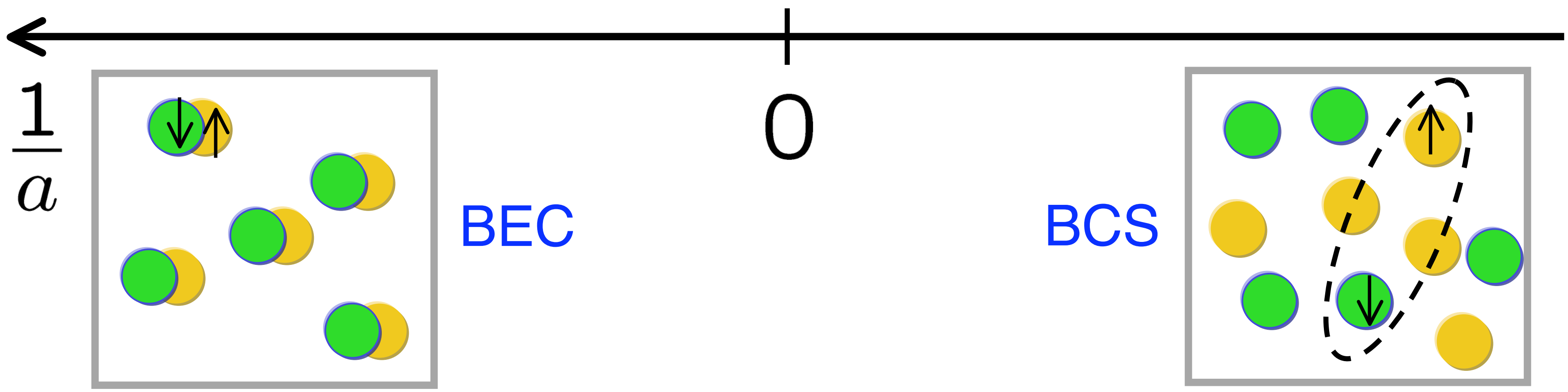
Polarised Fermi Superfluids

Phase Diagram & Dynamics

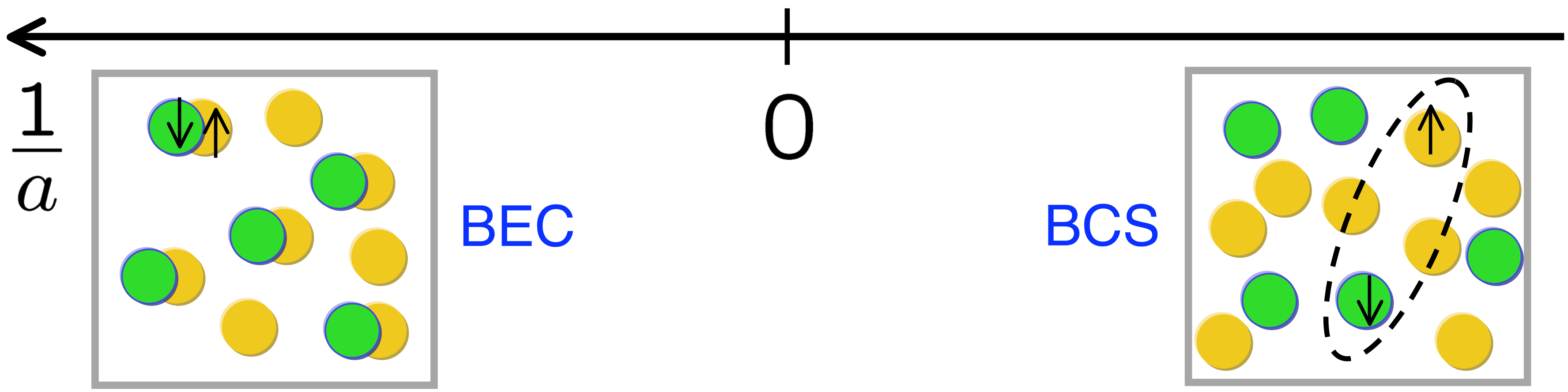
F.M. Marchetti

KITP, Santa Barbara, 23 May 2007

Fermi Superfluids

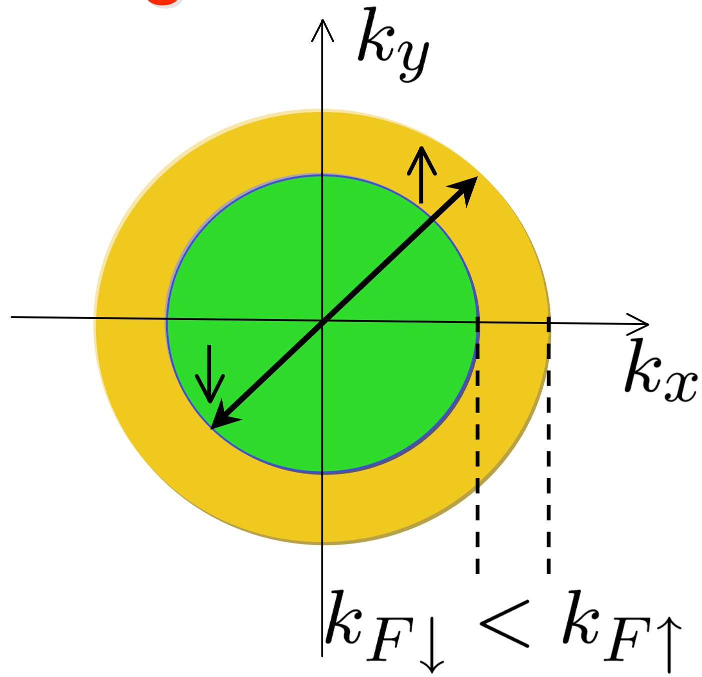


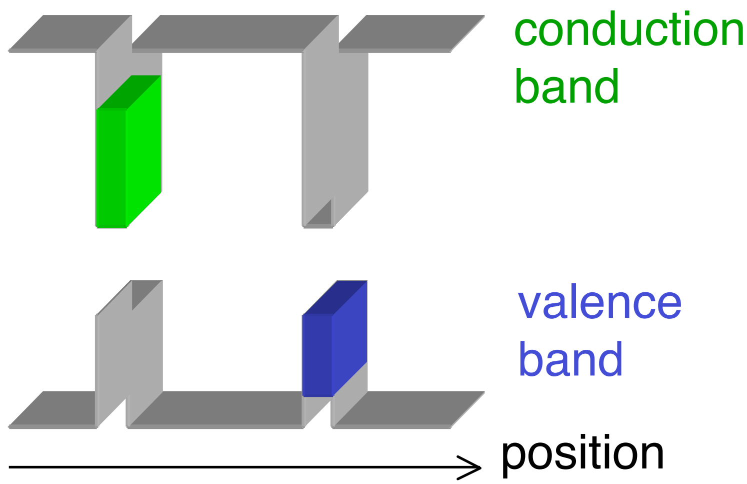
Polarised Fermi Superfluids

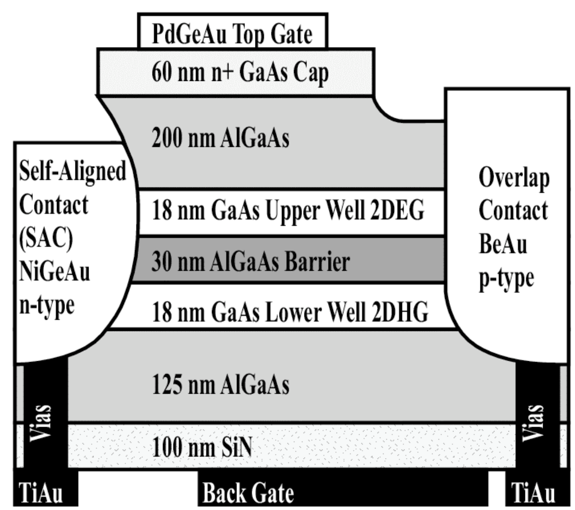


Why Interesting?

- Magnetised superconductors (Zeeman)
- Quantum Chromodynamics (and neutron stars)
- Electron-hole bilayers





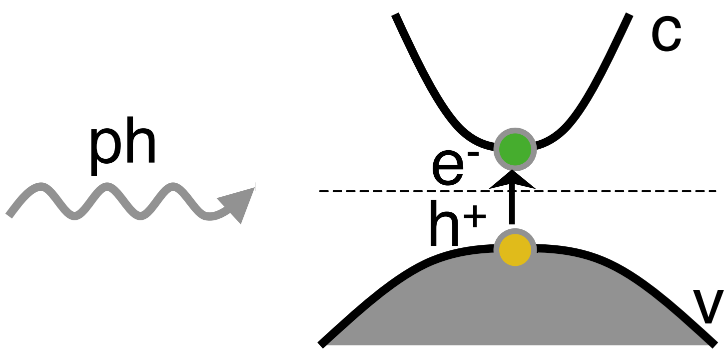


[J. A. Seamons *et al.*, *APL* **90**, 052103 (2007)]

Parenthesis on Excitonic Condensates

[L.V. Keldysh & Yu V. Kopaev, *Sov. Phys. Solid State* **6**, 2219 (1965)]

- Condensation of microcavity polaritons (1/2-exciton 1/2-photon quasi-particles)



[see J. Keeling's talk]

[J. Kasprzak *et al.*, *Nature* **443**, 409 (2006)]

Outline

- BEC-BCS crossover

1. Unbalanced populations: equal masses
   - Homogeneous phase diagram: T=0 & finite T
   - Trap
   - Experiments
2. Unequal masses (M. Parish tomorrow)
3. Dynamics of phase separation

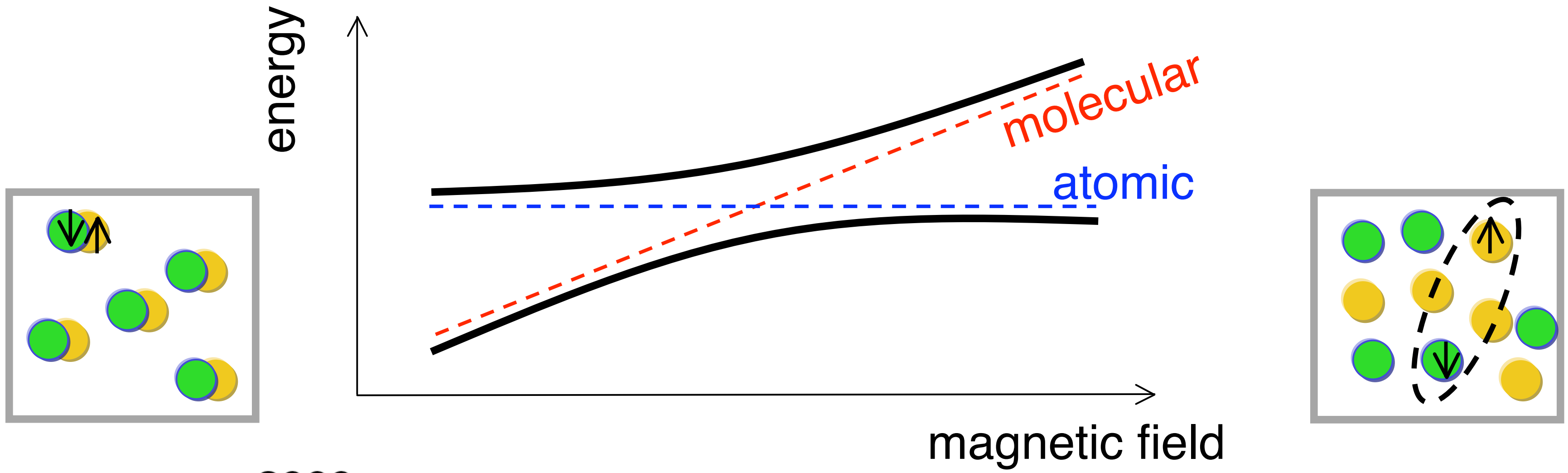
- Conclusions (and on-going work on Bose-Fermi mixtures)

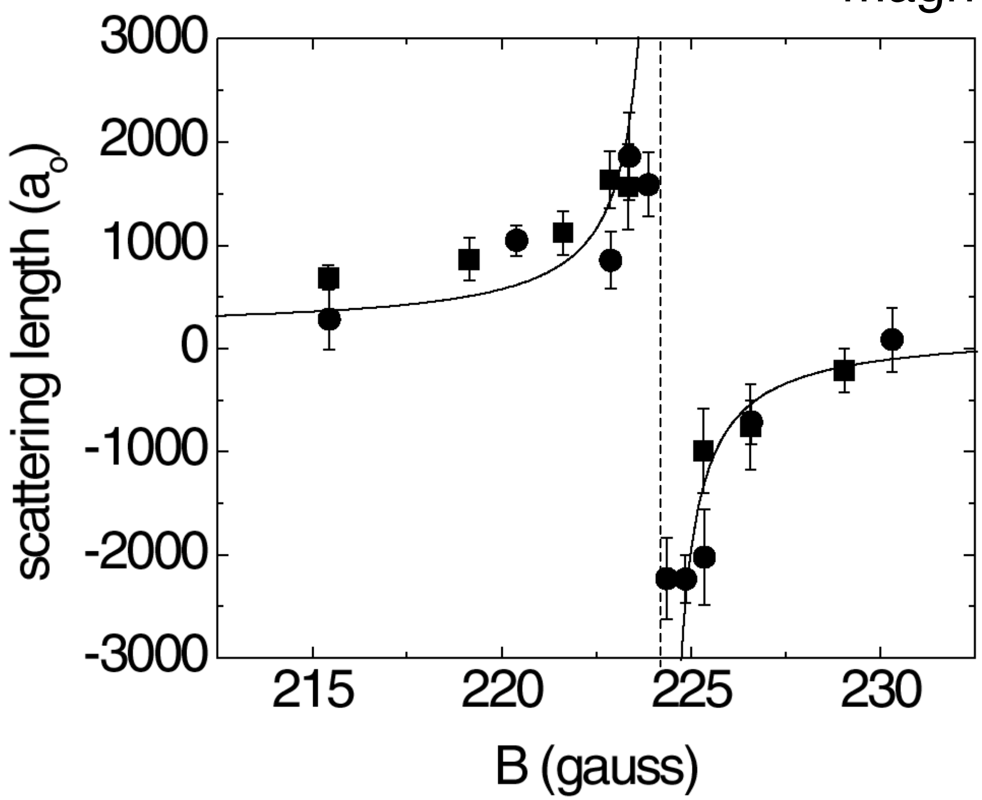
M.M. Parish   A. Lamacraft   B.D. Simons

1. [M.M. Parish, F.M. Marchetti, A. Lamacraft, & B.D. Simons, *Nature Physics* **3**, 124 (2007)]
2. [M.M. Parish, F.M. Marchetti, A. Lamacraft, & B.D. Simons, *Phys. Rev. Lett.* **98**, 160402 (2007)]
3. [A. Lamacraft & F.M. Marchetti, preprint cond-mat/0701692]

- Around 50 papers in the last 1+1/2 year…

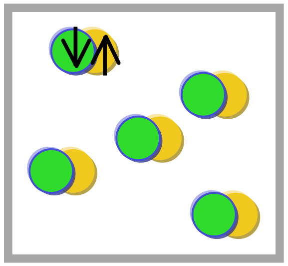
Feshbach Resonances

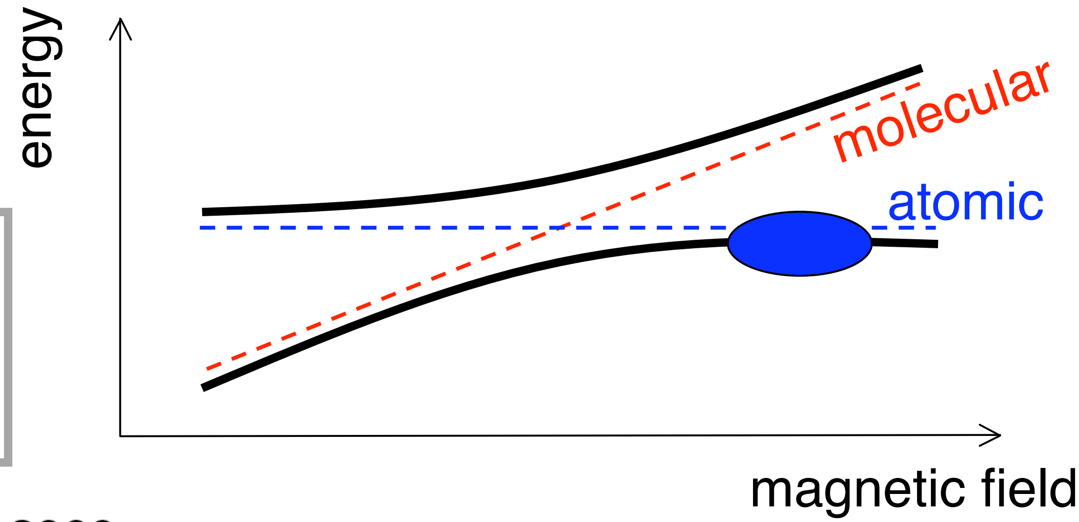


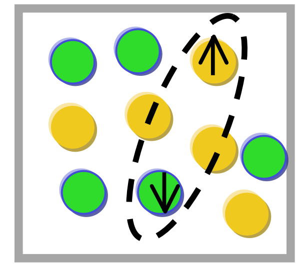


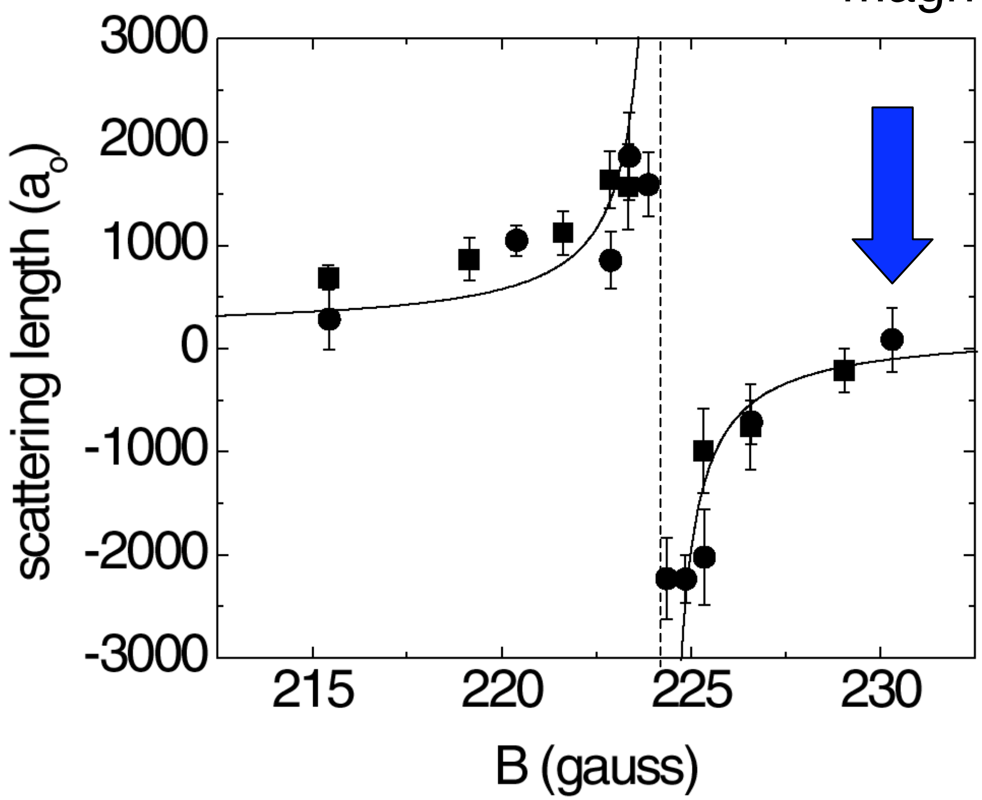
[C. A. Regal *et al.*, *Nature* **424**, 47 (2003)]

