Course in 3 lectures on

# Superfluidity in Ultracold Fermi Gases

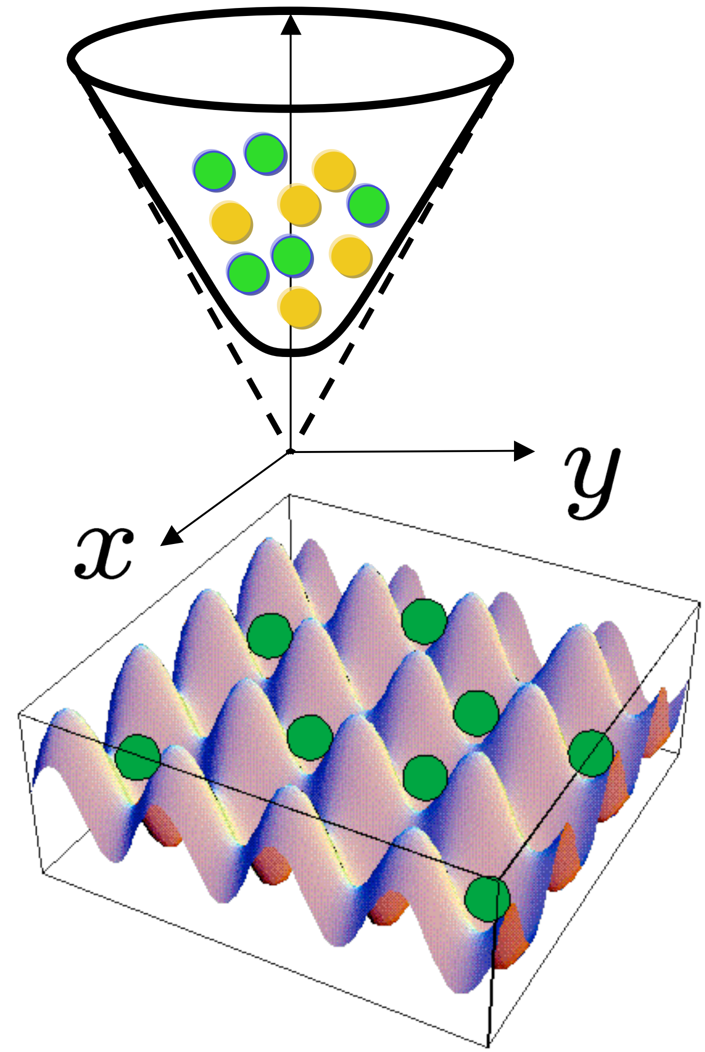
F.M. Marchetti

*Physics by the Lake*, Ambleside, 11, 12, 13 September 2007

# Motivation

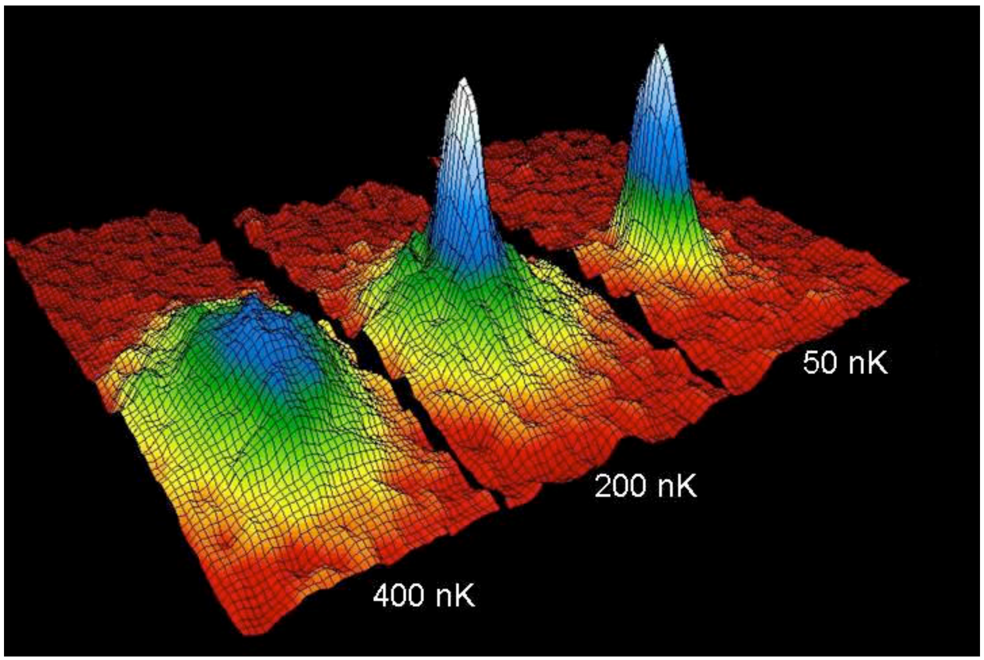
Search for novel phases of quantum coherent matter:
Why atomic gases?

- Tuning the interaction strength
- Mixtures of different statistics
- Optical lattices
- 1D, 2D
- External perturbations
- Versatile probing

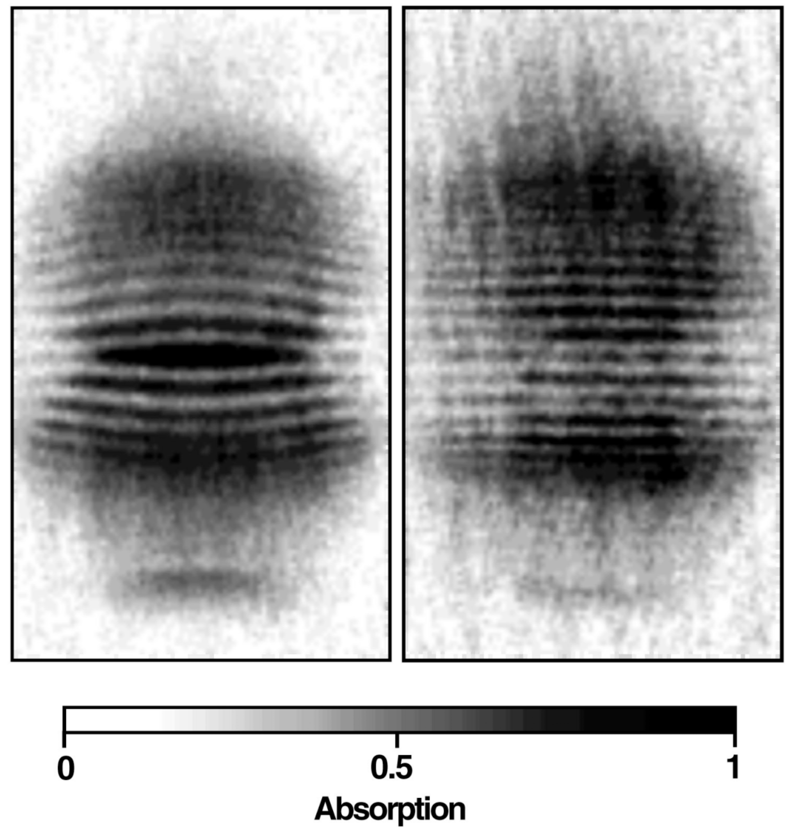


# Motivation

## Bosonic superfluids




[M. H. Anderson *et al*., Science **269**, 198 (1995)]




[M. R. Andrews *et al*., Science **275**, 637 (1997)]

