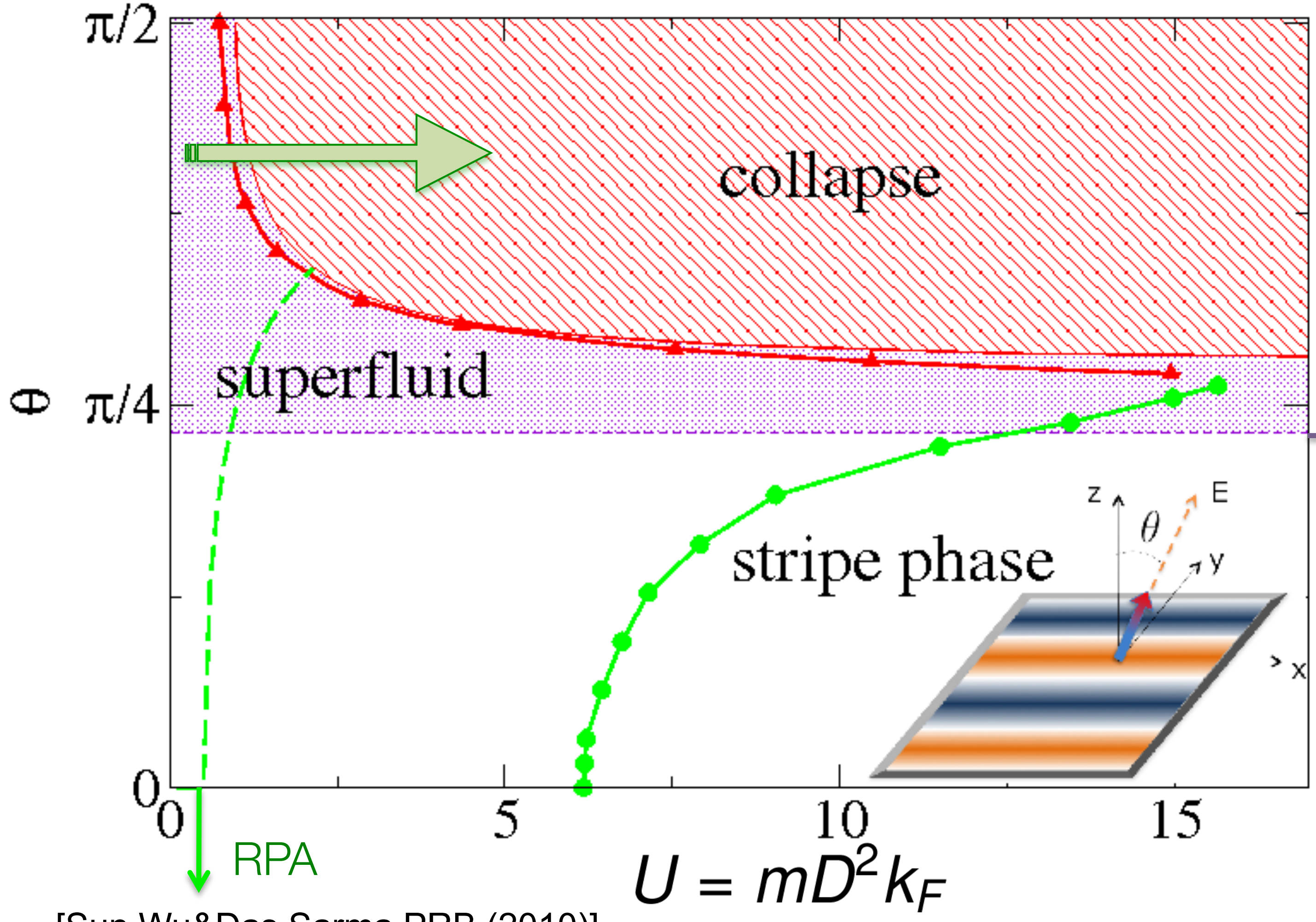
$$\phi = \frac{\pi}{2}$$



Phase diagram

No collapse within RPA [Sun,Wu&Das Sarma PRB (2010)]

[Marchetti&Parish (in prep.)]



[Bruun&Taylor PRL (2008)]

[Sun,Wu&Das Sarma PRB (2010)]

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Argonne National Laboratory

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