Feshbach Resonances



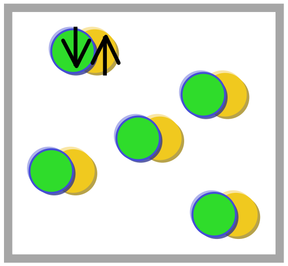


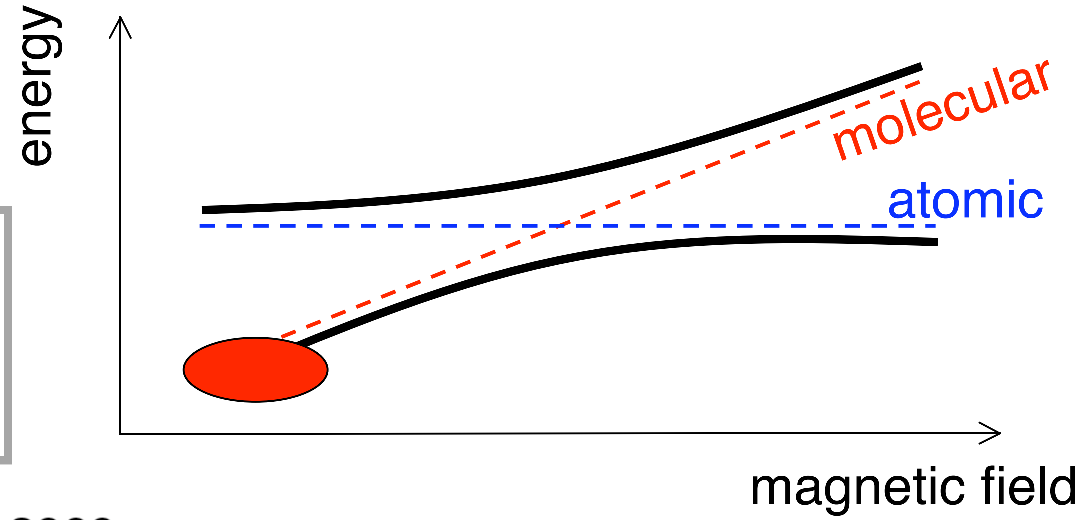


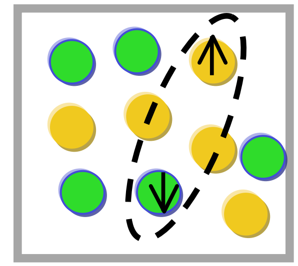


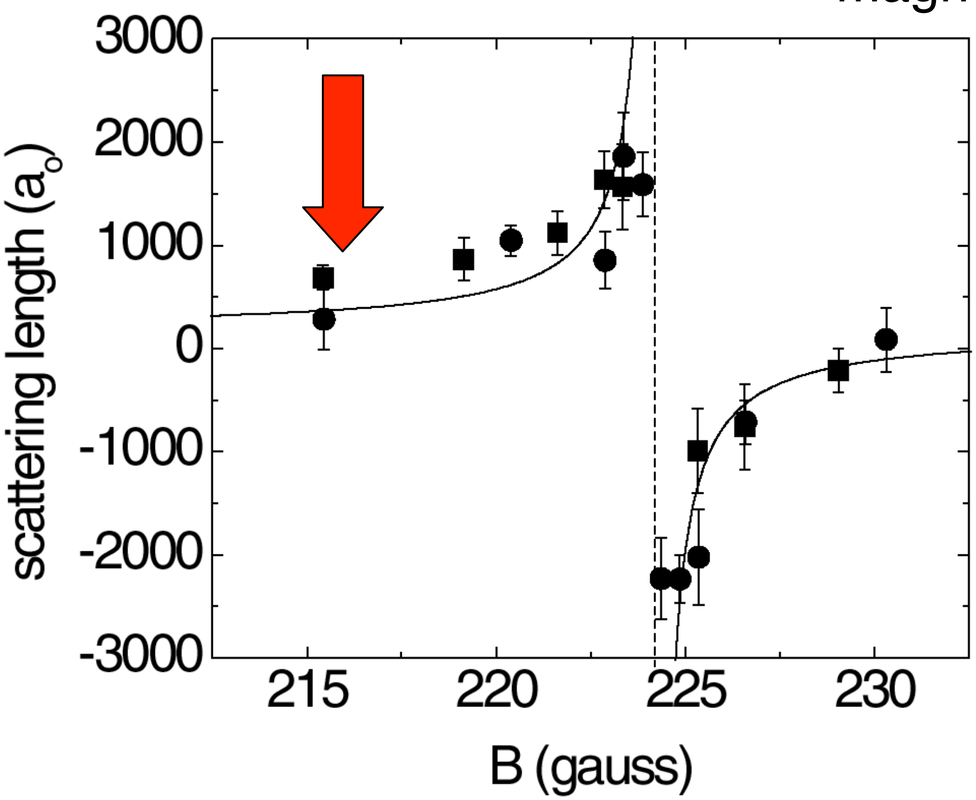
[C. A. Regal *et al.*, *Nature* **424**, 47 (2003)]

Feshbach Resonances







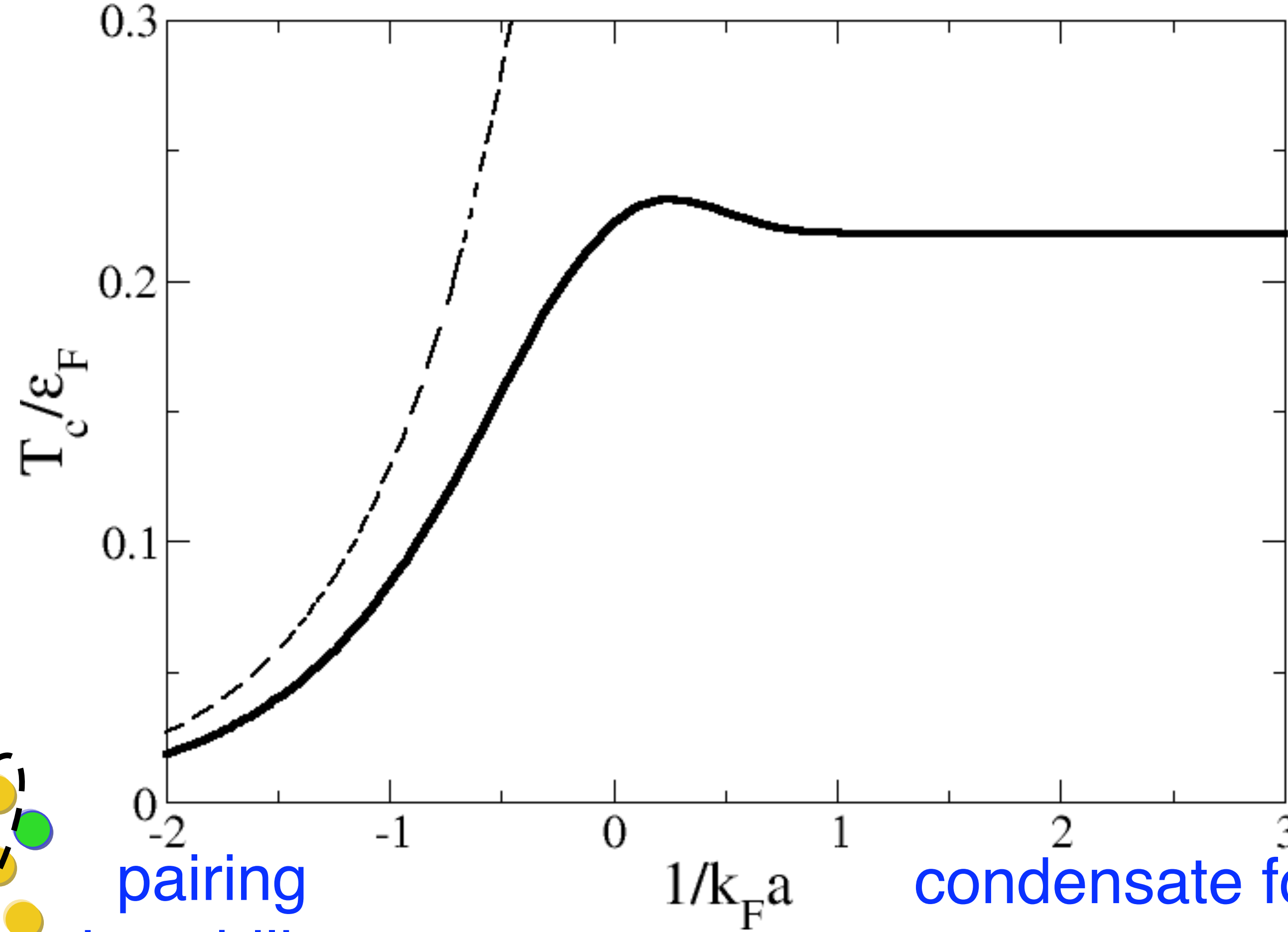


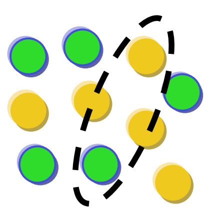
[C. A. Regal *et al.*, *Nature* **424**, 47 (2003)]

BEC-BCS Crossover

- At T=0 $$\prod_{\mathbf{k}} \left(u_{\mathbf{k}} + v_{\mathbf{k}} c^{\dagger}_{\mathbf{k},\uparrow} c^{\dagger}_{-\mathbf{k},\downarrow}\right)|0\rangle = e^{\lambda \sum_{\mathbf{k}} \varphi_{\mathbf{k}} c^{\dagger}_{\mathbf{k},\uparrow} c^{\dagger}_{-\mathbf{k},\downarrow}}|0\rangle$$

[P. Nozieres & S. Schmitt-Rink, J. Low temp. Phys. **59**, 195 (1985)]

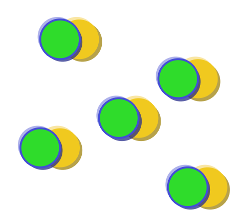




BCS

pairing instability

condensate forms out of preformed molecules



BEC

BEC-BCS Crossover

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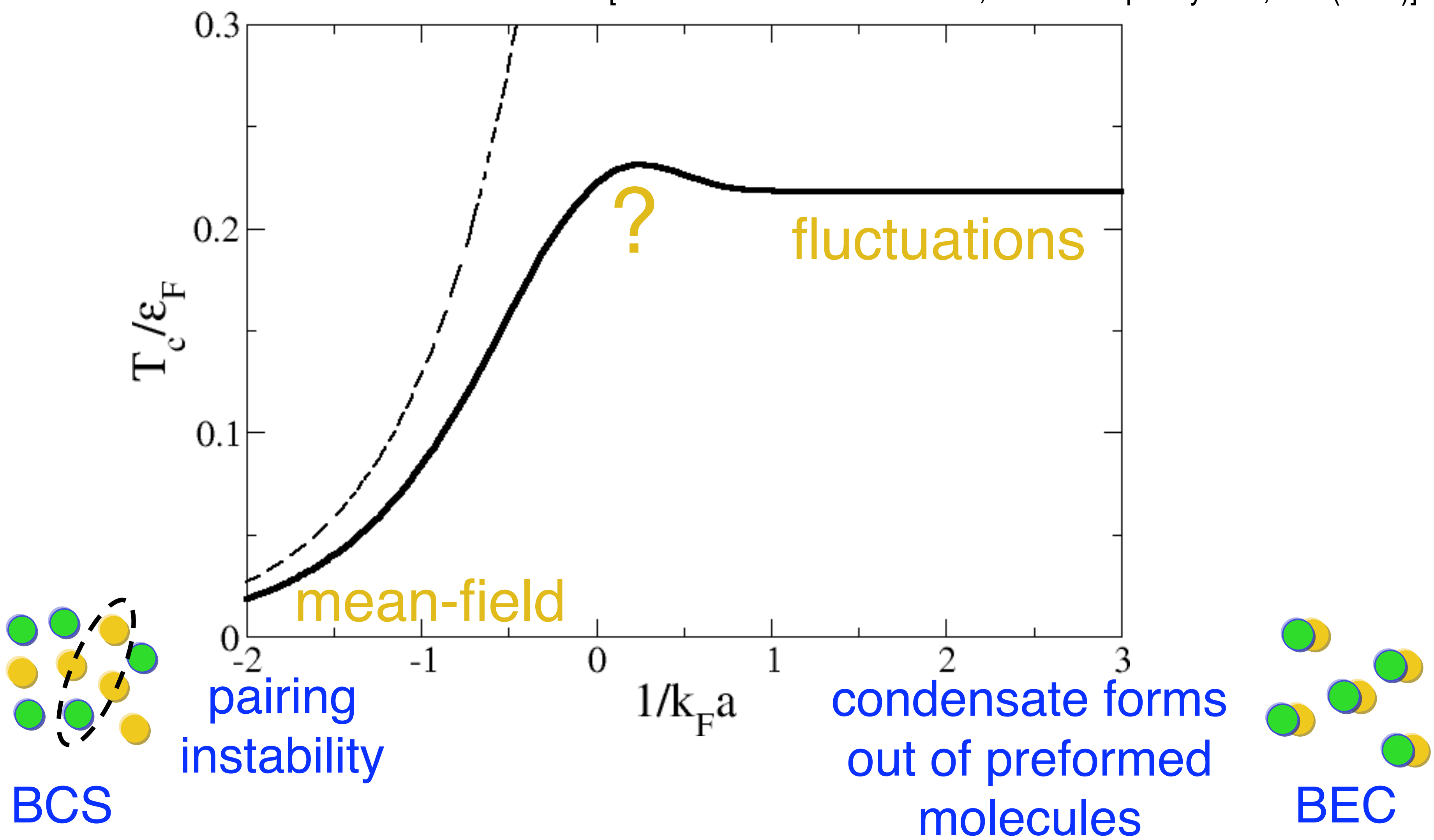
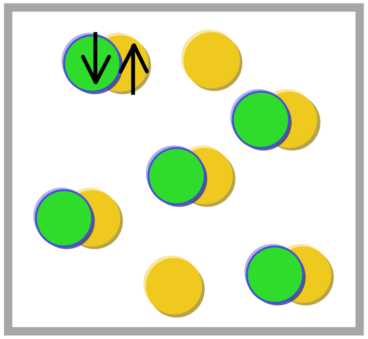
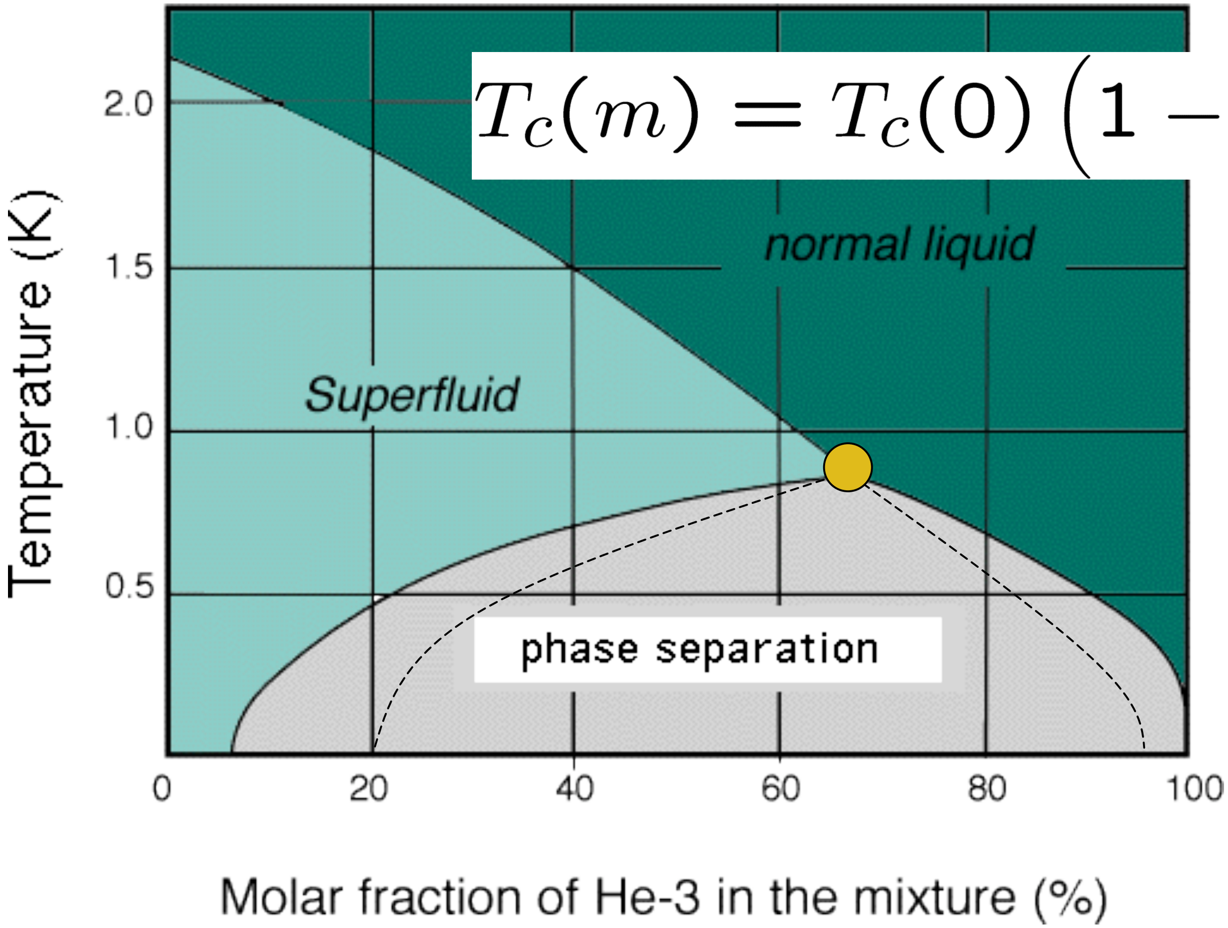