# Motivation

## Bosonic superfluids

- Nobel Prize 2001

C. Wieman & E. Cornell
(JILA)

W. Ketterle
(MIT)

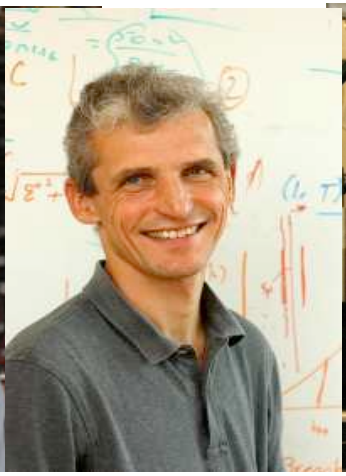
# Motivation

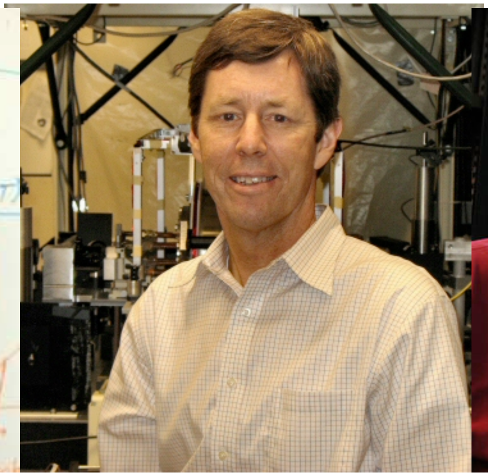
Fermionic superfluids

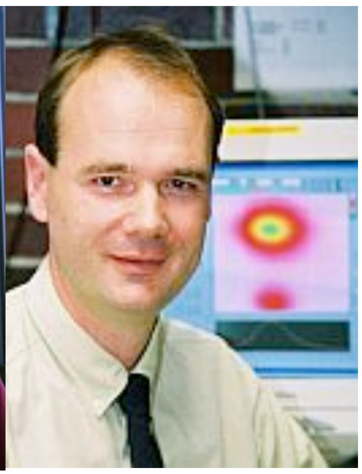
I. Weakly interacting Fermi gases

II. Feshbach resonances & BEC-BCS crossover

III. Polarised Fermi gases







D. Jin (JILA)

W. Ketterle (MIT)

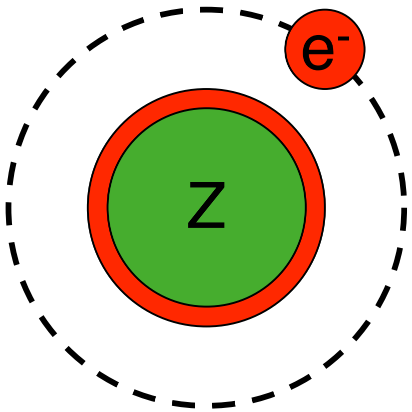
R. Hulet (Rice)

C. Salomon (ENS)

R. Grimm (Innsbruck)

# I. Weakly interacting Fermi gases

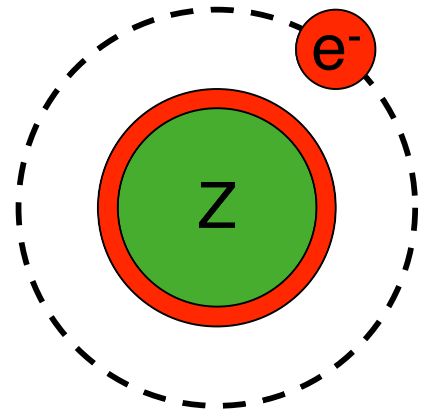
# Alkali atoms




- Electronic spin S=J=1/2, nuclear spin I
- Z odd, N determines the statistics
  - A=Z+N odd for bosons
  - even for fermions

| | | |
|---|---|---|
| bosons | $^{85}Rb$ | I=5/2 |
| | $^{87}Rb$ | I=3/2 |
| | $^{23}Na$ | I=3/2 |
| | $^{7}Li$ | I=3/2 |
| fermions | $^{40}K$ | I=4 |
| | $^{6}Li$ | I=1 |

# Hyperfine levels & Zeeman splitting

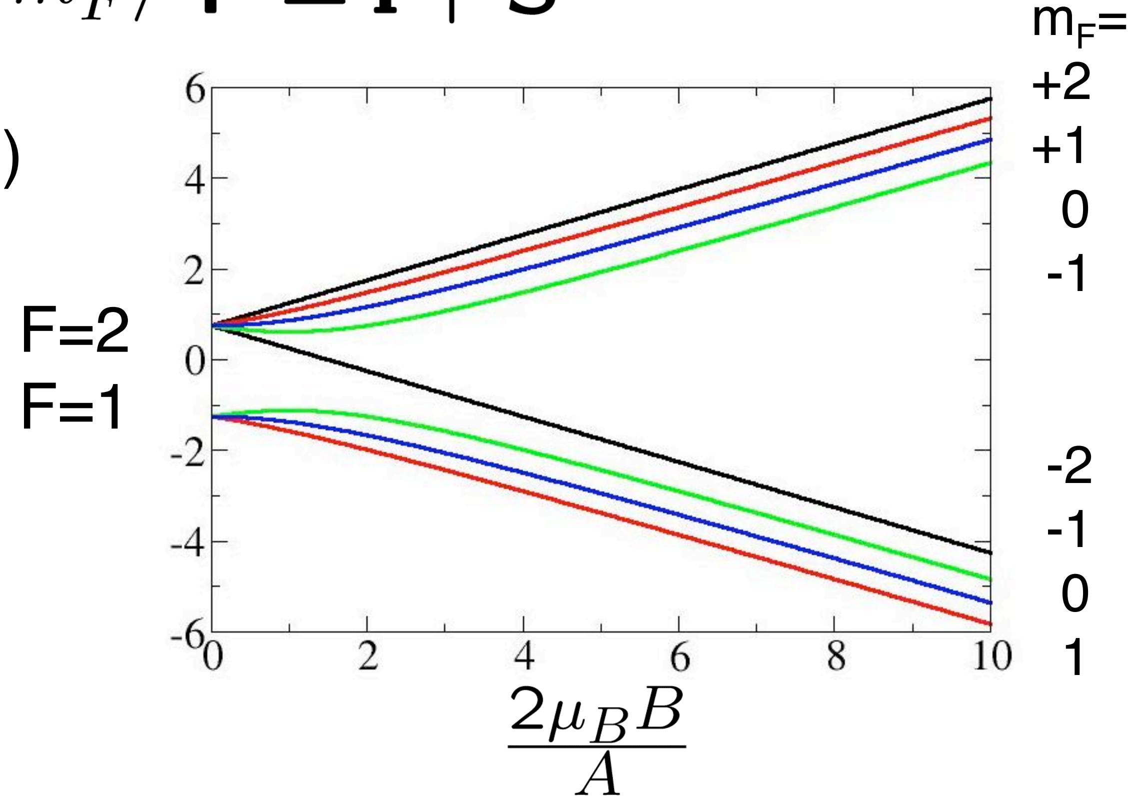



- Electronic spin S=J=1/2, nuclear spin I

$$\hat{H} = A\hat{\mathbf{I}}\cdot\hat{\mathbf{S}} + 2\mu_B B\hat{S}_z$$

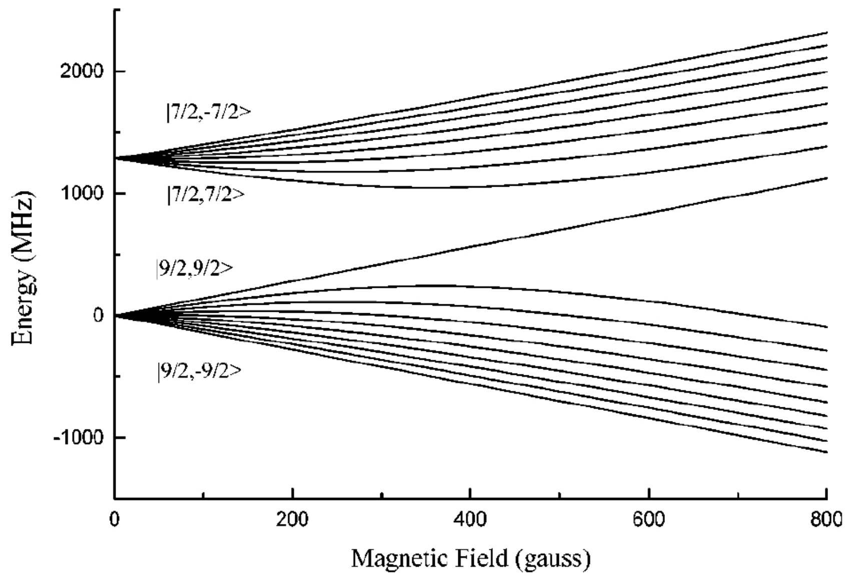
- Hyperfine levels $|F, m_F\rangle$ $\hat{\mathbf{F}} = \hat{\mathbf{I}} + \hat{\mathbf{S}}$

