Course in 3 lectures on

Superfluidity in Ultracold Fermi Gases

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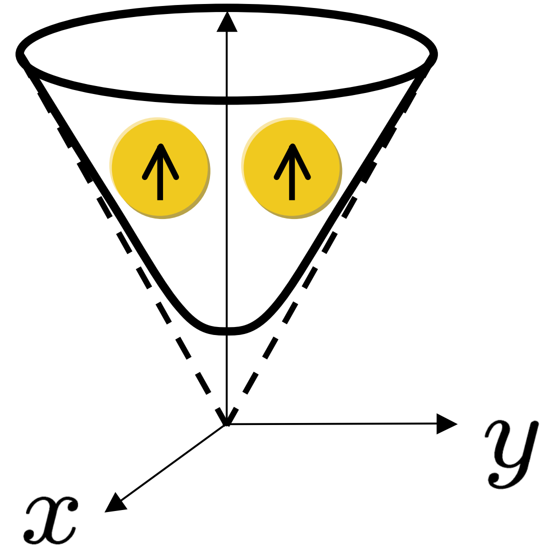
Physics by the Lake, Ambleside, 11, 12, 13 September 2007

During lecture I. …

1) Fully polarised gas = ideal Fermi gas

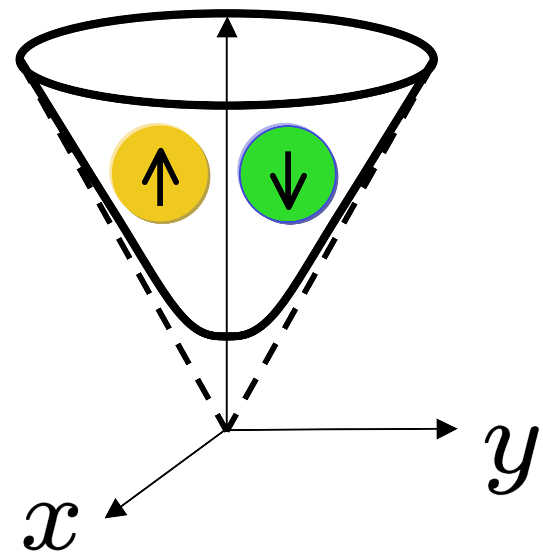
- Emergence of quantum degeneracy
- Distribution of the gas
- Energy of the gas
- Fermi pressure



2) Two-component mixtures can interact!

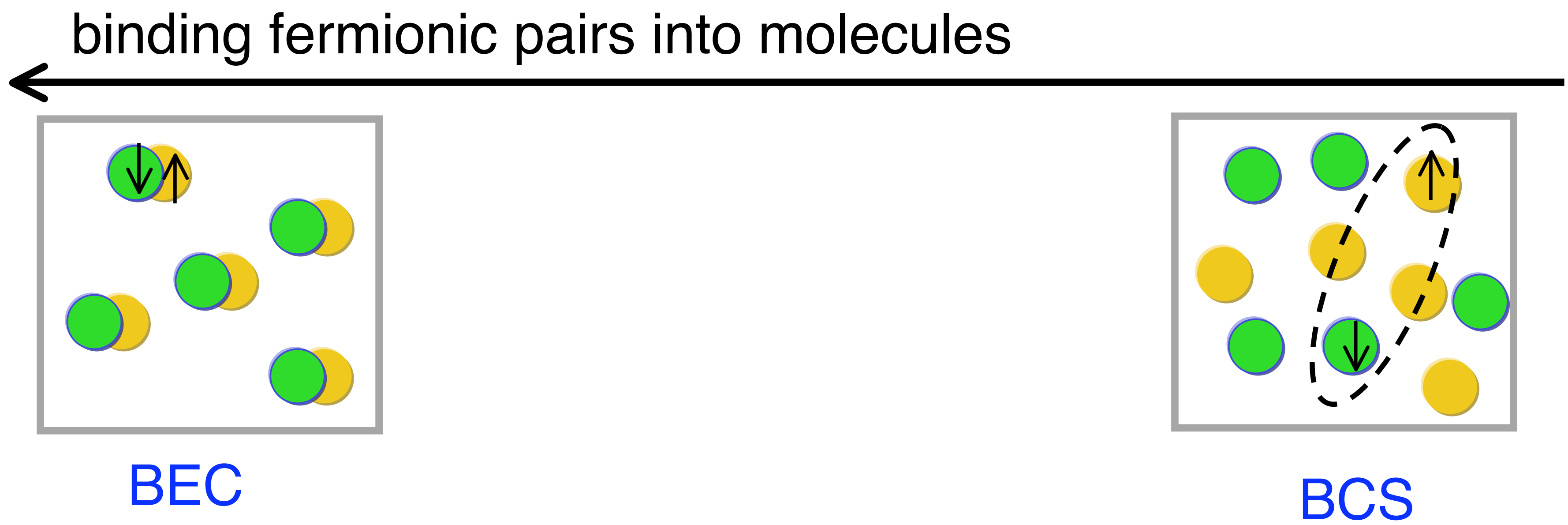
- Today: how & how tuning is possible

$$\frac{1}{k_F a}$$



3) Pairing instability & BCS theory

Why tuning the interaction is interesting?