Polarised Fermi Gases



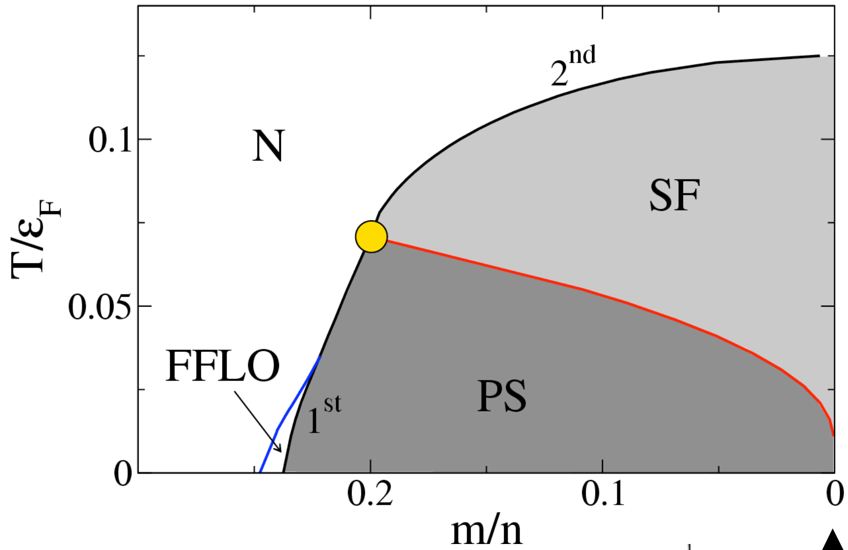
- Bose-Fermi mixture in the BEC limit
- ^{3}He-^{4}He mixture

$$T_c(m) = T_c(0)\left(1 - \frac{m}{n}\right)^{\alpha}$$



Phase Diagram

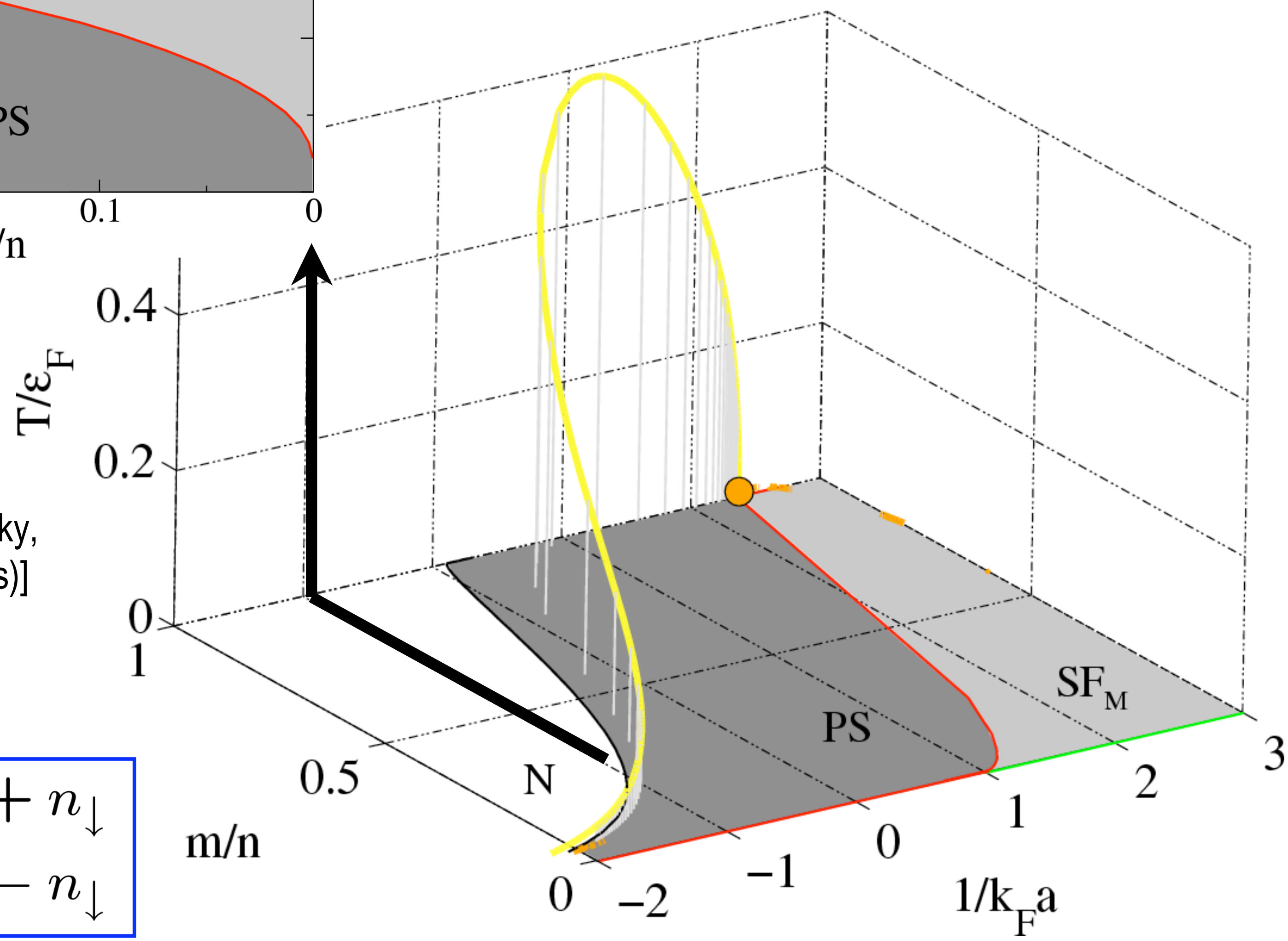
[M. Parish, F.M. Marchetti *et al.*, *Nature Physics* **3**, 124 (2007)]



$$\frac{1}{k_F a} = -1$$

[D. Sheehy & L. Radzihovsky, *Annals of Physics* (in press)]

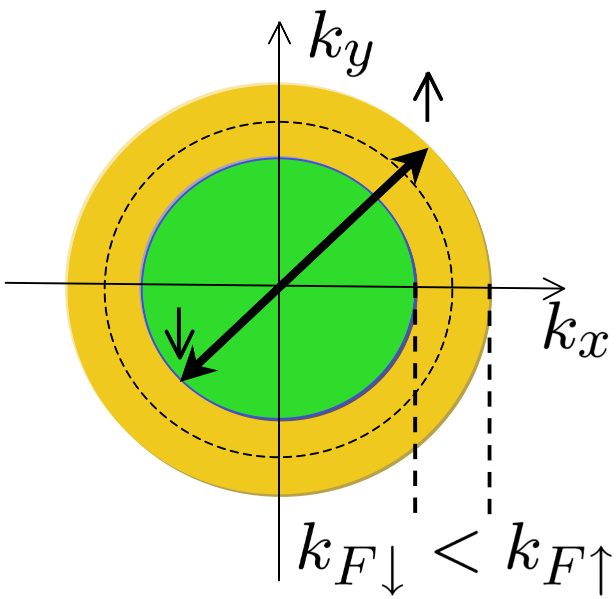
T=0 ⟶



$$n = n_\uparrow + n_\downarrow$$
$$m = n_\uparrow - n_\downarrow$$

Single Channel Model

$$\hat{\mathcal{H}} = \sum_{\mathbf{k},\sigma=\uparrow,\downarrow} (\epsilon_{\mathbf{k}} - \mu_\sigma) c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{g}{V} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} c^\dagger_{\mathbf{k}+\mathbf{q}/2\uparrow} c^\dagger_{-\mathbf{k}+\mathbf{q}/2\downarrow} c_{-\mathbf{k}'+\mathbf{q}/2\downarrow} c_{\mathbf{k}'+\mathbf{q}/2\uparrow}$$



Zeeman term

$$\mu = (\mu_\uparrow + \mu_\downarrow)/2$$
$$h = (\mu_\uparrow - \mu_\downarrow)/2$$

scattering length

$$\frac{1}{k_F a}$$

imbalance

$$n = n_\uparrow + n_\downarrow$$
$$m = n_\uparrow - n_\downarrow$$

$$\frac{1}{g} = \frac{m}{4\pi a} - \frac{1}{V} \sum_{\mathbf{k}} \frac{1}{2\epsilon_{\mathbf{k}}}$$

Mean-Field

$$\Delta = -g \sum_{\mathbf{k}} \langle c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \rangle$$

- Grand canonical free energy

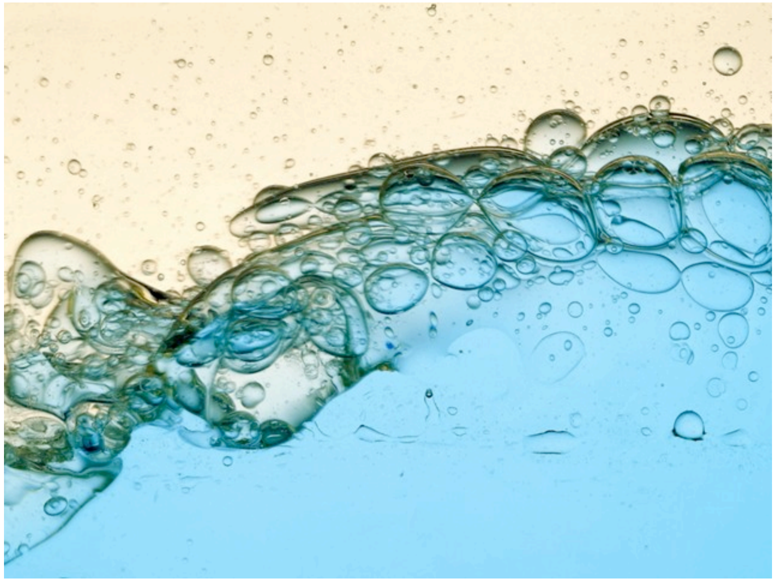
$$\Omega^{(0)}(\mu, h) = \min_{\Delta} f^{(0)}(\Delta; \mu, h) \Rightarrow f^{(0)}\left(\frac{\Delta}{|\mu|}; \frac{h}{|\mu|}\right)$$

$$\begin{cases} n = -\dfrac{\partial \Omega^{(0)}}{\partial \mu} \\ m = -\dfrac{\partial \Omega^{(0)}}{\partial h} \end{cases} \Rightarrow \frac{m}{n}$$

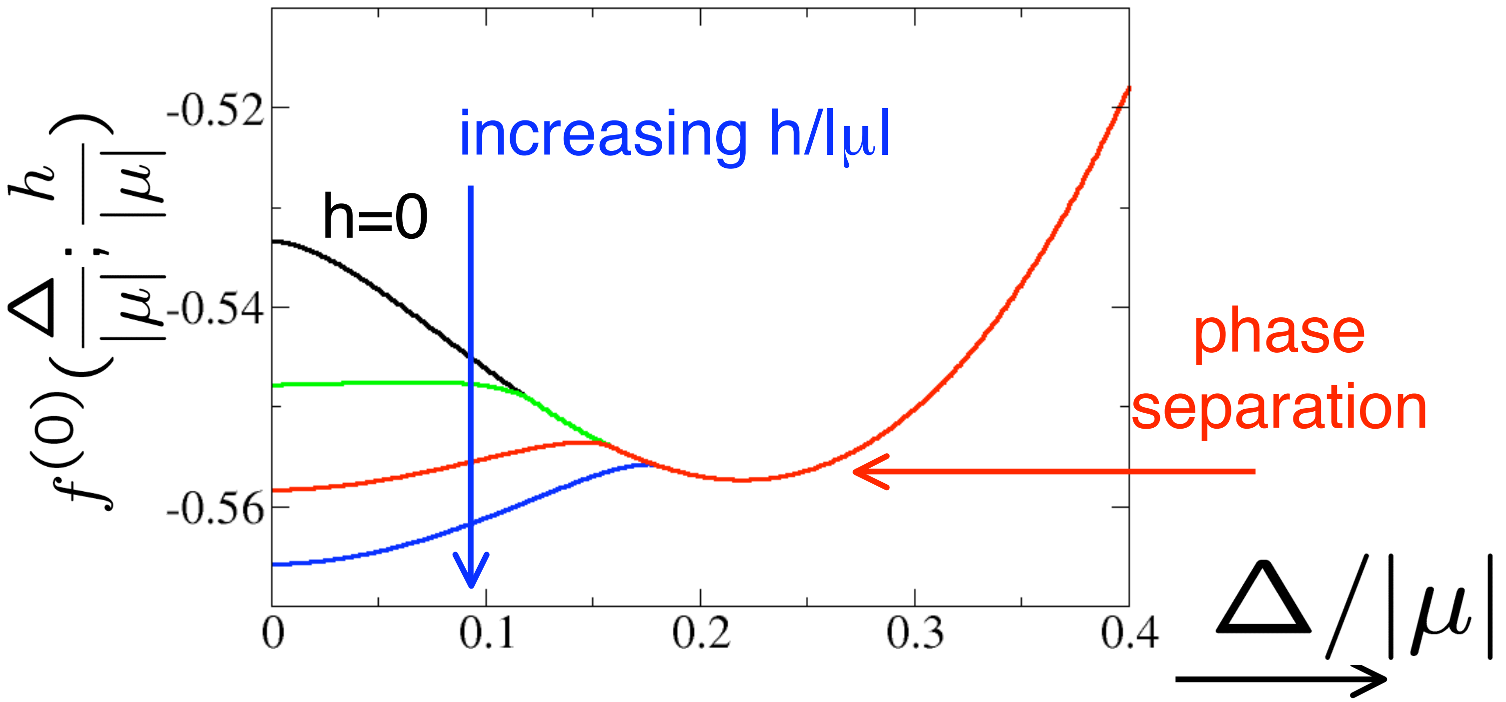
polarisation

1st Order Phase Transition

- $\frac{1}{k_F a} < \left(\frac{1}{k_F a}\right)_{\text{tricrit}}$
  and $\frac{T}{\varepsilon_F} < \left(\frac{T}{\varepsilon_F}\right)_{\text{tricrit}}$



(oil&water)



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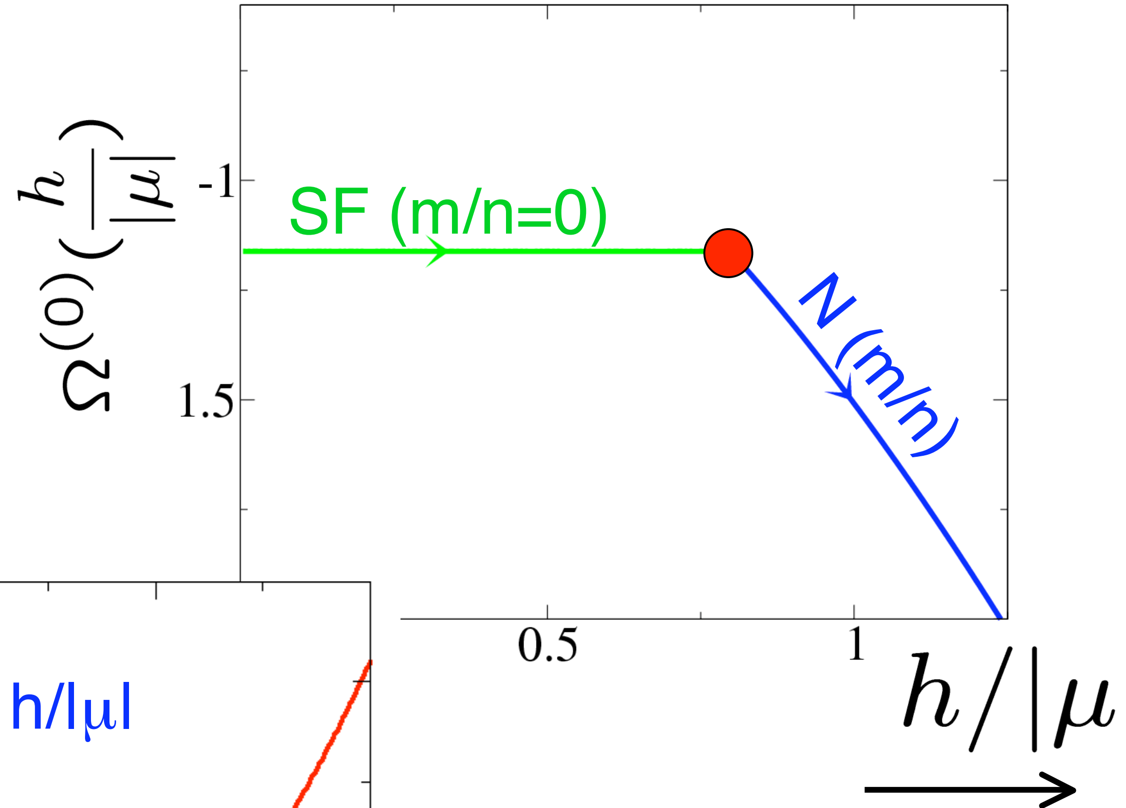
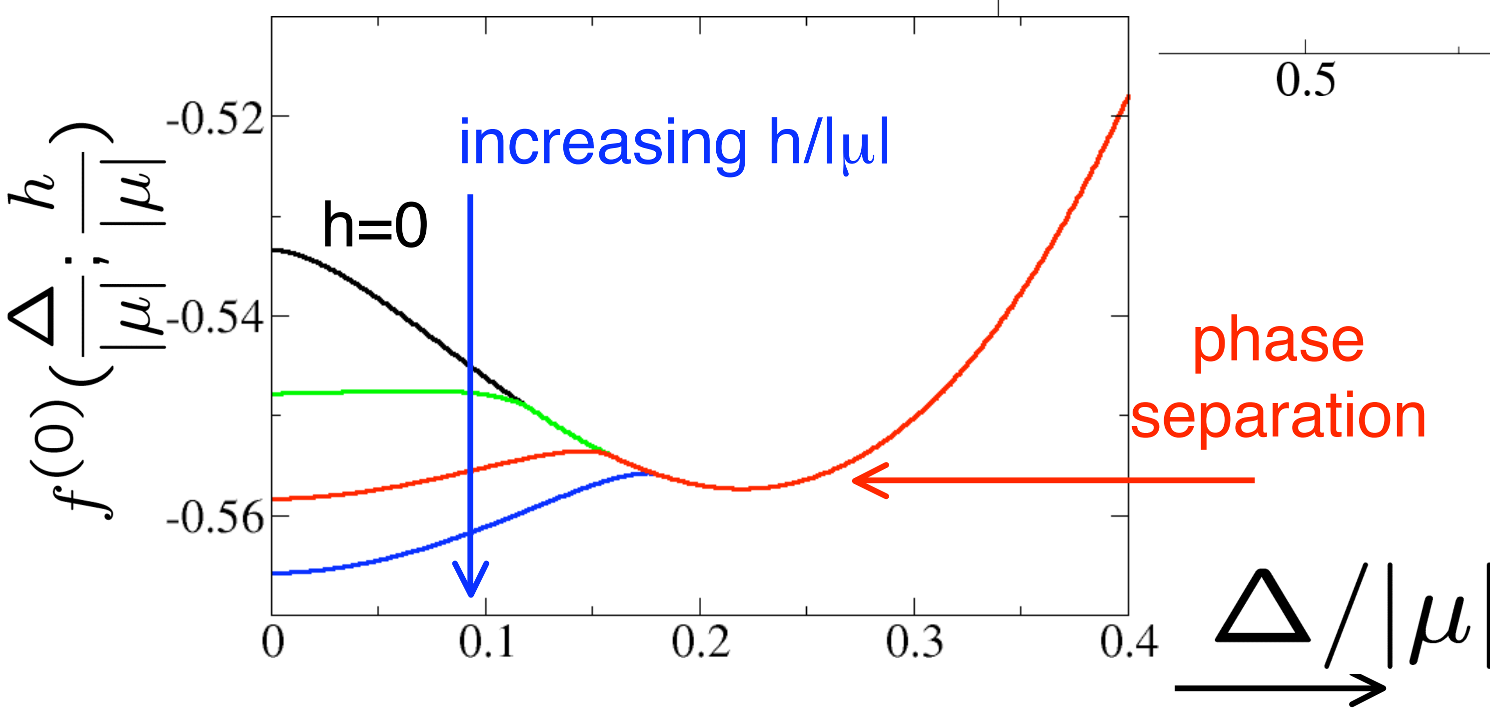
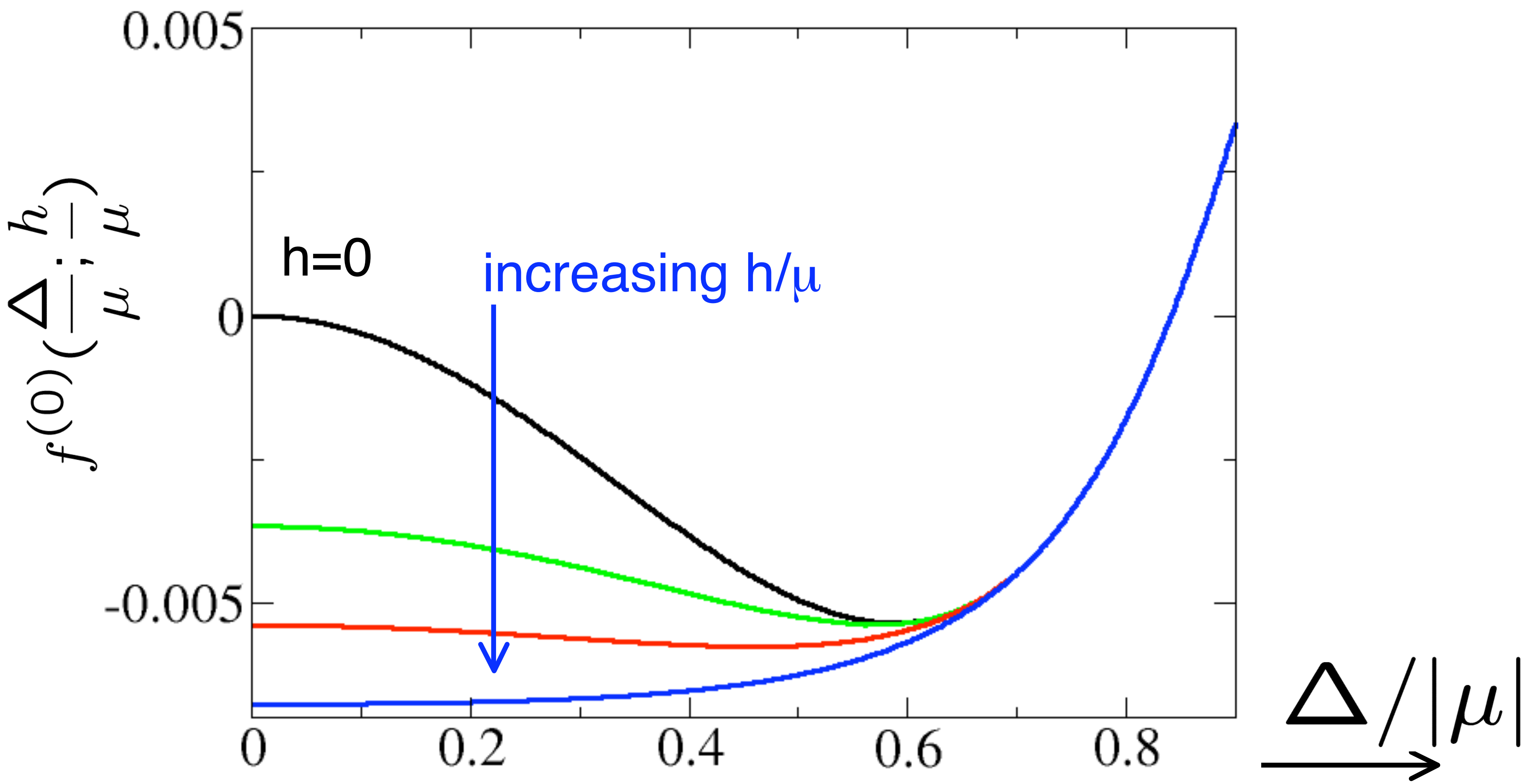