- I=3/2 ($^{87}$Rb, $^{23}$Na, $^{7}$Li)

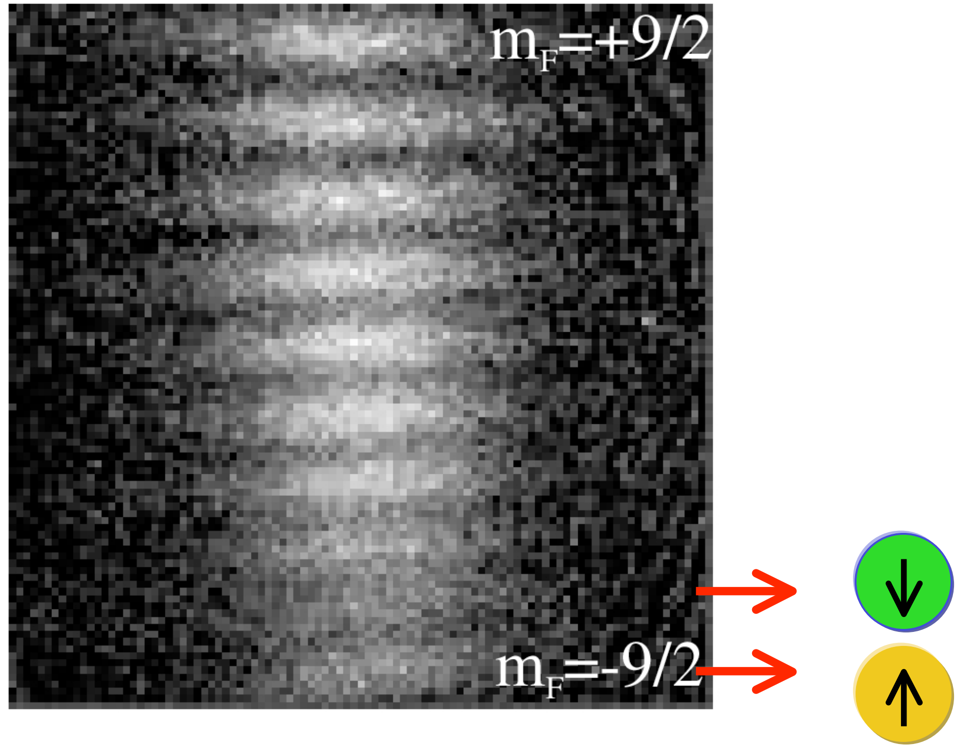


# Hyperfine levels & Zeeman splitting

- I=4 ($^{40}$K)



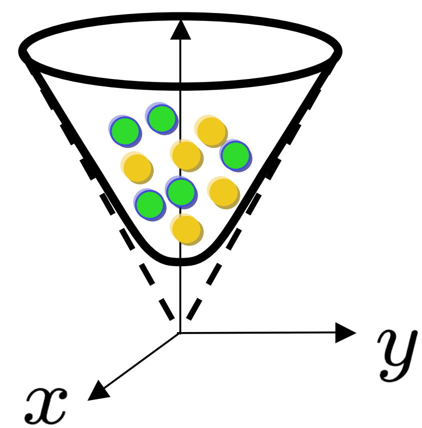

[J. L. Bohn *et al.* *PRA* **59**, 3660 (1999)]




[T. Loftus *et al.* *PRL* **88**, 173201 (2002)]

- Control the populations of atoms in different hyperfine states $\Delta E_{\text{hyp}} \gg k_B T$

- Magnetic trapping



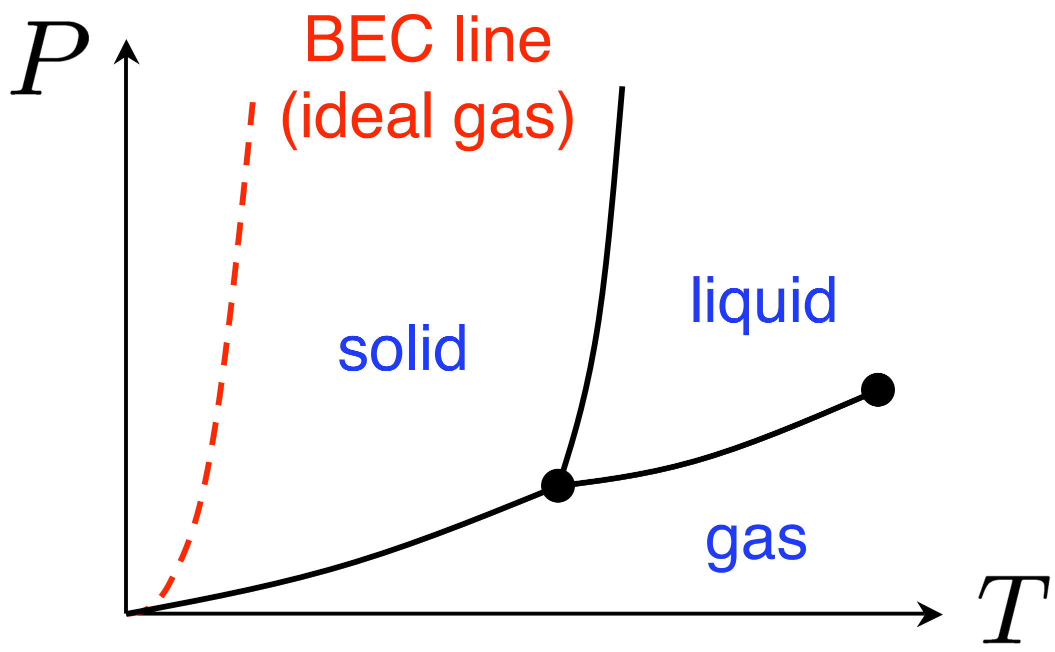
# Why dilute and ultracold?

Metastability vs. true thermal equilibrium

- Low density:
  three-body recombination rate << two-body scattering rate

$$n \sim 10^{13} - 10^{15} \mathrm{cm}^{-3}$$

- E.g.



# Why dilute and ultracold?

Metastability vs. true thermal equilibrium

- Low density:
  three-body recombination rate << two-body scattering rate

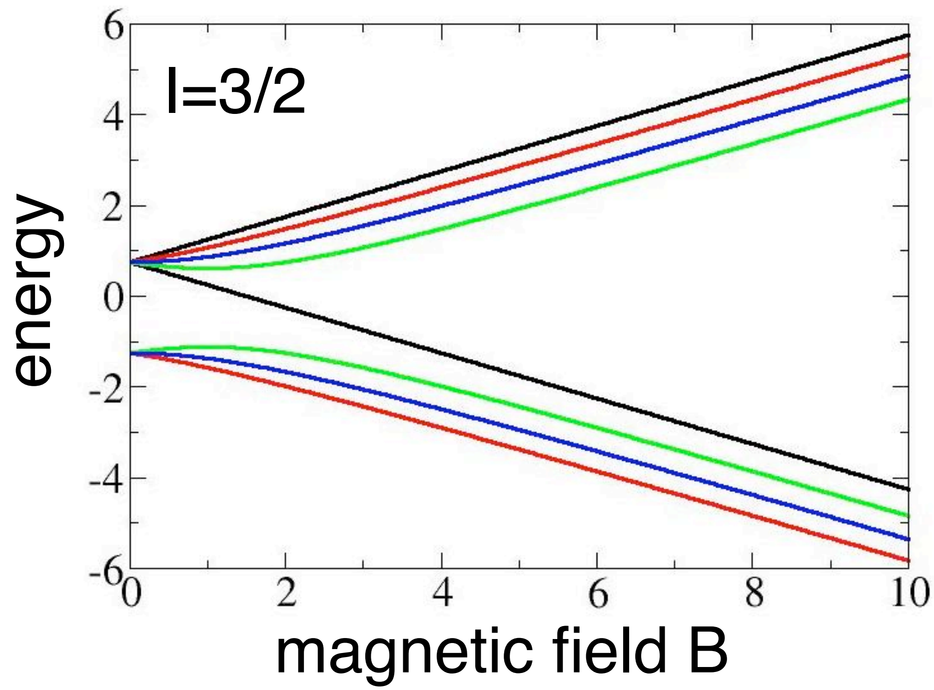
$$n \sim 10^{13} - 10^{15} \text{cm}^{-3}$$