2nd Order Phase Transition

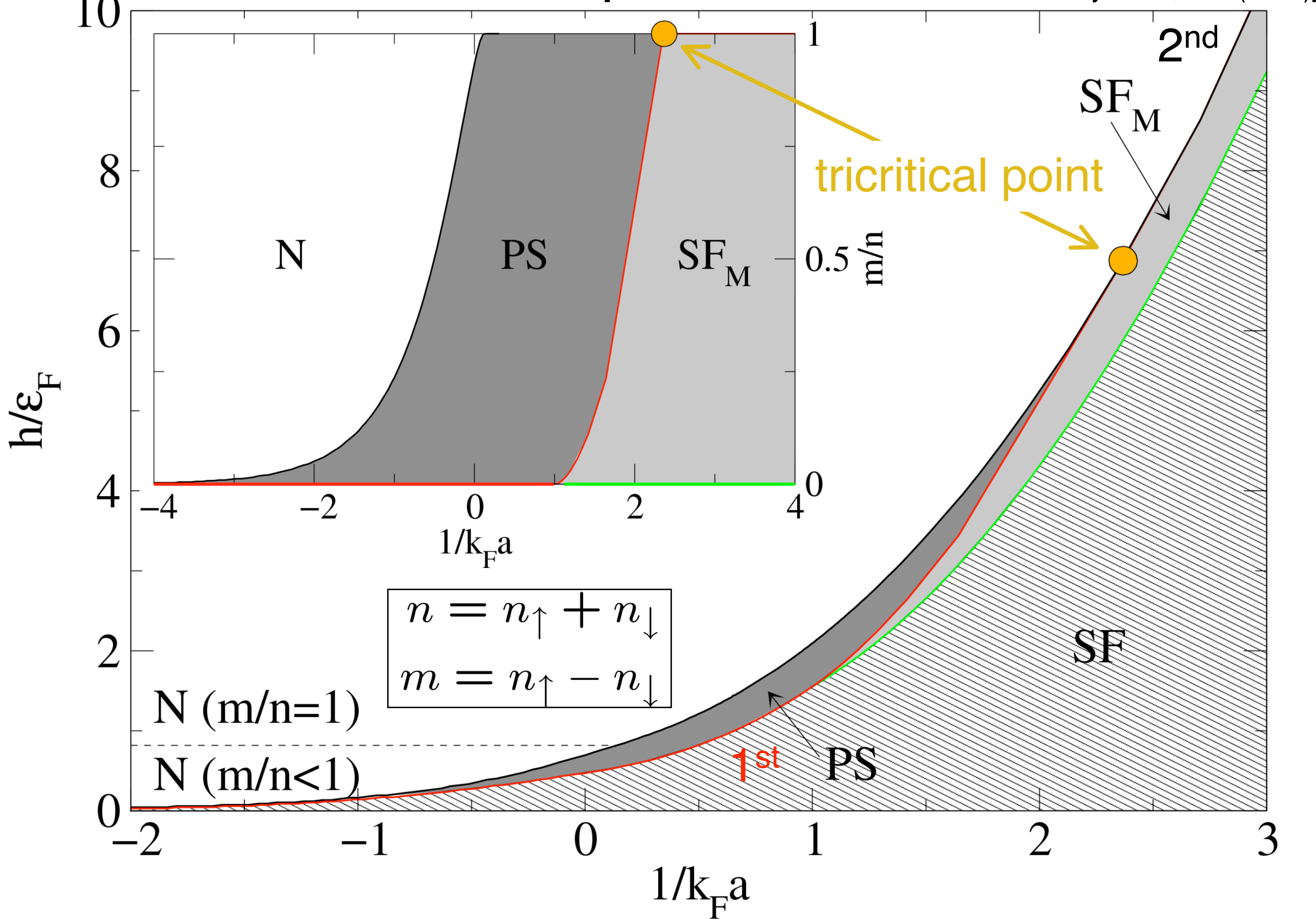
- $$\frac{1}{k_F a} > \left(\frac{1}{k_F a}\right)_{\text{tricrit}}$$

  or $$\frac{T}{\varepsilon_F} > \left(\frac{T}{\varepsilon_F}\right)_{\text{tricrit}}$$



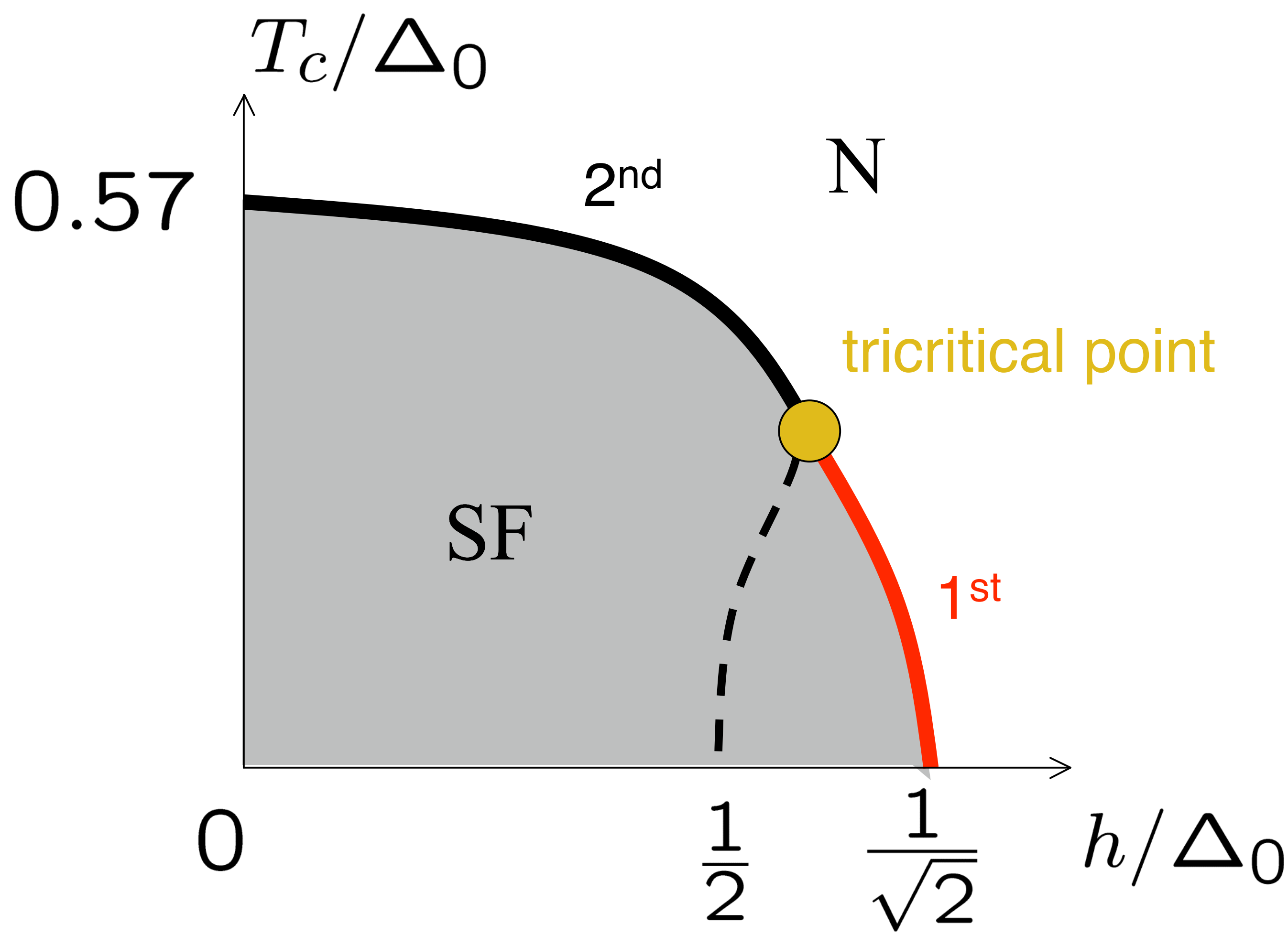
T=0 Phase Diagram

[M. Parish, F.M. Marchetti *et al.*, *Nature Physics* **3**, 124 (2007)]



Finite T: BCS Limit

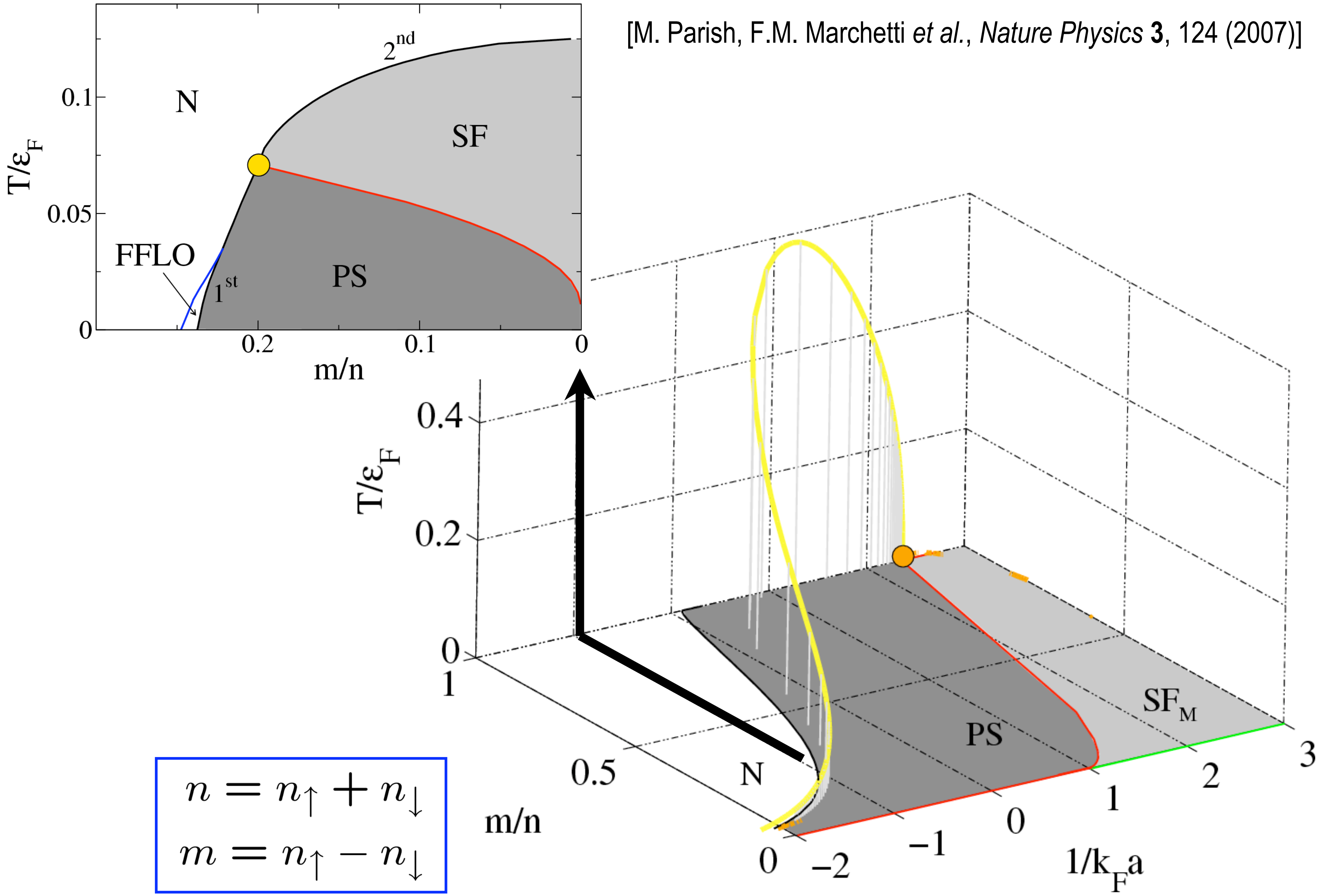
[G. Sarma, J. Phys. Chem. Solids **24** 1029 (1963)]



- Tricritical point: both at T=0 & at finite T

Finite T Phase Diagram

[M. Parish, F.M. Marchetti *et al.*, *Nature Physics* **3**, 124 (2007)]



$$n = n_\uparrow + n_\downarrow$$
$$m = n_\uparrow - n_\downarrow$$

Adding Pair Fluctuations (finite T)

[P. Nozieres & S. Schmitt-Rink, J. Low temp. Phys. **59**, 195 (1985)]

- One loop correction to mean-field T_c ($\Delta = 0$)

$$\Omega(\mu, h) = \Omega^{(0)}(\mu, h) + \Omega^{(1)}(\mu, h)$$

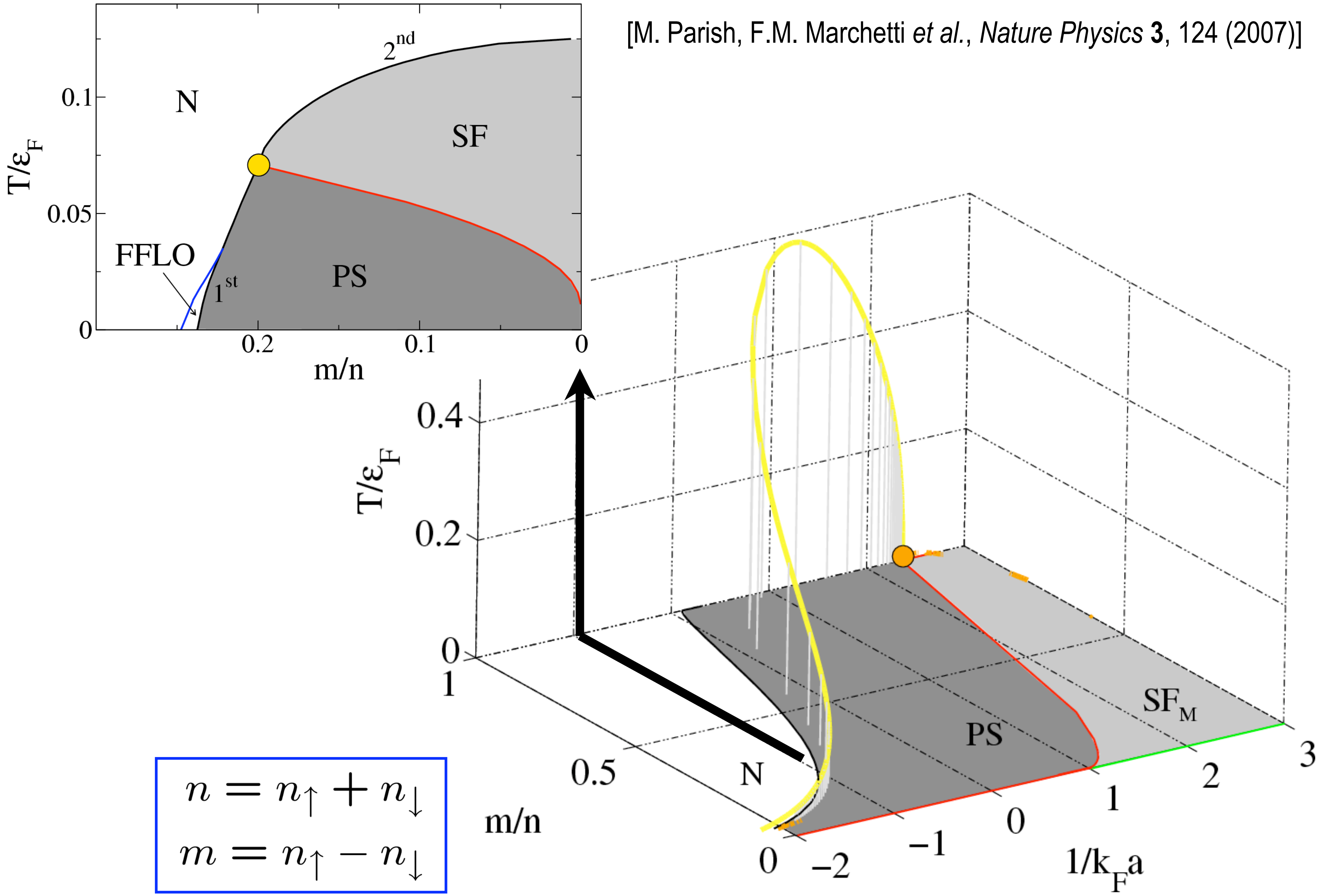
$$n = -\frac{\partial \Omega}{\partial \mu} = n^{(0)} + n^{(1)}$$
$$m = -\frac{\partial \Omega}{\partial h} = m^{(0)} + m^{(1)}$$

condensed pairs + qp's

thermal pairs

Finite T Phase Diagram

[M. Parish, F.M. Marchetti *et al.*, *Nature Physics* **3**, 124 (2007)]