- Quantum degeneracy $n\lambda_T^3 = n \left( \frac{2\pi}{mk_B T} \right)^{3/2} \geq 1$

- Low temperature:

$$T \sim 500\text{nK} - \mu\text{K}$$

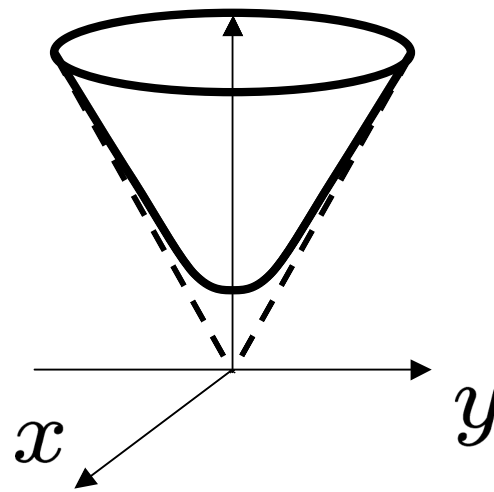
# Trapping the atoms, e.g. magnetically




$$E_\alpha \simeq \text{const} - \mu_\alpha B$$

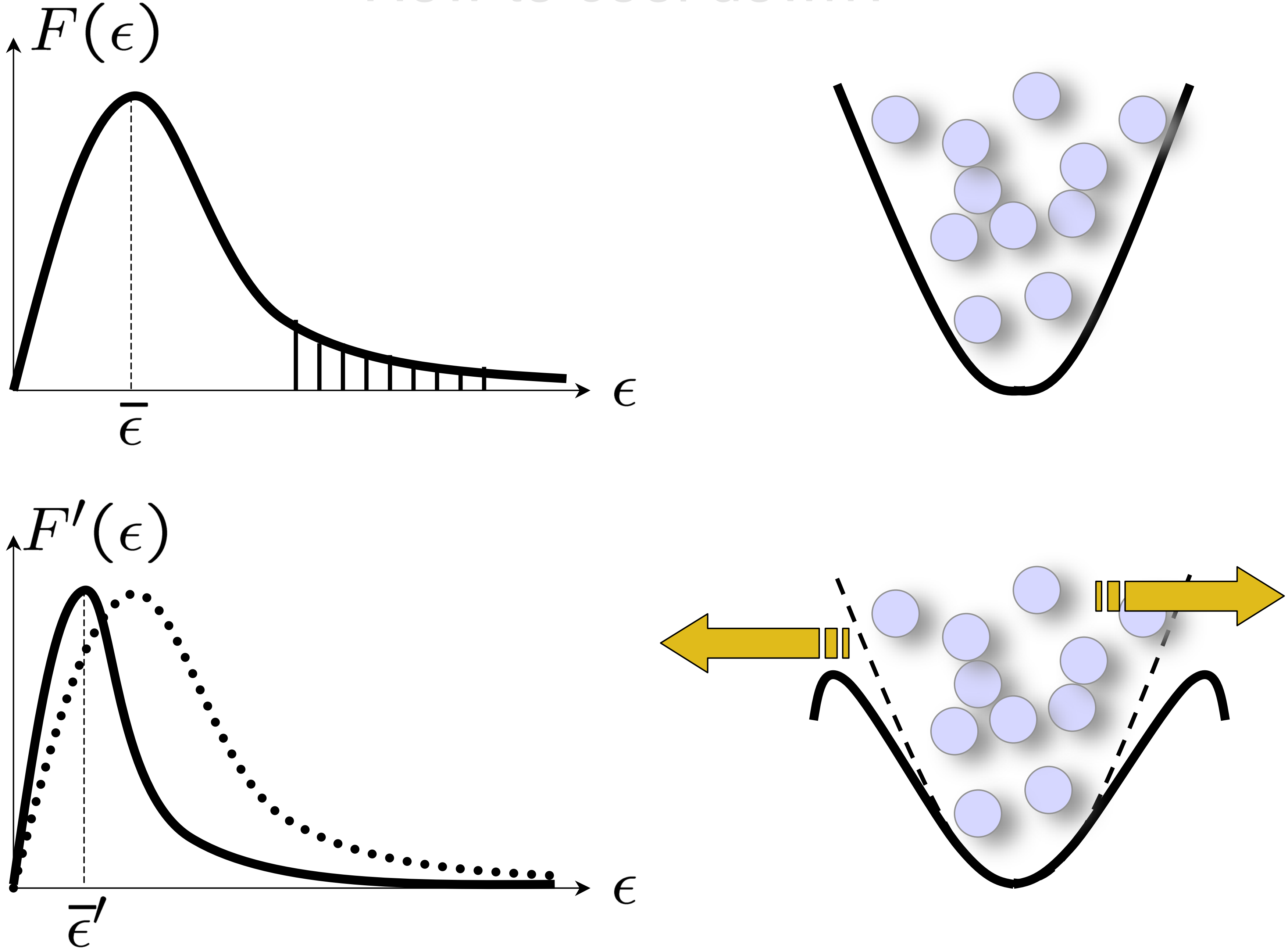
- $\mu_\alpha > 0$ high-field seeking states
- $\mu_\alpha < 0$ low-field seeking states

$$V(\mathbf{r}) = \frac{1}{2}\bar{\omega}^2 r^2$$




- Atoms in an inhomogeneous field experience a spatially-varying potential

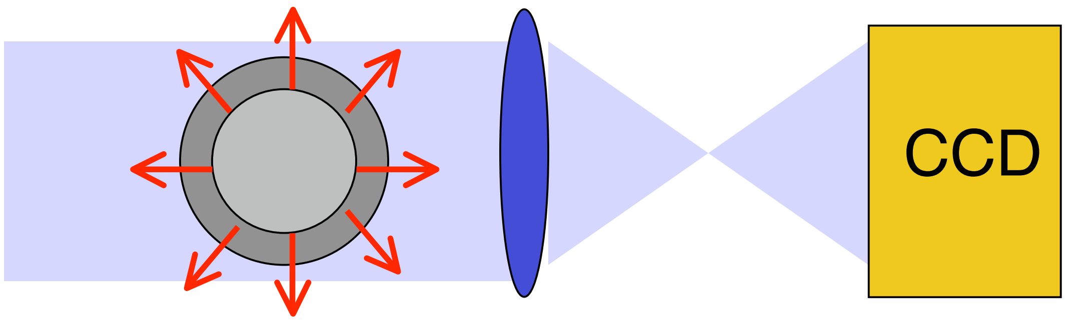
# How to cool down?



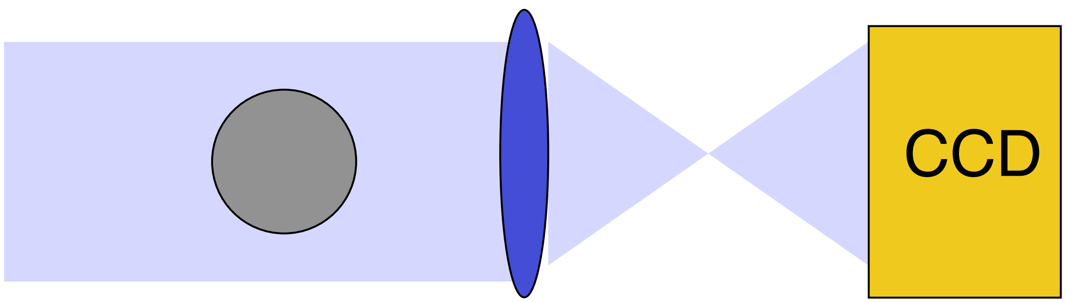
# Imaging atoms

1. Time of flight: momentum distribution

$$\lim_{t\bar{\omega}\gg 1} n(\mathbf{r},t) \propto n(\mathbf{p}=\frac{m\mathbf{r}}{t})$$




2. In-situ imaging: space distribution $n(\mathbf{r})$



# Recap on alkali atoms

I. Fermions or bosons

II. Different ‘spin’ (hyperfine) state

III. Gas if very dilute (two-body collisions, but not three)

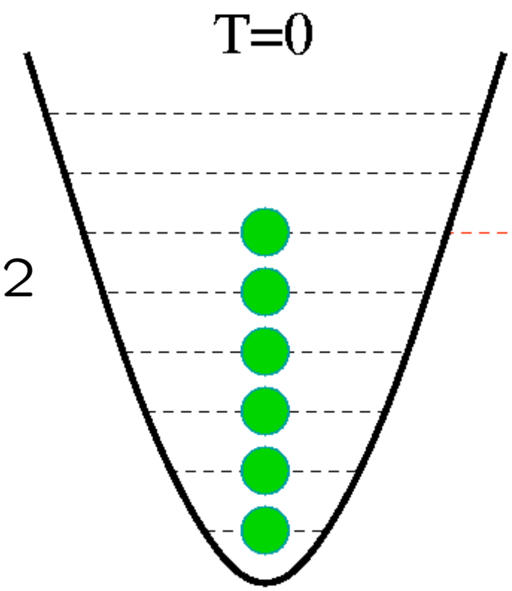
IV. Quantum degeneracy: very cold

V. Can be imaged (thermodynamic properties of the gas from density distribution)

# The Ideal Fermi Gas

Emergence of quantum degeneracy at T<T$_F$

$V(\mathbf{r}) = \frac{1}{2}\bar{\omega}^2 r^2$




$$k_B T_F = \varepsilon_F = \bar{\omega}(6N)^{1/3}$$

(problem 1)

- $k_B T_{\text{BEC}} \simeq k_B T_F$
- … but it is a crossover

# The Ideal Fermi Gas

## Space (and momentum) distribution

- Local density approximation

$$f_F(\mathbf{r},\mathbf{k}) = \frac{1}{\exp[\beta(\epsilon_{\mathbf{k}} + V(\mathbf{r}) - \mu)] + 1}$$

$$n(\mathbf{r}) = \int \frac{d\mathbf{k}}{(2\pi)^3} f_F(\mathbf{r},\mathbf{k})$$

$$n(\mathbf{k}) = \int d\mathbf{r}\, f_F(\mathbf{r},\mathbf{k})$$

- T=0: Fermi see in real space

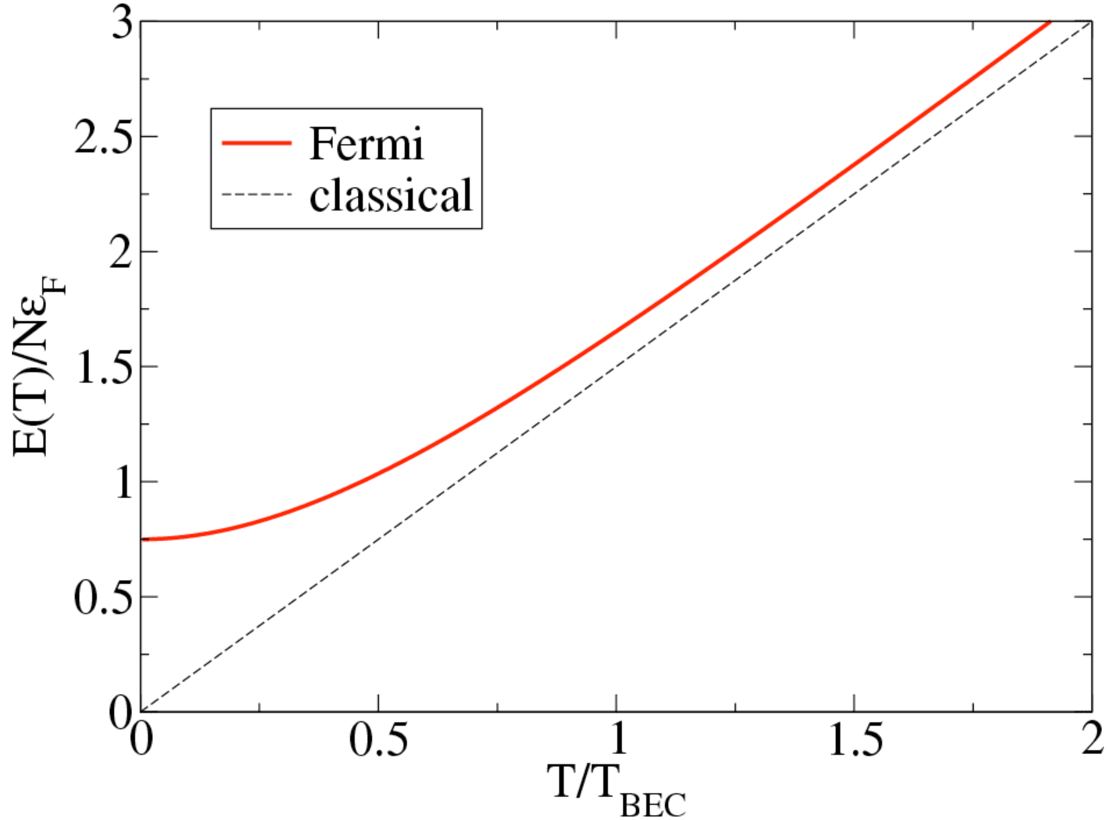
$$n_{T=0}(\mathbf{r}) = \begin{cases} \frac{1}{6\pi^2}\left[2m\left(\varepsilon_F - V(\mathbf{r})\right)\right]^{3/2} & V(\mathbf{r}) < \varepsilon_F \\ 0 & \text{otherwise} \end{cases}$$

…with Fermi radius $$R_F = \left(\frac{2\varepsilon_F}{m\bar{\omega}}\right)^{1/2}$$