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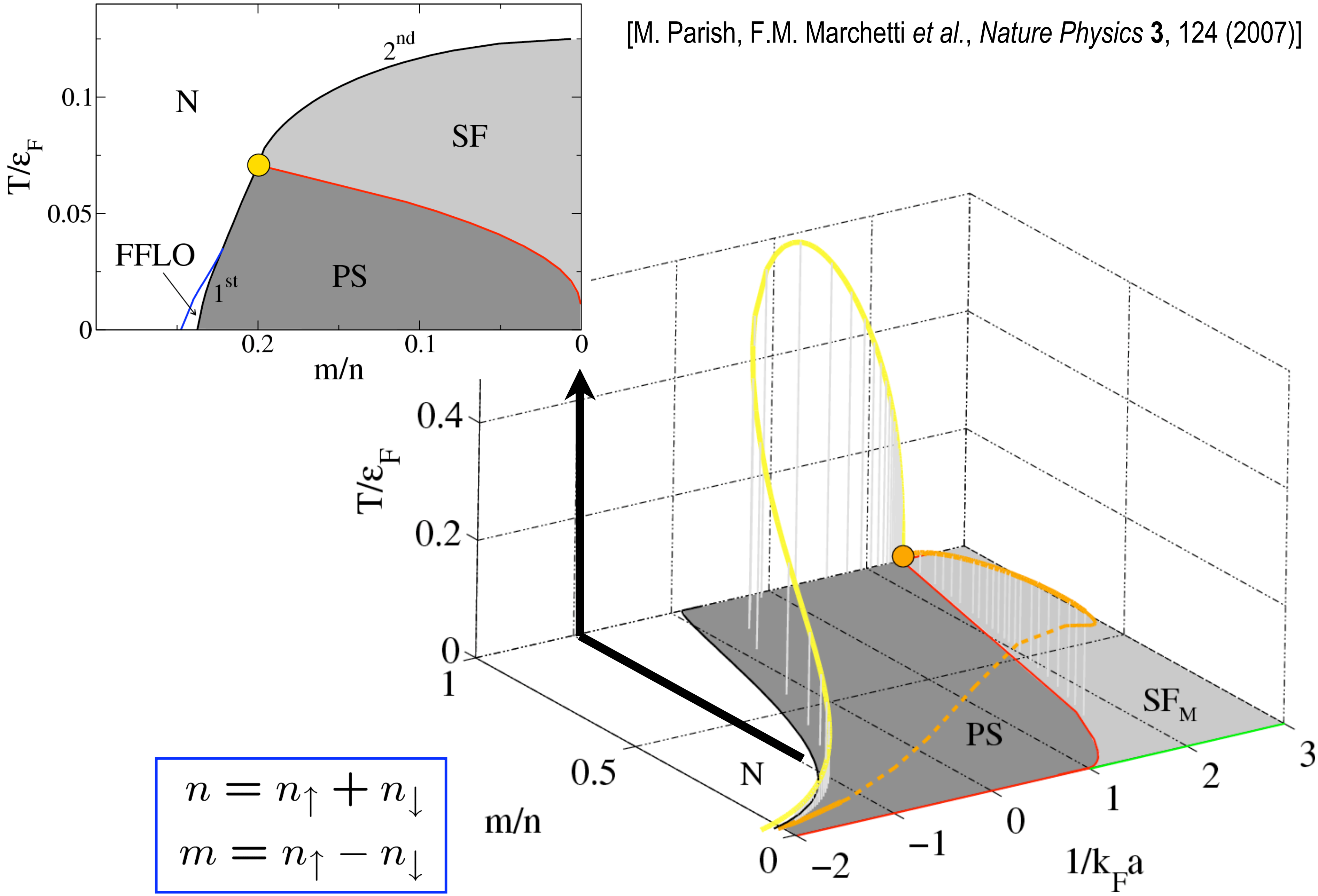
Finite T Phase Diagram

[M. Parish, F.M. Marchetti *et al.*, *Nature Physics* **3**, 124 (2007)]

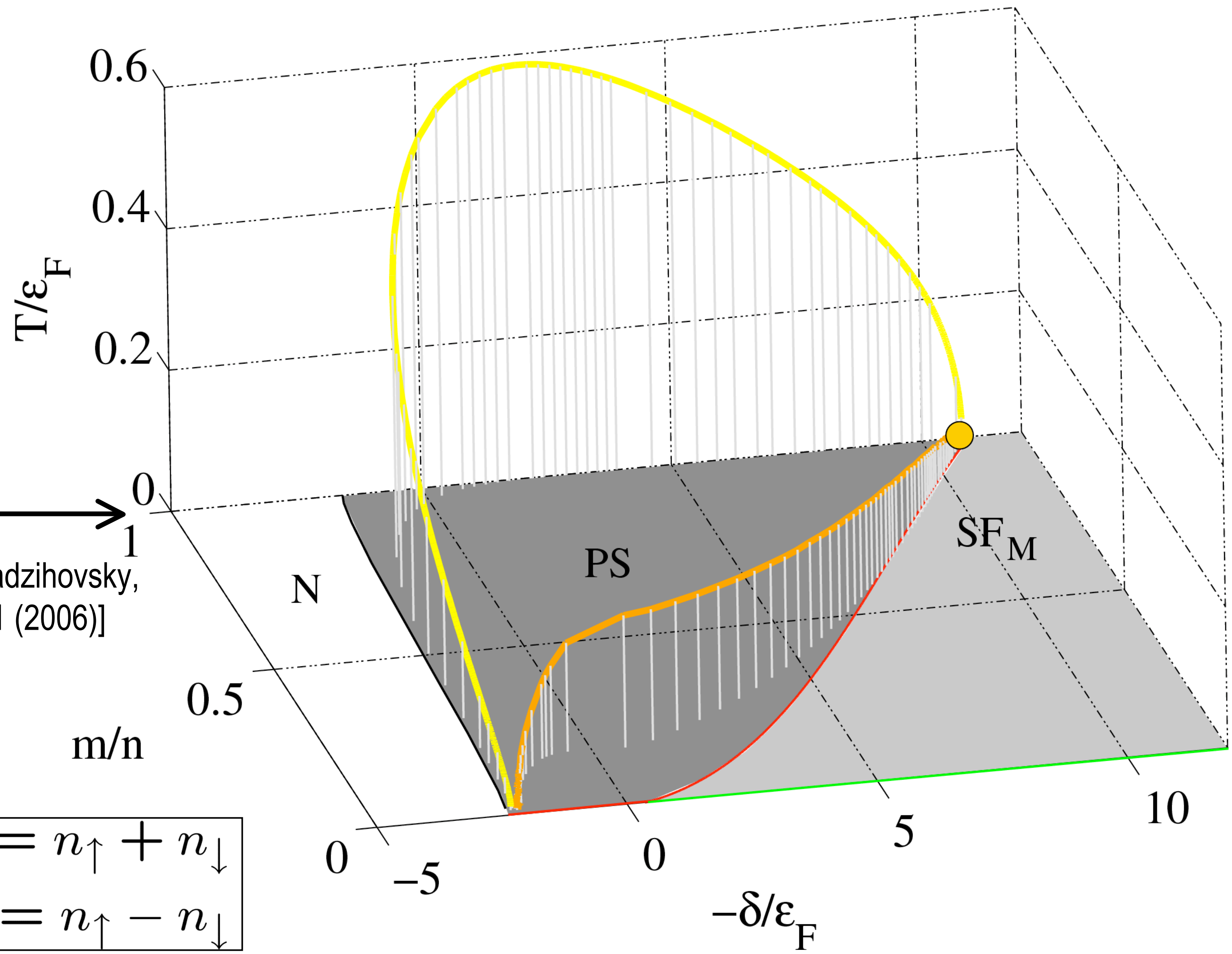


$$n = n_\uparrow + n_\downarrow$$
$$m = n_\uparrow - n_\downarrow$$

1- vs. 2-Channel Model

[A. Andreev *et al*., *PRL* **93**, 130402 (2004)]

[M. Parish, F.M. Marchetti *et al*., *Nature Physics* **3**, 124 (2007)]

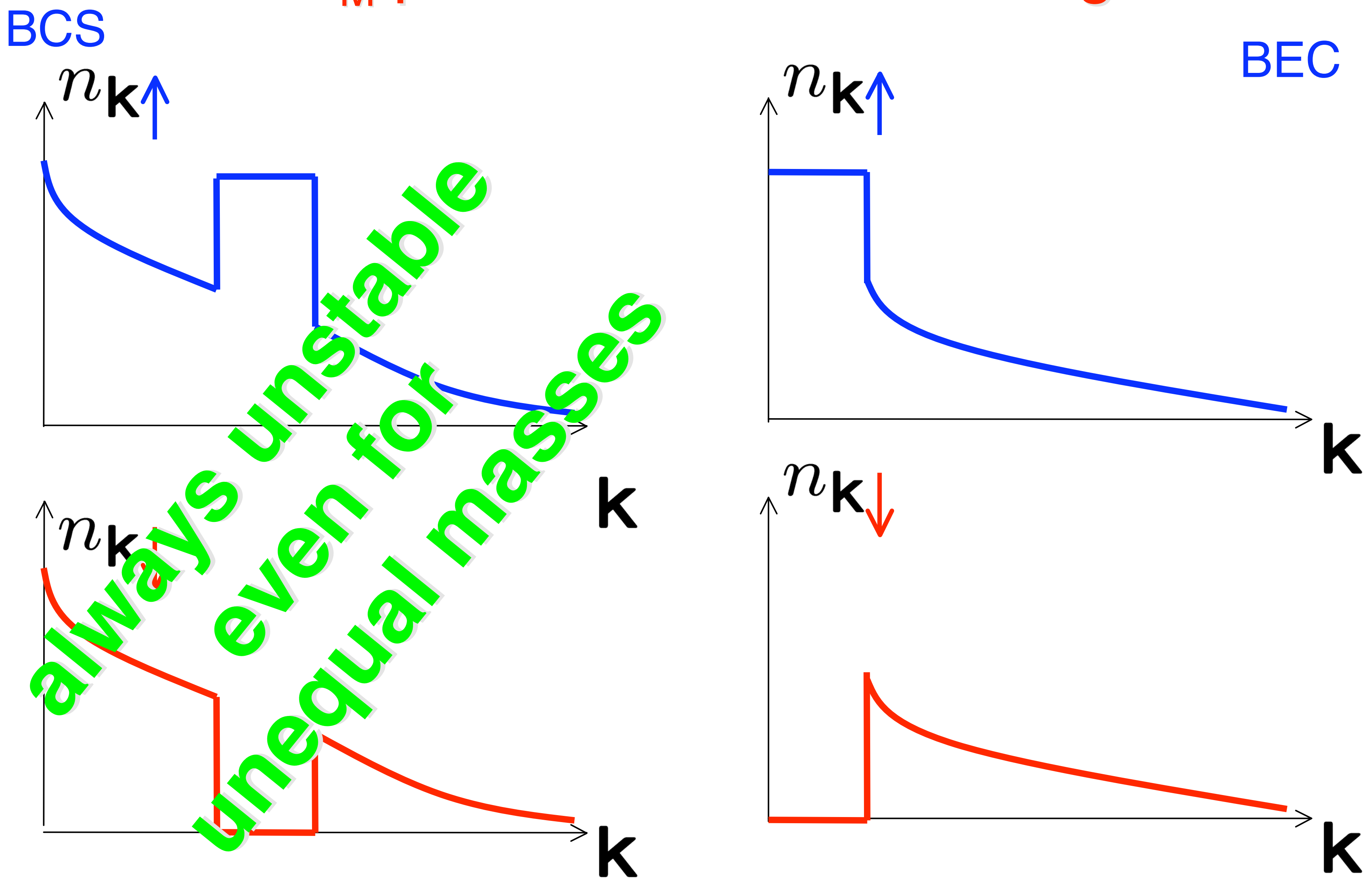


T=0 →

D. Sheehy & L. Radzihovsky, *PRL* **96**, 060401 (2006)]

$$n = n_\uparrow + n_\downarrow$$
$$m = n_\uparrow - n_\downarrow$$

SF_M phase: Breached Pairing

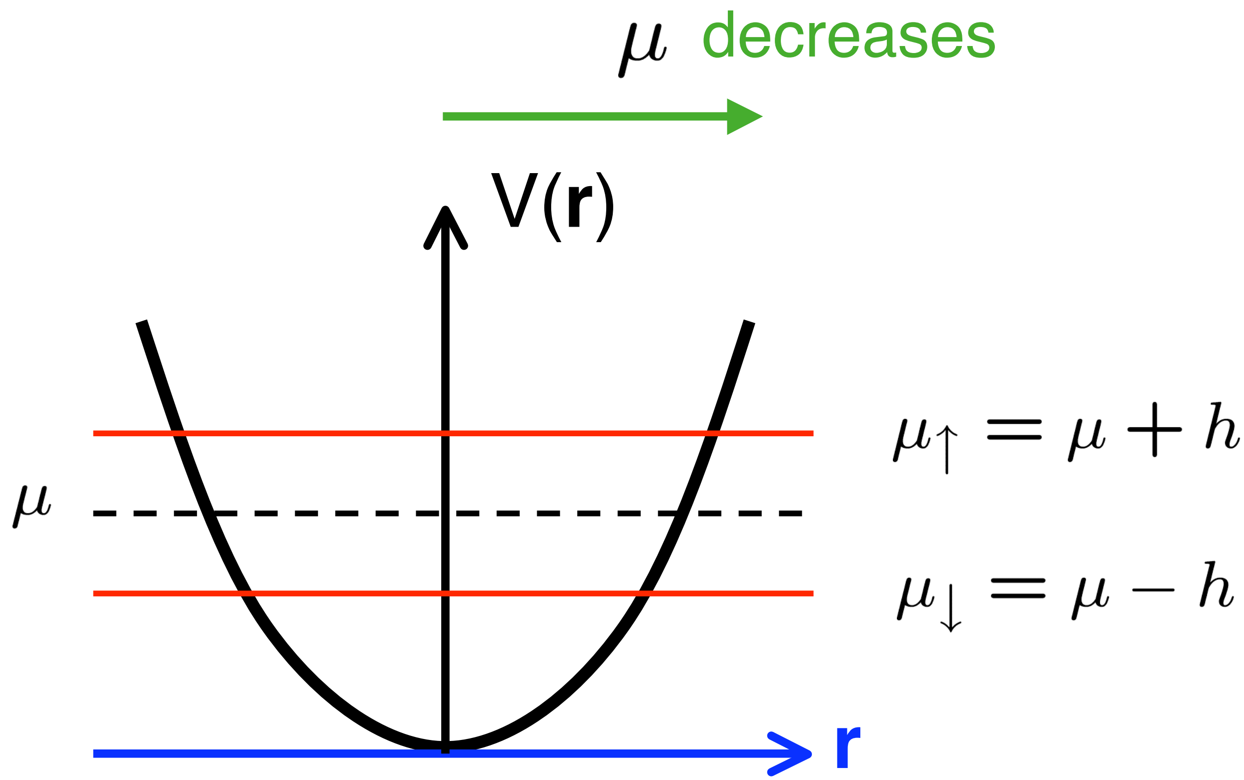


[W. V. Liu & F. Wilczek, PRL **90** 047002 (2003)]

[M. Parish, F.M. Marchetti *et al.*, *Nature Physics* **3**, 124 (2007)]
[M. Parish, F.M. Marchetti *et al.*, *PRL* **98**, 160402 (2007)]

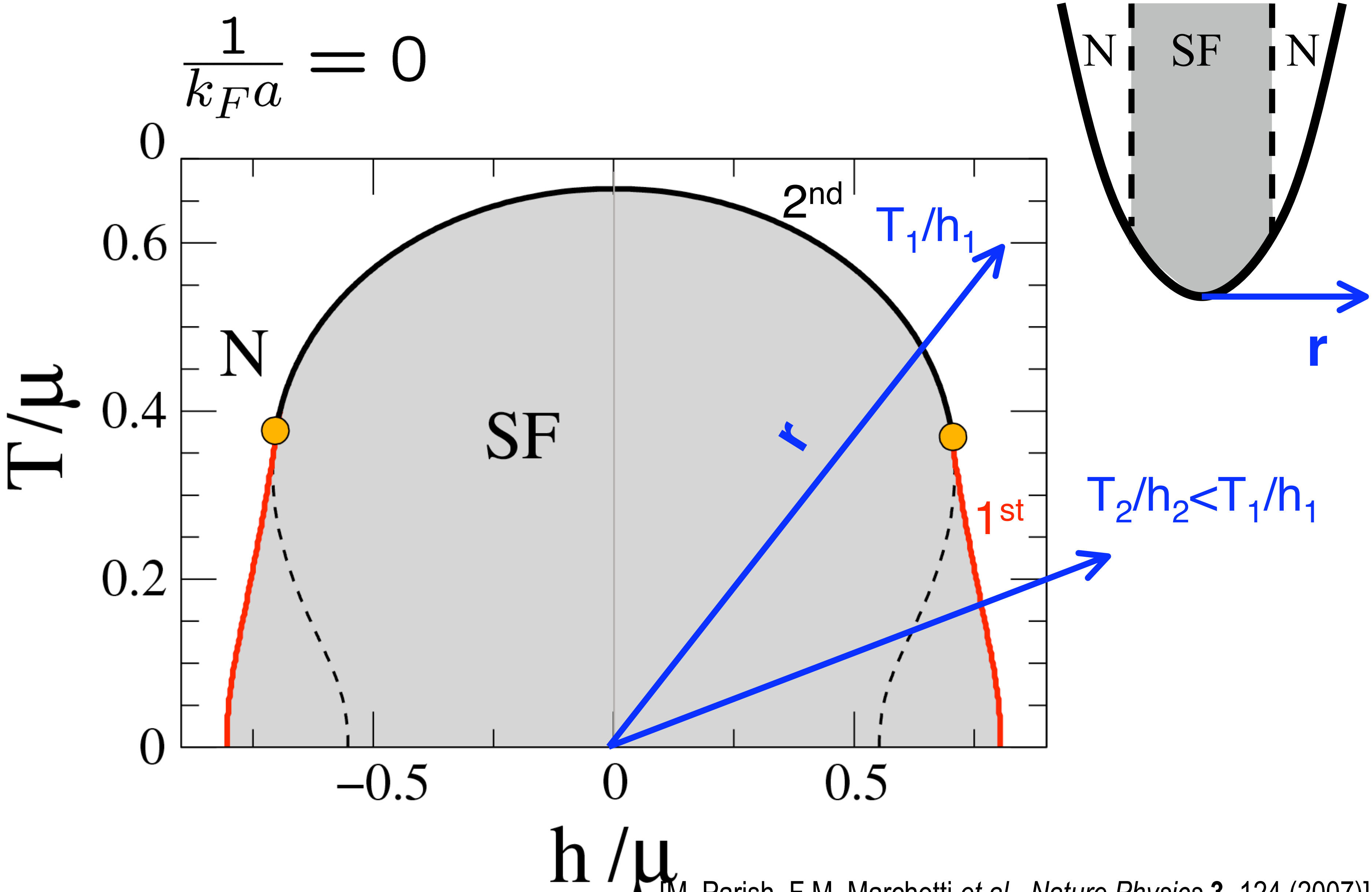
Trapped Fermi Gases

- LDA $\quad \mu_{\uparrow,\downarrow}(\mathbf{r}) = \mu_{\uparrow,\downarrow} - V(\mathbf{r})$