# The Ideal Fermi Gas

## Energy of the gas

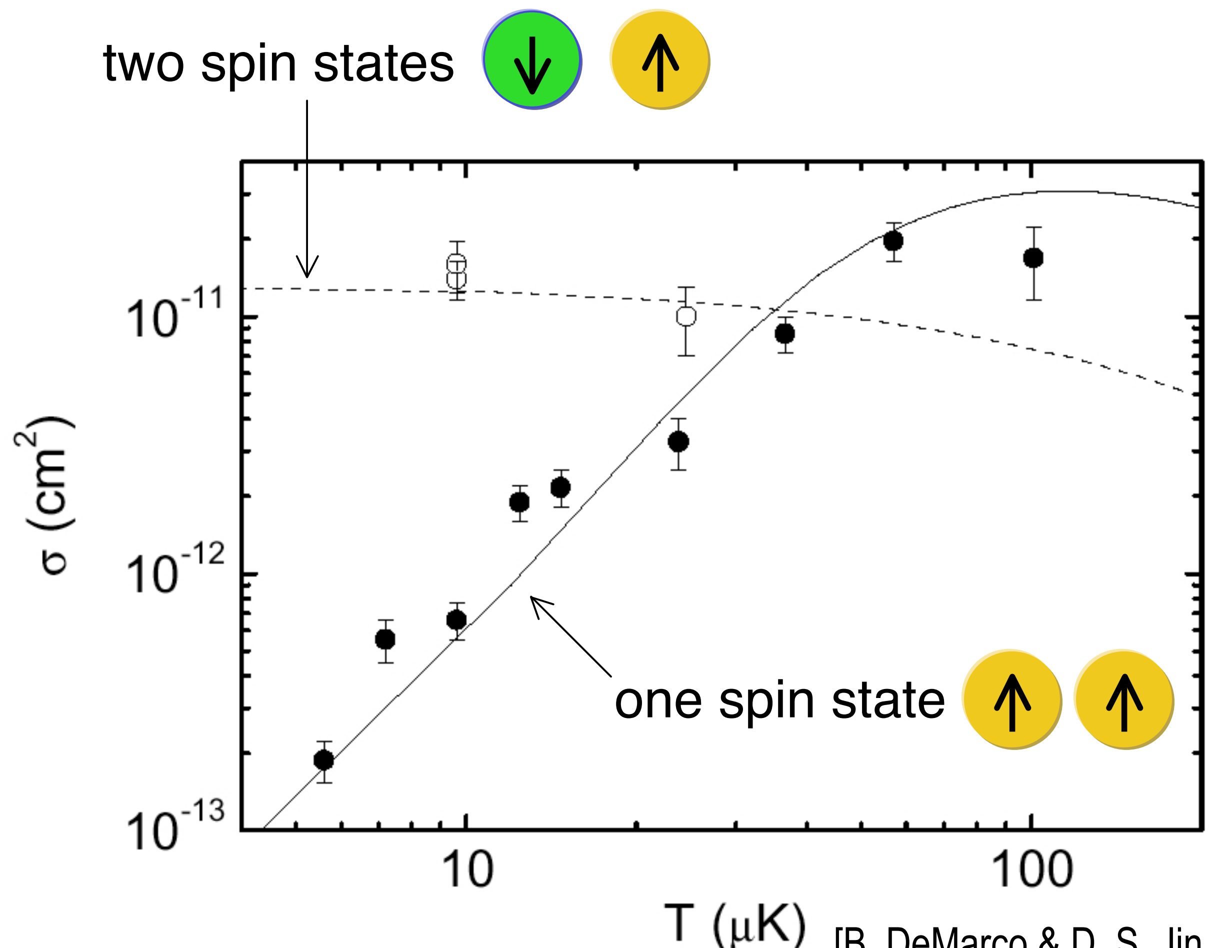
$$E(T) = \int \frac{d\mathbf{k}}{(2\pi)^3} \epsilon_{\mathbf{k}} n(\mathbf{k}) \rightarrow \frac{3}{4} N \varepsilon_F \quad \text{at } T = 0$$



# What to measure in experiments?

- At low T, identical fermions ↑ ↑ do not interact

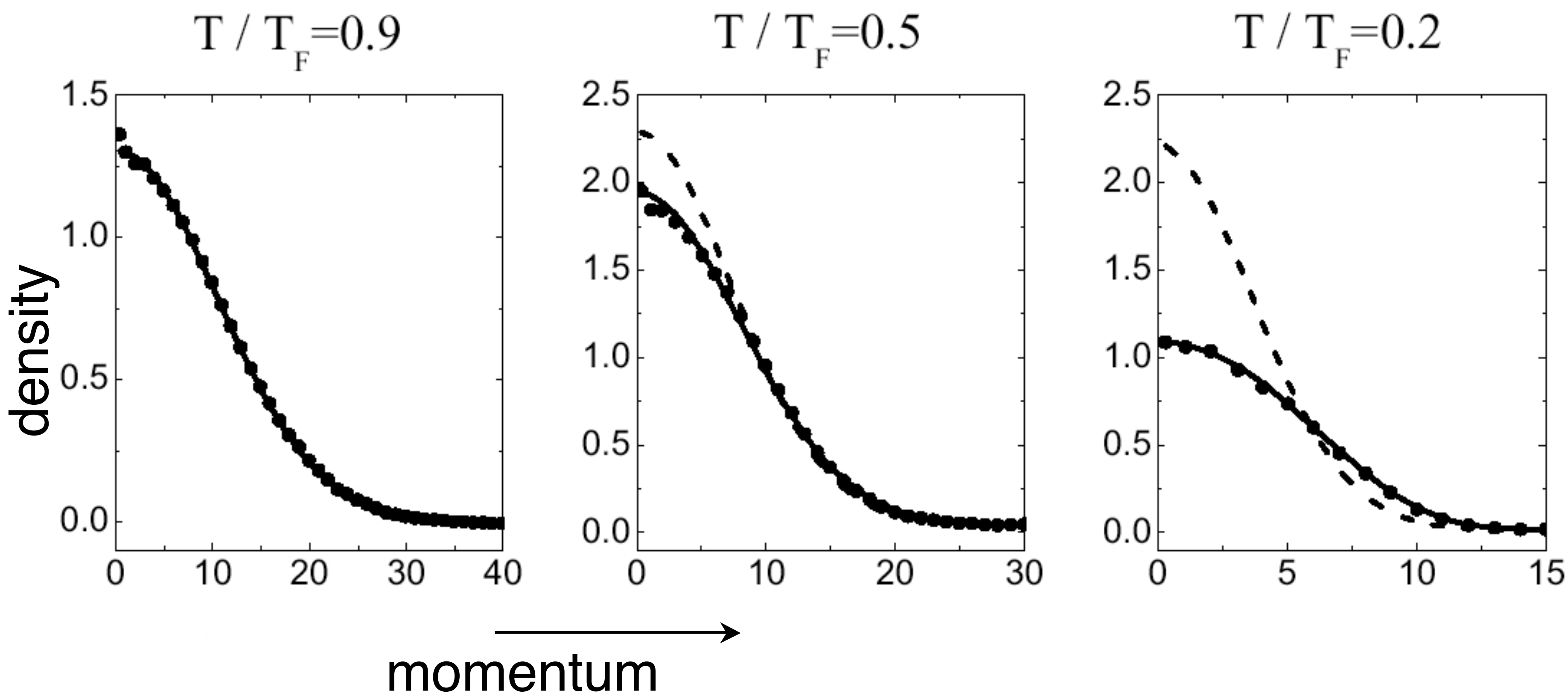
$$\Psi(1,2) = \psi(\mathbf{r}_1, \mathbf{r}_2)\chi_{\text{spin}}$$




[B. DeMarco & D. S. Jin, *Science* **285**, 1703 (1999)]
[D. S. Jin *et al.*, *ICAP proceedings* (2000)]