Phase Diagram for Trapped Gases

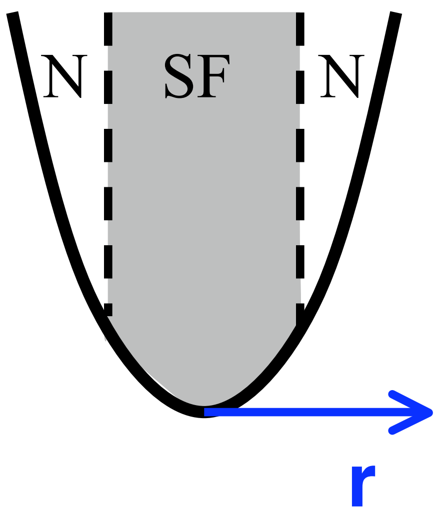
$$\frac{1}{k_F a} = 0$$



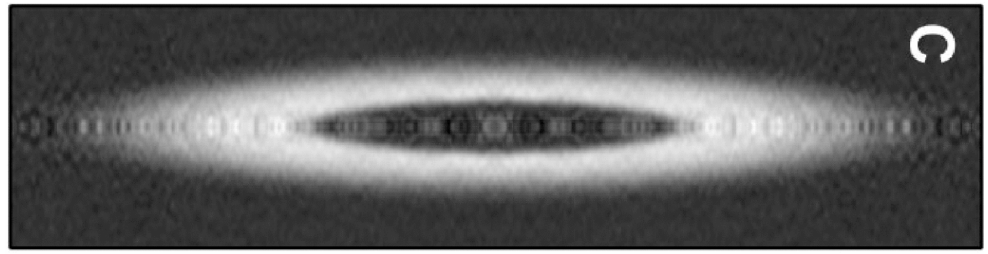
[M. Parish, F.M. Marchetti *et al.*, *Nature Physics* **3**, 124 (2007)]

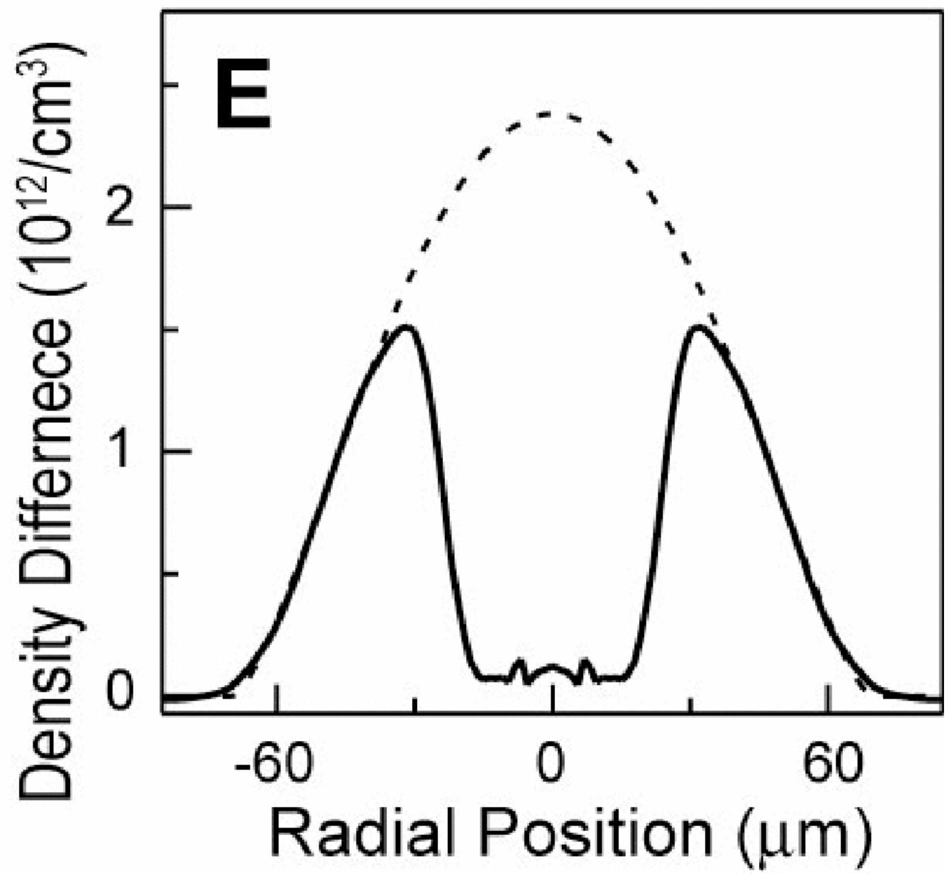
Experiments on Imbalanced Fermi Clouds

- In-situ imaging of phase separation (3D density distribution $n_{\uparrow,\downarrow}(\mathbf{r})$)



$$n_\uparrow(\mathbf{r}) - n_\downarrow(\mathbf{r})$$



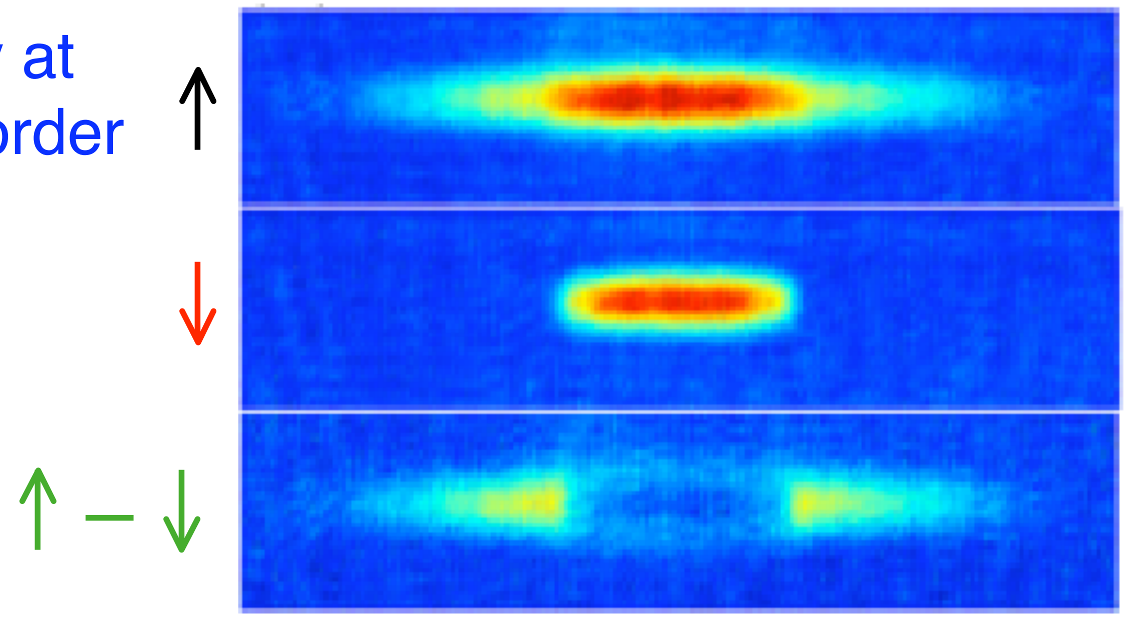


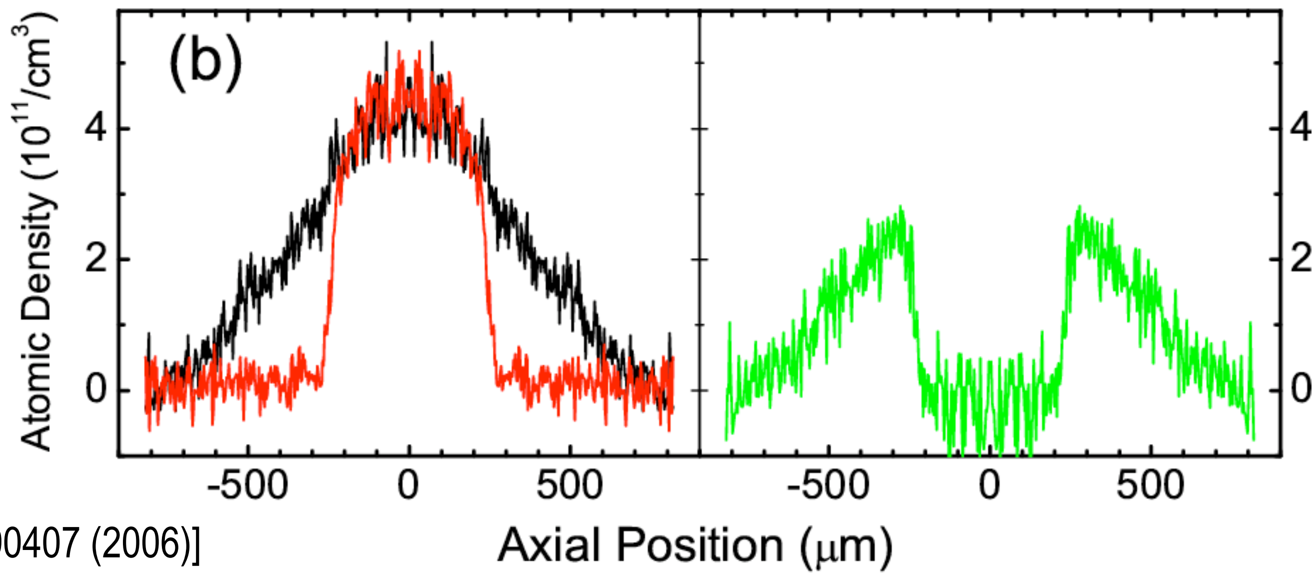
[Y. Shin *et al.*, PRL **97**, 030401 (2006)]

Experiments on Imbalanced Fermi Clouds

- Sharp phase boundary at low temperatures (1^{st} order transition)

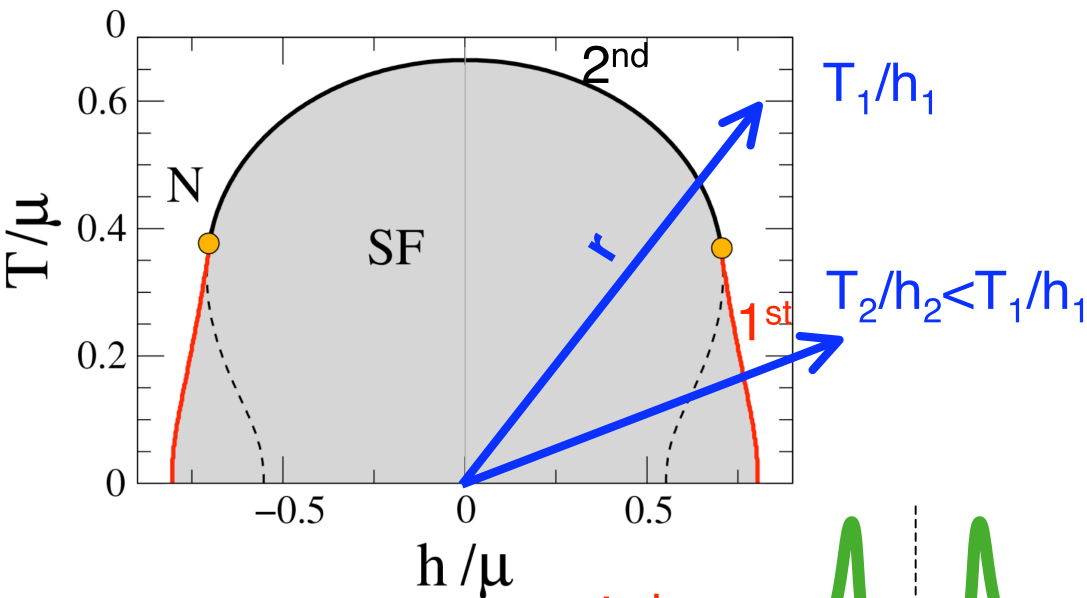
$T < 0.05\ T_F$
$m/n = 0.35$



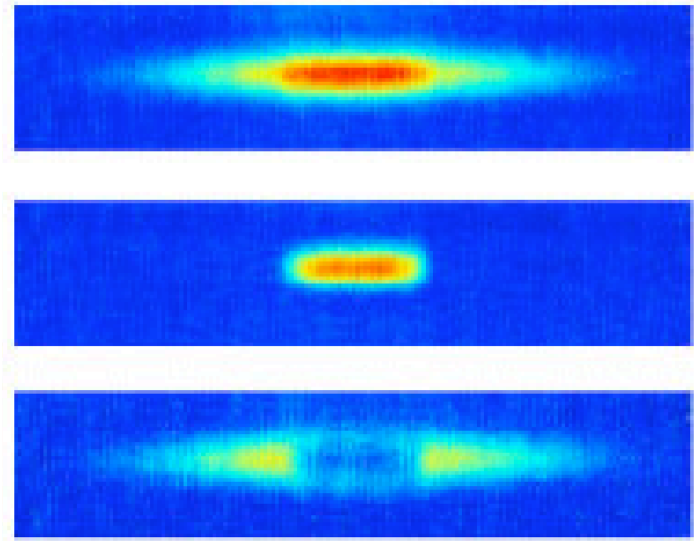


[G. B. Partridge *et al*., PRL **97**, 190407 (2006)]

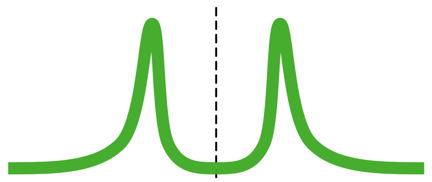
Temperature Dependence of Phase Separation



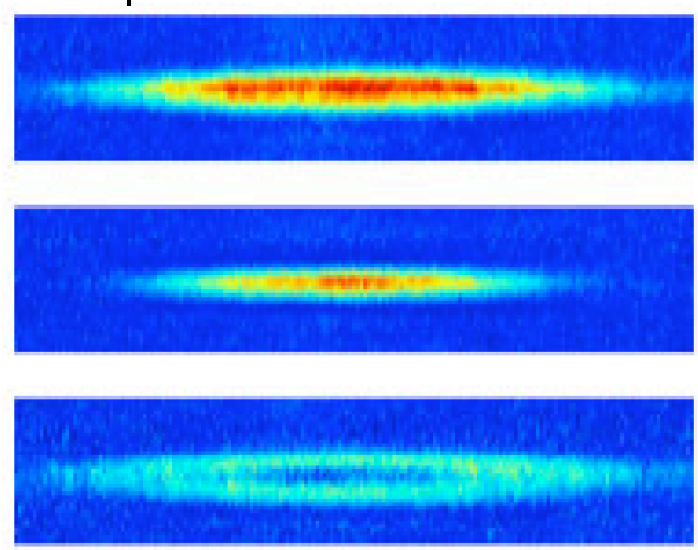
T/T$_F$<0.05



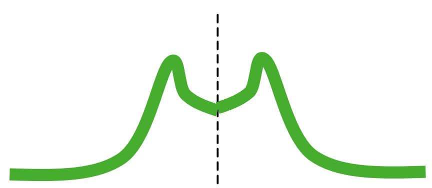
1st



T/T$_F$=0.2

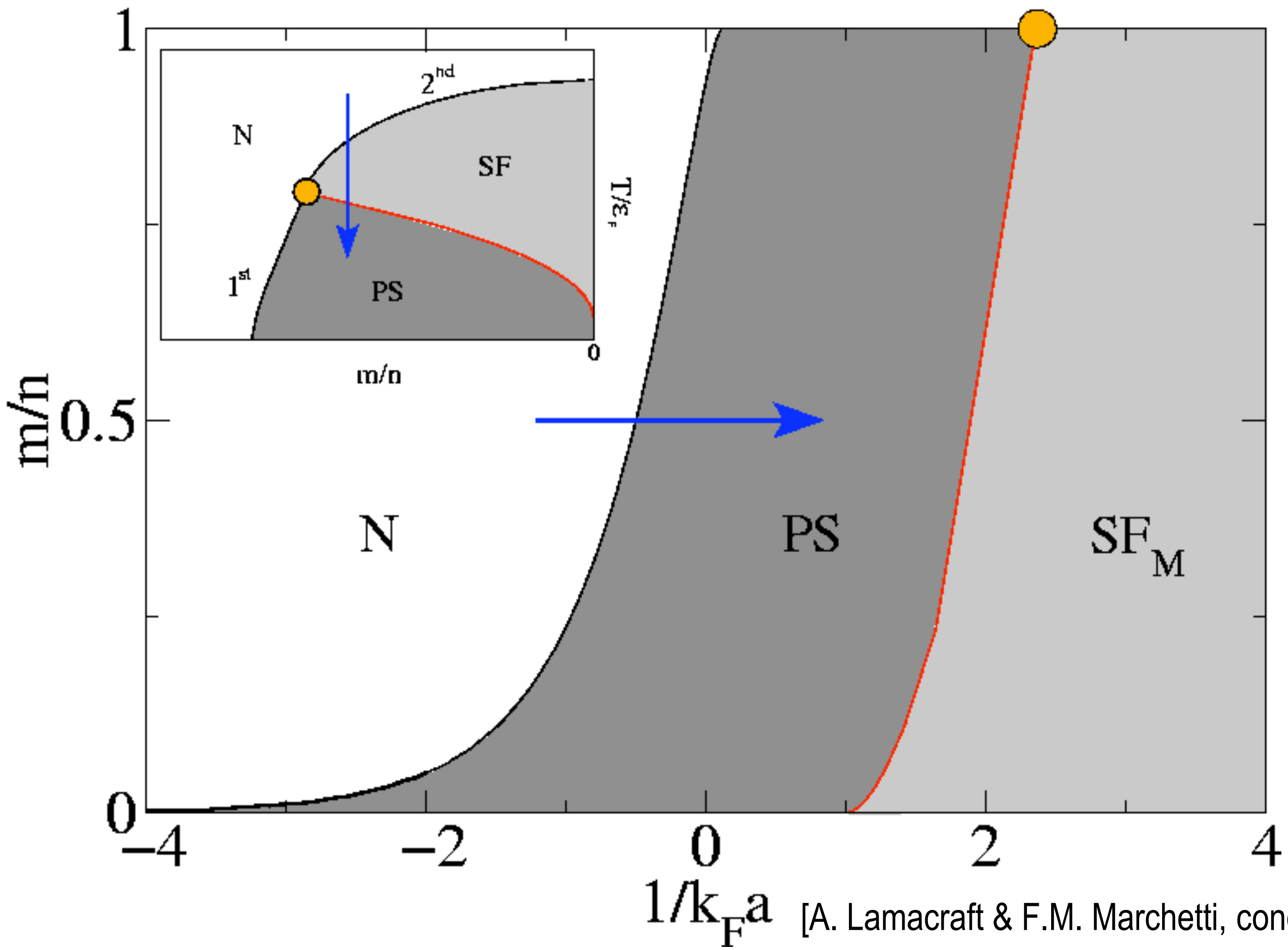


2nd



[G. B. Partridge *et al.*, PRL **97**, 190407 (2006)]