# What to measure in experiments?

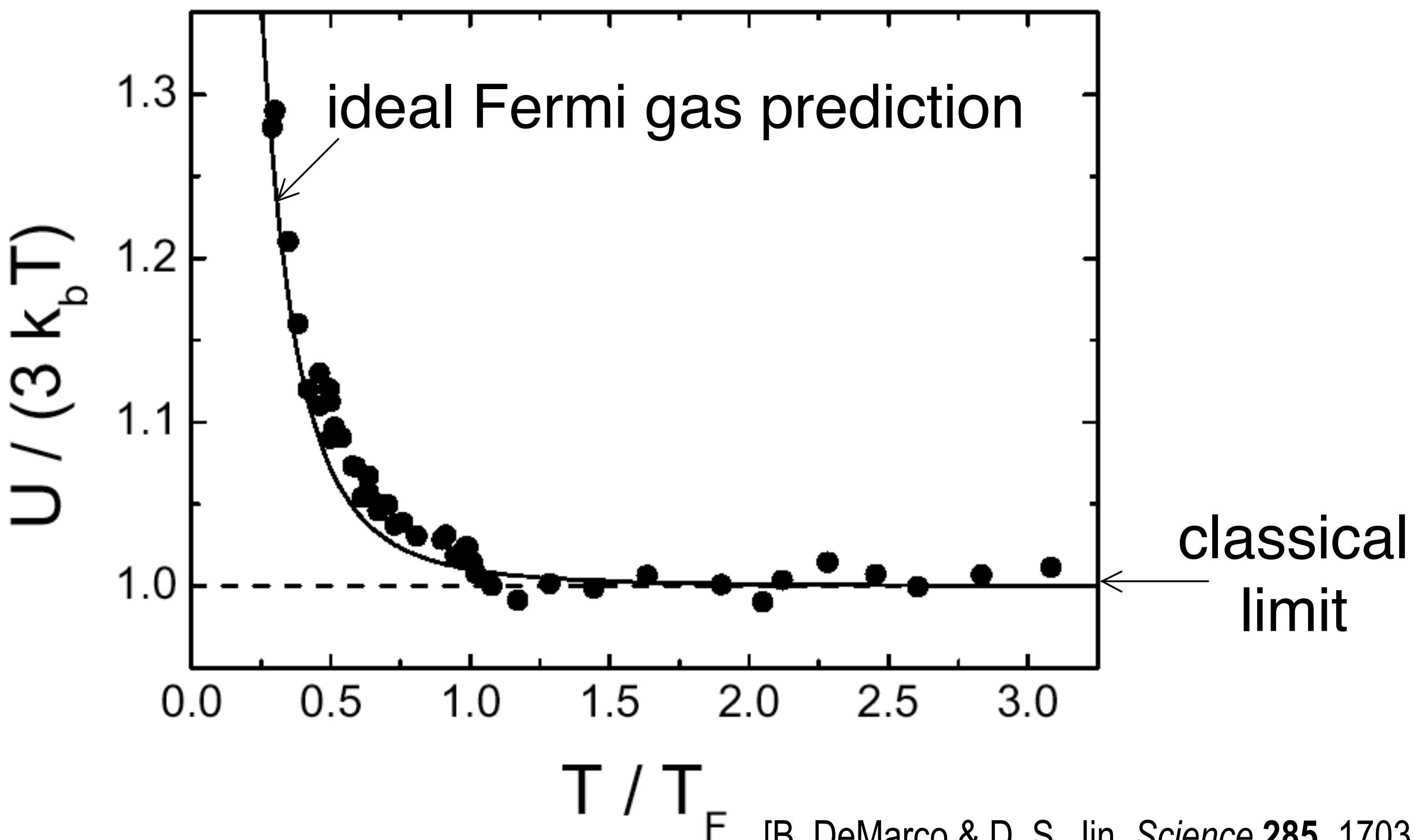
- Momentum distribution:
  emergence of a non-Gaussian component at low $T/T_F$




[B. DeMarco & D. S. Jin, *Science* **285**, 1703 (1999)]
[D. S. Jin *et al.*, *ICAP proceedings* (2000)]

# What to measure in experiments?

- Energy of the gas:
  deviation from the classical behaviour at low $T/T_F$

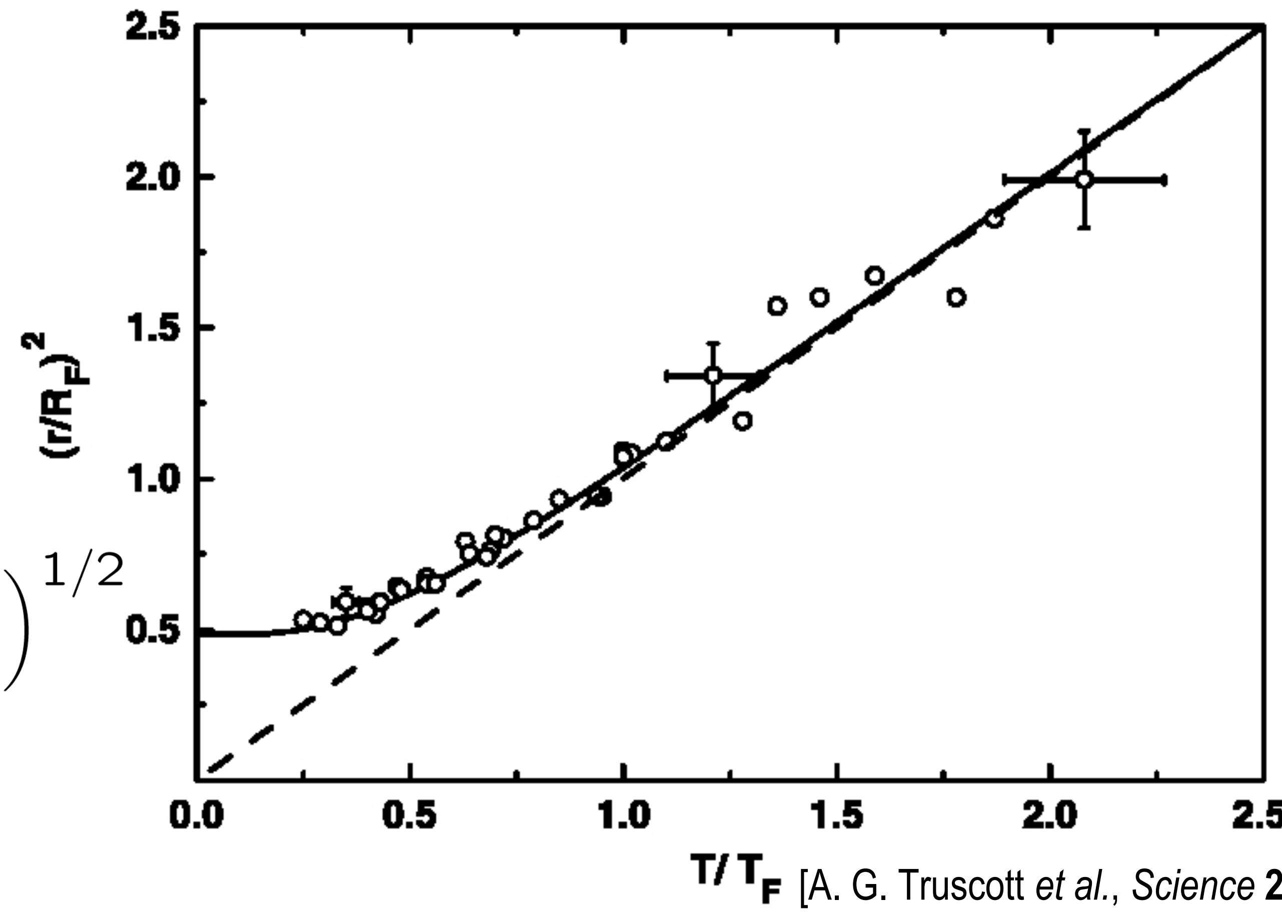



[B. DeMarco & D. S. Jin, *Science* **285**, 1703 (1999)]
[D. S. Jin *et al.*, *ICAP proceedings* (2000)]

# What to measure in experiments?

- Radius of the cloud:
  evidence of Fermi pressure at low $T/T_F$

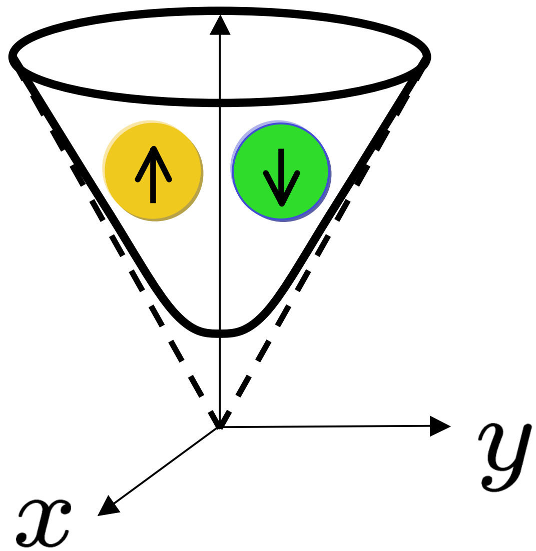
$$R_F = \left(\frac{2\varepsilon_F}{m\bar{\omega}}\right)^{1/2}$$




[A. G. Truscott *et al.*, *Science* **291**, 2570 (2001)]