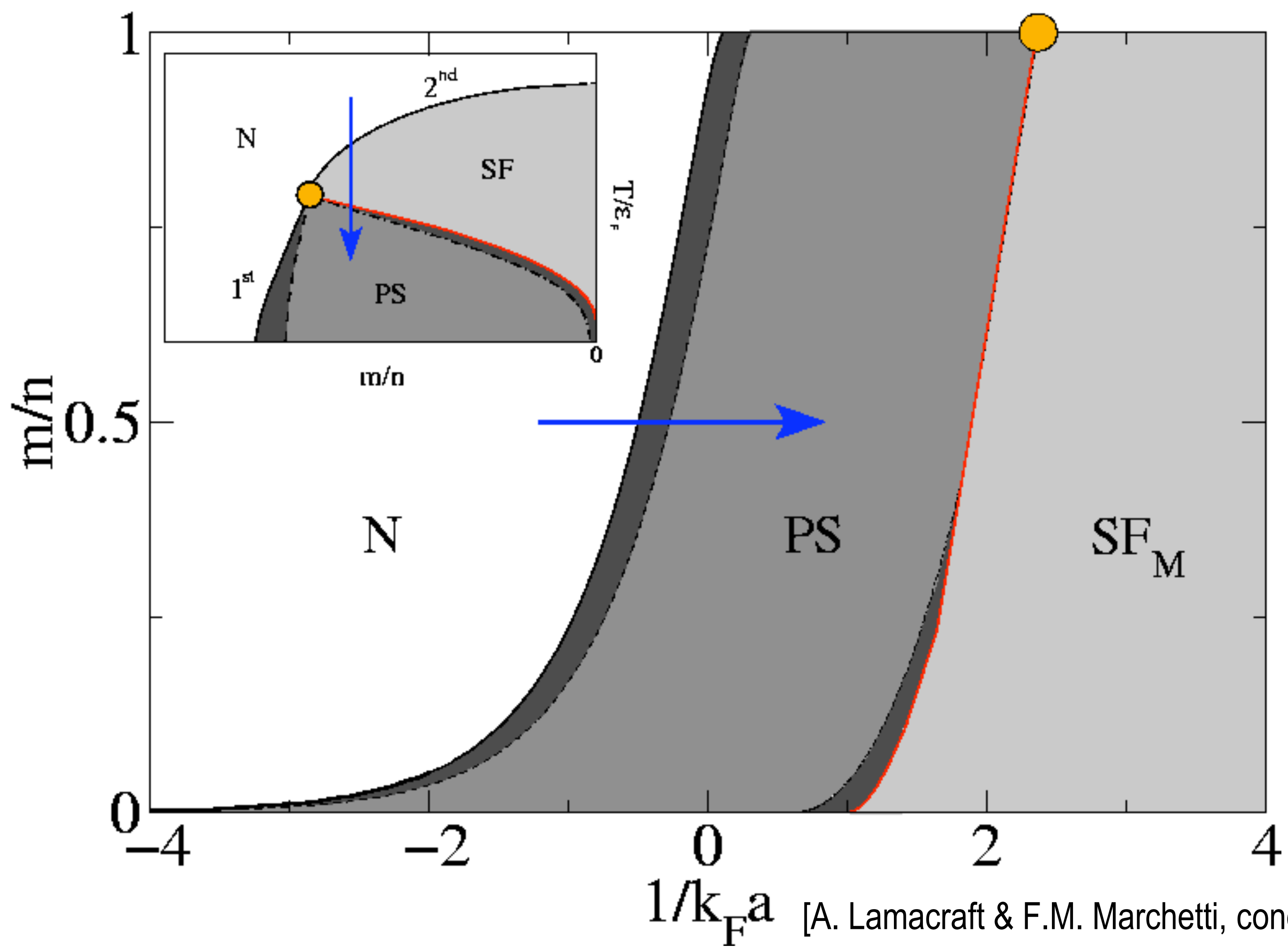
Dynamics of Phase Separation



[A. Lamacraft & F.M. Marchetti, cond-mat/0701692]

Dynamics of Phase Separation

- Spinodal: phase separation <u>starts</u> via a linear instability

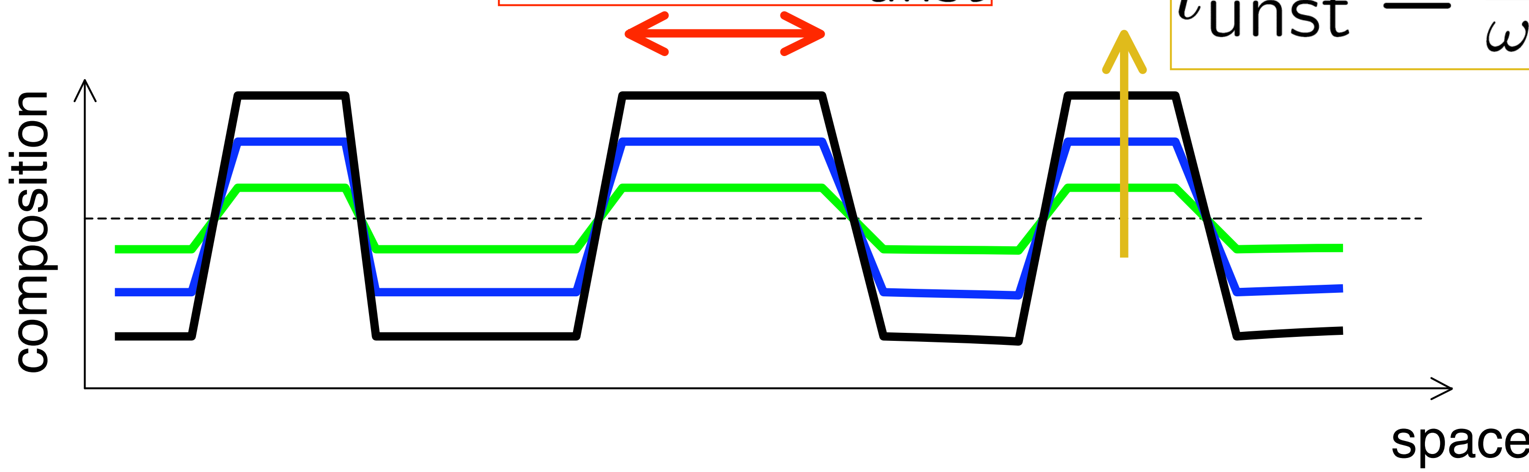


[A. Lamacraft & F.M. Marchetti, cond-mat/0701692]

Spinodal Decomposition

- Early time dynamics

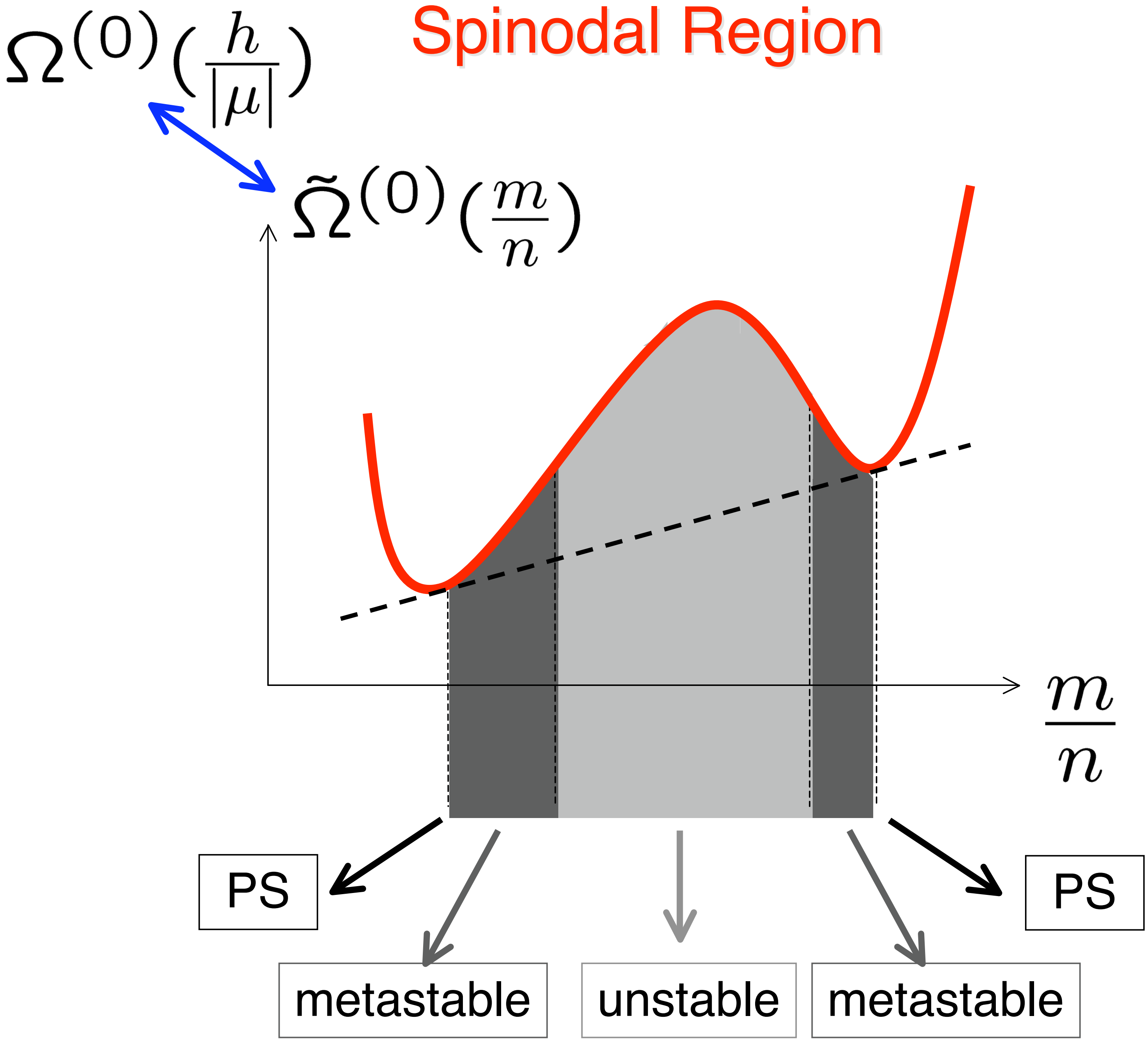
$$\ell_{\text{unst}} = \frac{1}{\mathbf{q}_{\text{unst}}} \qquad t_{\text{unst}} = \frac{1}{\omega_{\text{unst}}}$$



- We have to find:

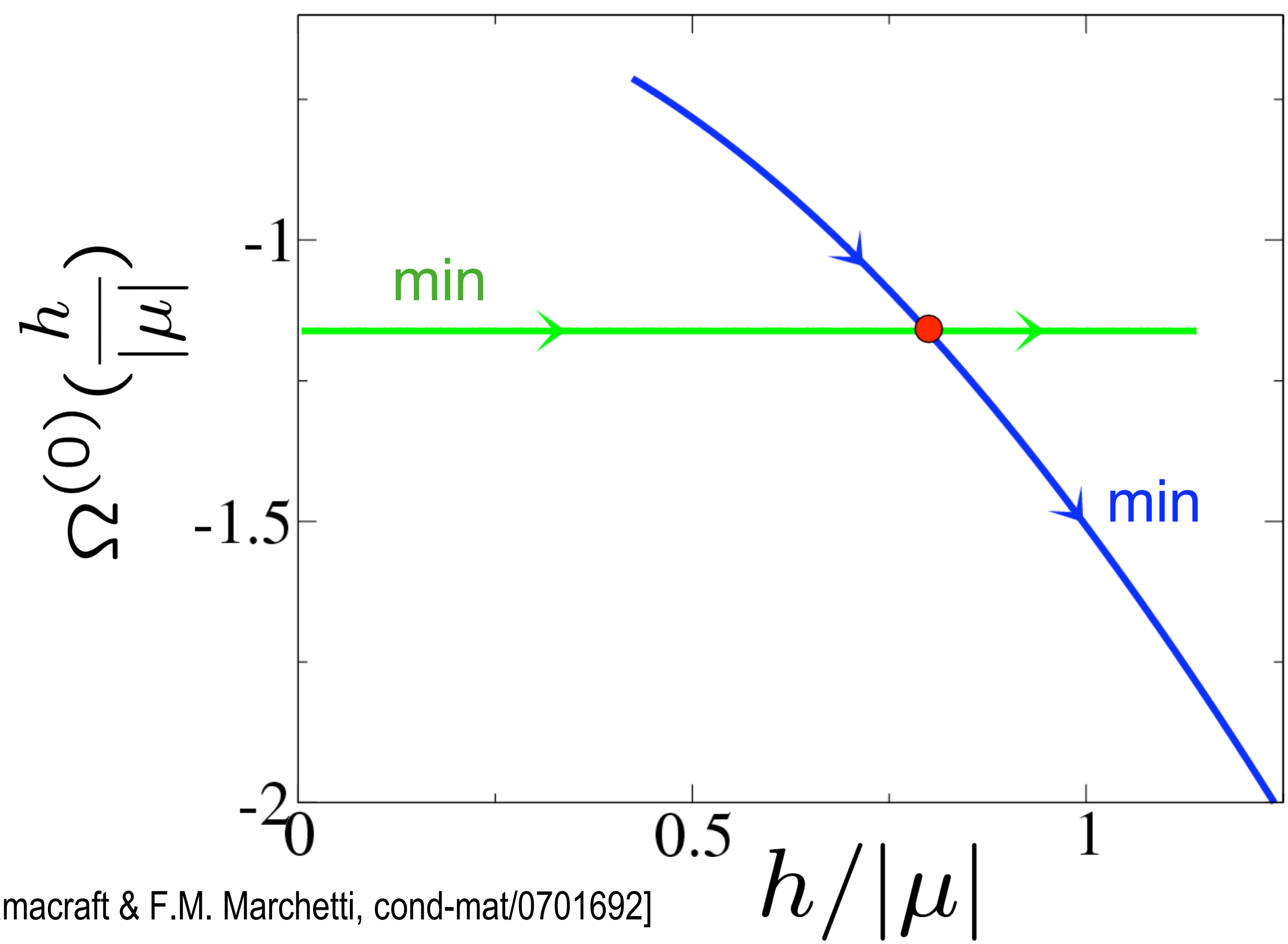
1. Spinodal region
2. Unstable density modes
3. Most unstable mode → $\mathbf{q}_{\text{unst}}, \ \omega_{\text{unst}}$

Spinodal Region



Spinodal Region

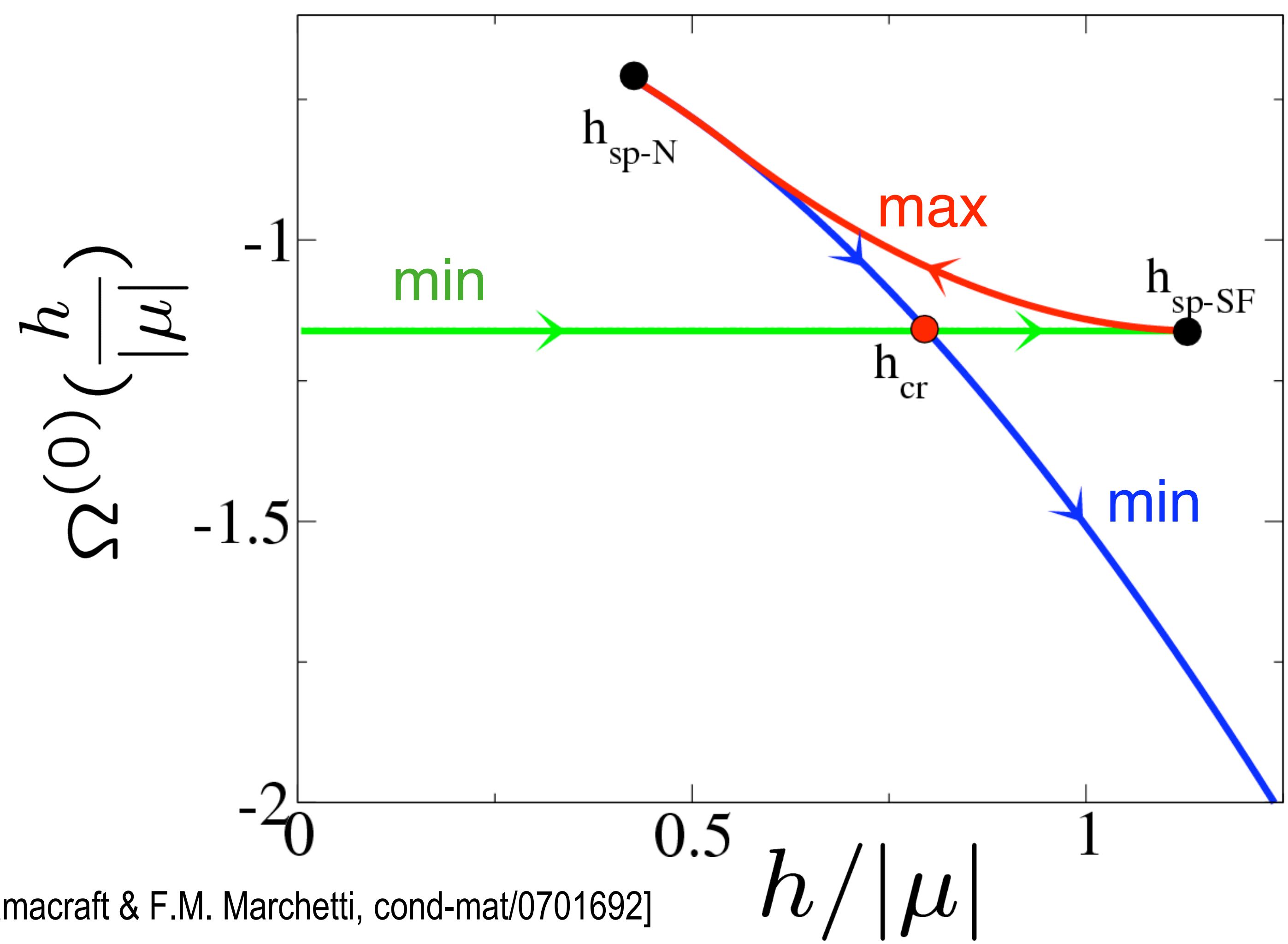
$$\partial^2\tilde{\Omega}^{(0)}(\tfrac{m}{n}) < 0 \quad \Leftrightarrow \quad \partial^2\Omega^{(0)}(\tfrac{h}{|\mu|}) > 0$$



[A. Lamacraft & F.M. Marchetti, cond-mat/0701692]

Spinodal Region

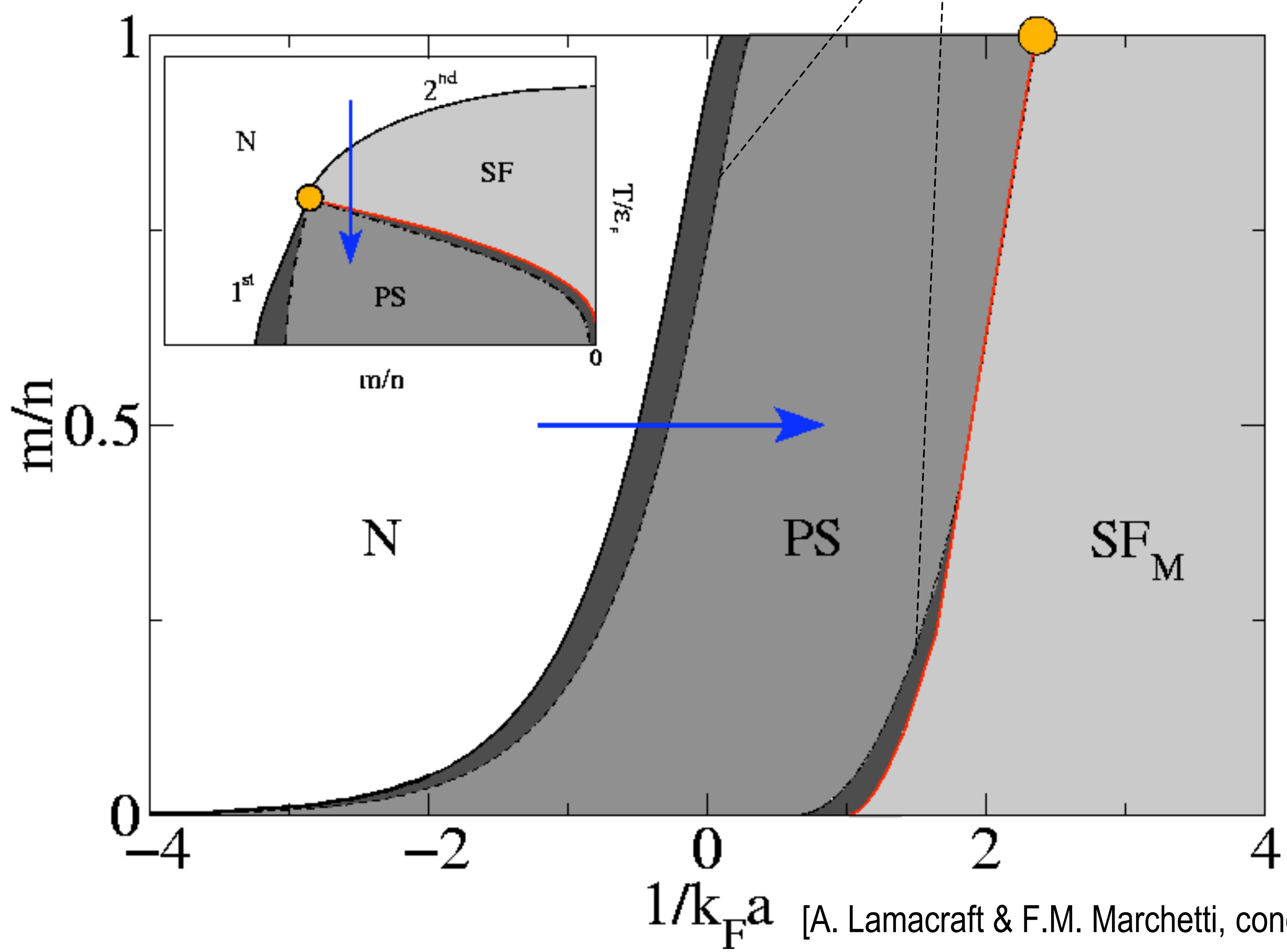
$$\partial^2 \tilde{\Omega}^{(0)}\left(\frac{m}{n}\right) < 0 \quad \Leftrightarrow \quad \partial^2 \Omega^{(0)}\left(\frac{h}{|\mu|}\right) > 0$$



[A. Lamacraft & F.M. Marchetti, cond-mat/0701692]

Spinodal Region

$$\partial^2_\Delta f^{(0)}(\Delta;\mu,h) = 0$$



[A. Lamacraft & F.M. Marchetti, cond-mat/0701692]

Unstable Modes

- Matrix response function ($\partial^2 \tilde{\Omega}^{(0)}(\frac{m}{n})$)

$$\det \widehat{\chi}^{-1}(\mathbf{q}, \varepsilon(\mathbf{q})) = 0$$

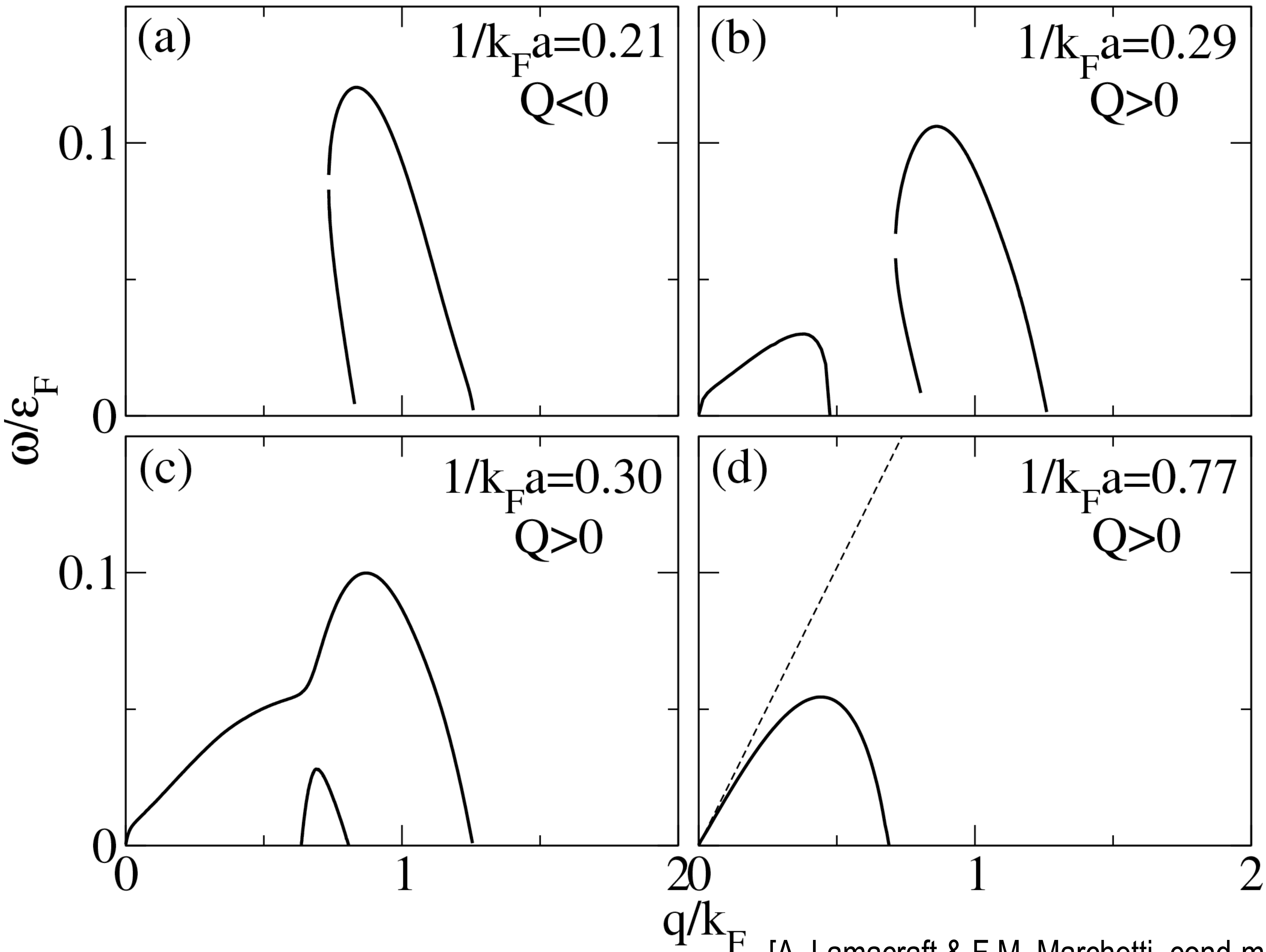
- Unstable modes $-i\varepsilon(\mathbf{q}) = \omega(\mathbf{q}) > 0$
  (exponentially growing)

- Sufficient condition

$$\det \widehat{\mathcal{D}}^{-1}(\mathbf{q}, \varepsilon(\mathbf{q})) = 0$$

(order parameter propagator)

Unstable Modes

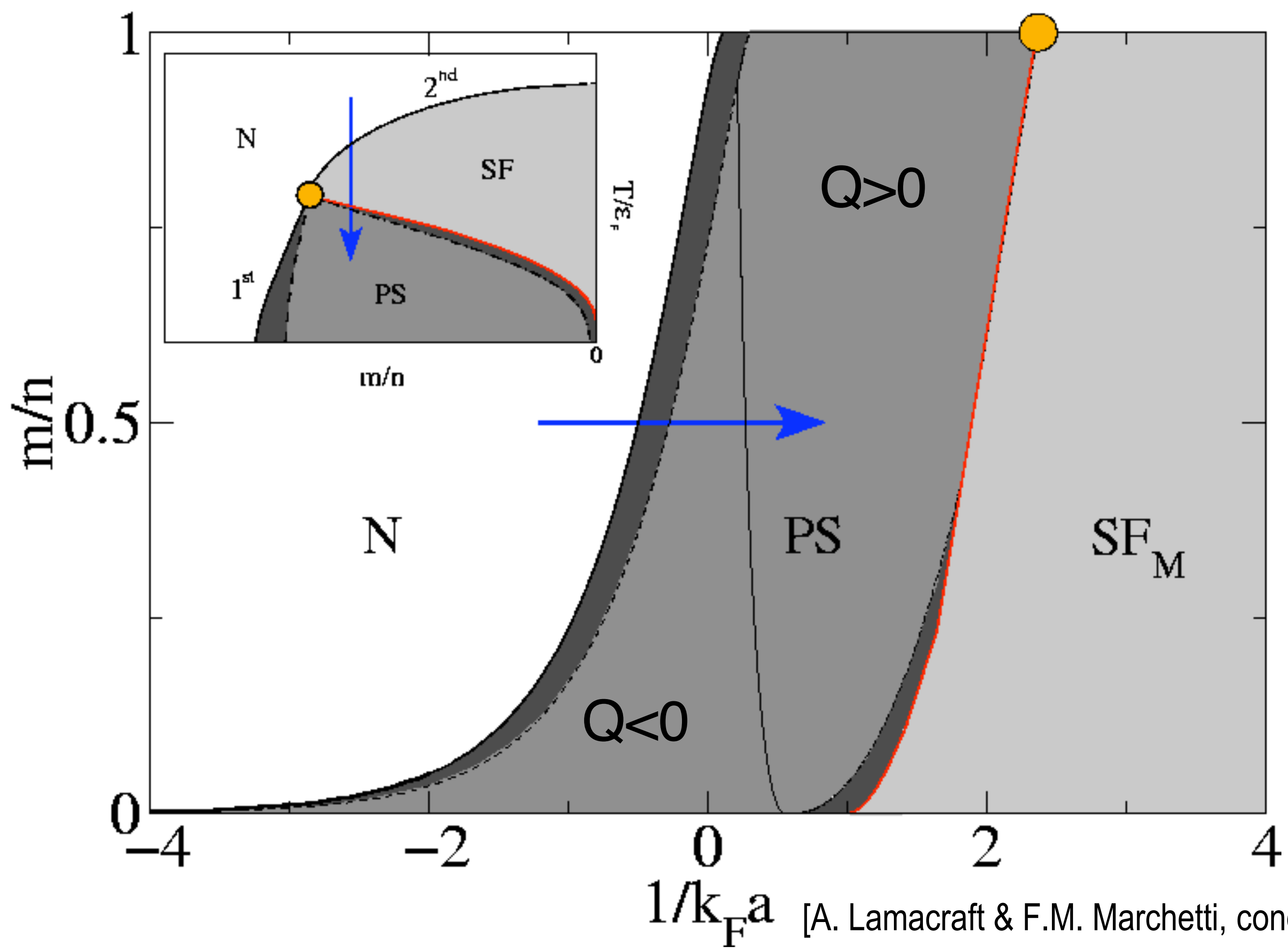


[A. Lamacraft & F.M. Marchetti, cond-mat/0701692]

Superfluid Stiffness

- Superfluid (phase) stiffness: Q> 0 or Q< 0

[C.-H. Pao *et al.*, *PRB* **73**, 132506 (2006)]

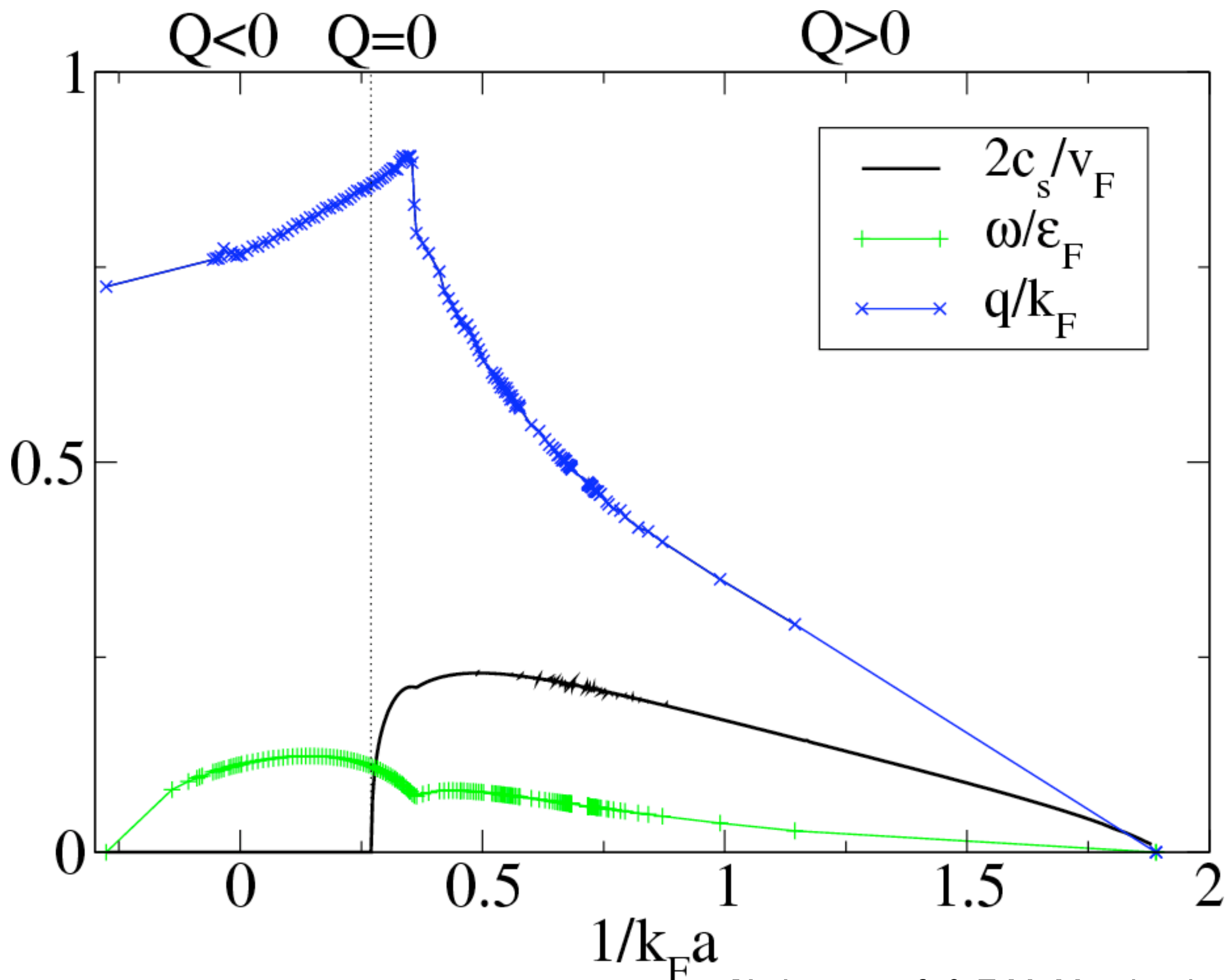


[A. Lamacraft & F.M. Marchetti, cond-mat/0701692]

Most Unstable Modes

- Characteristic length and time scales (for $T_F = 1\mu\text{K}$)

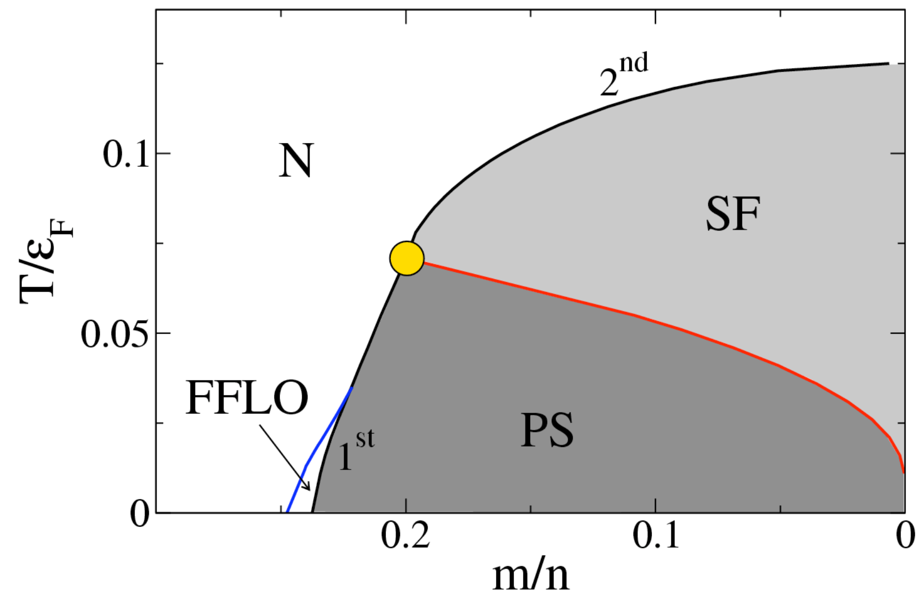
$$\ell_{\text{unst}} \simeq 0.1\mu\text{m} \qquad t_{\text{unst}} \simeq 400\mu\text{s}$$



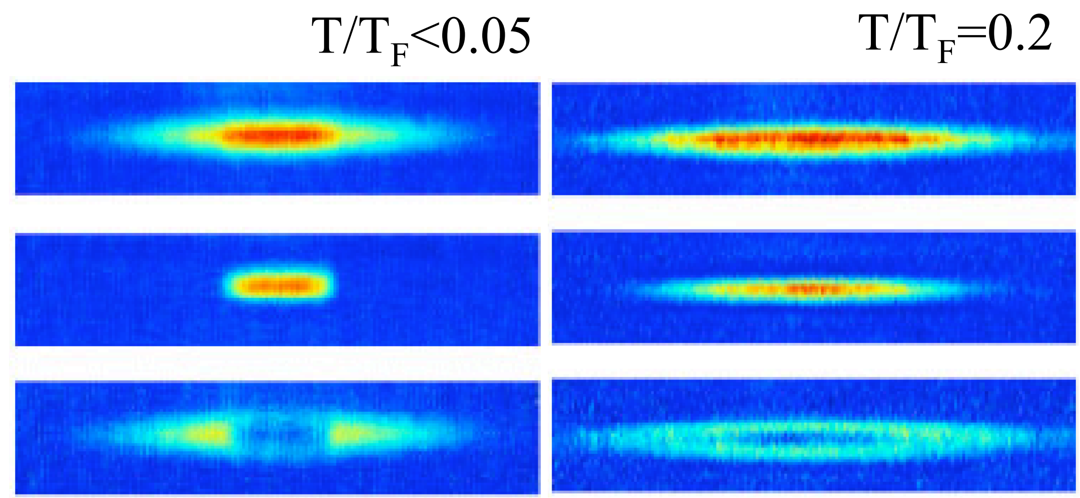
[A. Lamacraft & F.M. Marchetti, cond-mat/0701692]

Conclusions

1. Phase diagram of polarised Fermi superfluids



2. Probing the order of the transition in experiments



3. Dynamics of phase separation