# Pairing instability

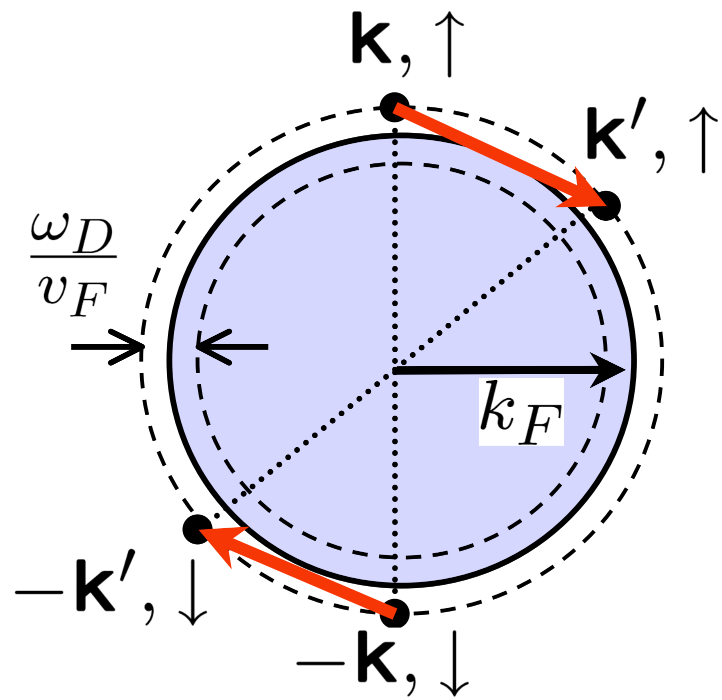
What happens for a two-component mixture?




- Interactions at low T now possible
- Lecture II: tuning the interaction (Feshbach resonance)
- What do we already know for electrons in a metal…
  (Derek Lee lectures!)

# Reminder: the one-pair Cooper problem

[L. N. Cooper, *Phys. Rev.* **104**, 1189 (1956)]




- Two electrons interact above a (non-interacting) filled Fermi sea
- A bound state always exists for an (arbitrarily) weak attractive potential

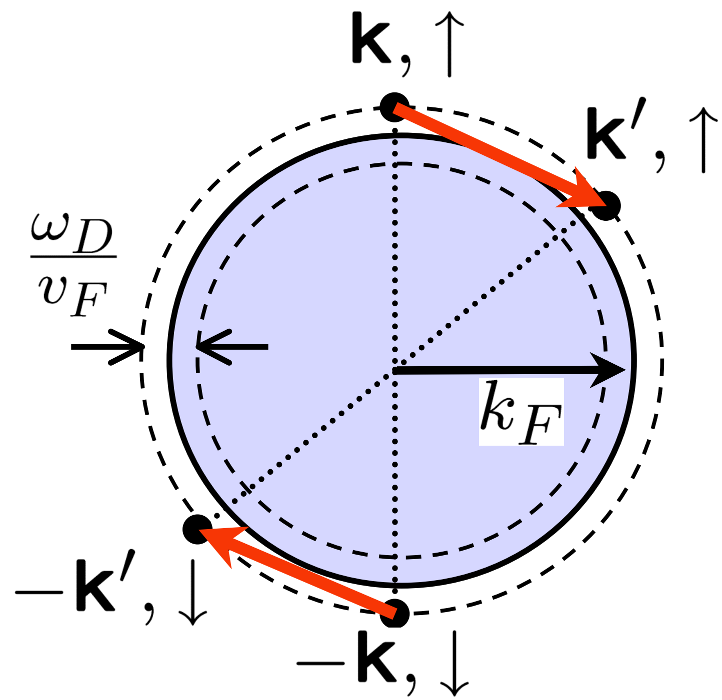
$$\epsilon \simeq -2\omega_D e^{-\frac{2}{\lambda\mathcal{N}(\varepsilon_F)}}$$

- Cooper suggested that the instability of the normal (metallic) phase, because of electrons binding into pairs, was associated with the occurrence of superconductivity
- N.B. $\epsilon \simeq k_B T_{\rm BCS}$ Cooper pairs form and condensed at the same temperature scale (pairing instability)!

(see Derek Lee 4th lecture!)

# Reminder: the one-pair Cooper problem

[L. N. Cooper, *Phys. Rev.* **104**, 1189 (1956)]




- Mean square radius same order of the coherence length (size of the Cooper pair)

$$\overline{r^2} \simeq \frac{4}{3}\frac{v_F^2}{\epsilon^2} \propto \xi^2$$

- However an isolated pair model cannot be fully appropriate for superconductors, where there are typically $10^{11}$ other electrons within a 'coherence volume' $(\overline{r^2})^{3/2}$

→ BCS theory

(see Derek Lee 4th lecture!)

# Reminder: BCS theory

[J. Bardeen, L.N. Cooper, and J.R. Schrieffer, *Phys. Rev.* **106**, 162 (1957)]

$$\hat{\mathcal{H}} = \sum_{\mathbf{k},\sigma=\uparrow,\downarrow} \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{g}{V} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} c^\dagger_{\mathbf{k}+\mathbf{q}/2\uparrow} c^\dagger_{-\mathbf{k}+\mathbf{q}/2\downarrow} c_{-\mathbf{k}'+\mathbf{q}/2\downarrow} c_{\mathbf{k}'+\mathbf{q}/2\uparrow}$$

- Mean-field Hamiltonian (alternative to the variational approach)

$$\hat{H} - \varepsilon_F \hat{N} \simeq \sum_{\mathbf{k}} \begin{pmatrix} c^\dagger_{\mathbf{k}\uparrow} & c_{-\mathbf{k}\downarrow} \end{pmatrix} \begin{pmatrix} \xi_{\mathbf{k}} & -\Delta \\ -\Delta & -\xi_{\mathbf{k}} \end{pmatrix} \begin{pmatrix} c_{\mathbf{k}\uparrow} \\ c^\dagger_{-\mathbf{k}\downarrow} \end{pmatrix} + \sum_{\mathbf{k}} \xi_{\mathbf{k}} - \frac{\Delta^2}{g} V$$

$$\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \varepsilon_F$$

…where the order parameter $\Delta \equiv -\frac{g}{V} \sum_{\mathbf{k}} \langle \psi | c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} | \psi \rangle$

(see Derek Lee 4th lecture!)

# Reminder: BCS theory

[J. Bardeen, L.N. Cooper, and J.R. Schrieffer, *Phys. Rev.* **106**, 162 (1957)]

- Problem 2: Show that the mean-field Hamiltonian is diagonalised by the quasi-particle operators

$$\begin{pmatrix} \gamma_{\mathbf{k}\uparrow} \\ \gamma^\dagger_{-\mathbf{k}\downarrow} \end{pmatrix} = \begin{pmatrix} \cos\theta_{\mathbf{k}} & \sin\theta_{\mathbf{k}} \\ \sin\theta_{\mathbf{k}} & -\cos\theta_{\mathbf{k}} \end{pmatrix} \begin{pmatrix} c_{\mathbf{k}\uparrow} \\ c^\dagger_{-\mathbf{k}\downarrow} \end{pmatrix}$$

(preserves anticommutation relations) & that

$$\hat{H} - \varepsilon_F \hat{N} \simeq \sum_{\mathbf{k},\sigma=\uparrow,\downarrow} E_{\mathbf{k}} \gamma^\dagger_{\mathbf{k}\sigma} \gamma_{\mathbf{k}\sigma} + \sum_{\mathbf{k}} (\xi_{\mathbf{k}} - E_{\mathbf{k}}) - \frac{\Delta^2}{g} V$$

$$E_{\mathbf{k}} = \sqrt{\xi^2_{\mathbf{k}} + \Delta^2}$$

- Ground state uniquely given by

$$|\psi\rangle = \prod_{\mathbf{k}} \gamma_{-\mathbf{k}\downarrow} \gamma_{\mathbf{k}\uparrow} |0\rangle \propto \prod_{\mathbf{k}} \left( \cos\theta_{\mathbf{k}} + \sin\theta_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} \right) |0\rangle$$