II. Feshbach resonances & BEC-BCS crossover

Interactions between atoms

Dilute gases: two-body collisions

particle separation >> scattering length

$n^{-1/3} \sim 100\text{nm}$ $a \sim 5\text{nm}$

1) Essential to ensure thermalization (equilibrium & cooling)
2) Can be tuned!

$$\left[-\frac{-\nabla^2}{2m_r} + U(\mathbf{r})\right]\psi_{\mathbf{k}}(\mathbf{r}) = \frac{k^2}{2m_r}\psi_{\mathbf{k}}(\mathbf{r})$$

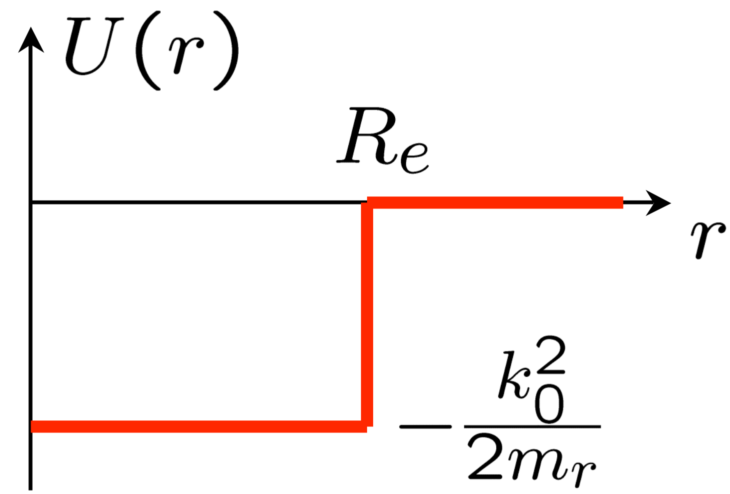
$$\lim_{r\to\infty, k\to 0} \psi_{\mathbf{k}}(\mathbf{r}) \propto 1 - \frac{a}{r}$$ (low-energy asymptotics)

$$a \simeq \frac{2m_r}{4\pi}\int d\mathbf{r}U(\mathbf{r})$$

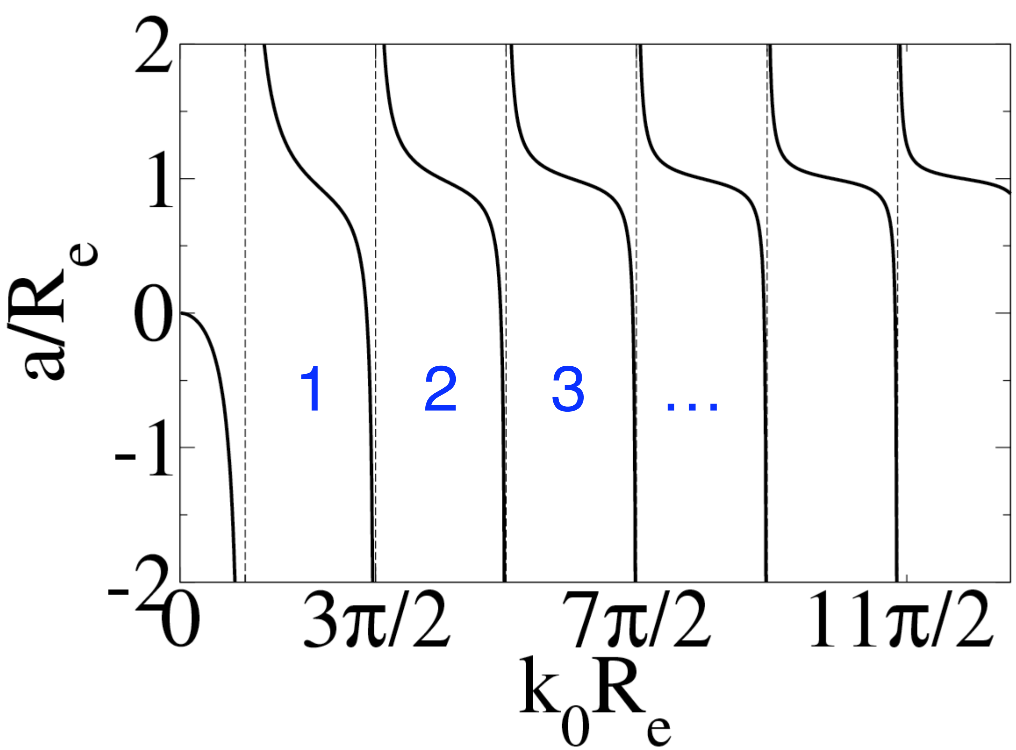
- a<0 attractive effective interaction
- a>0 repulsive

E.g., square well potential

$$\psi_{\mathbf{k}}(\mathbf{r}) = \frac{u_k(r)}{r}$$



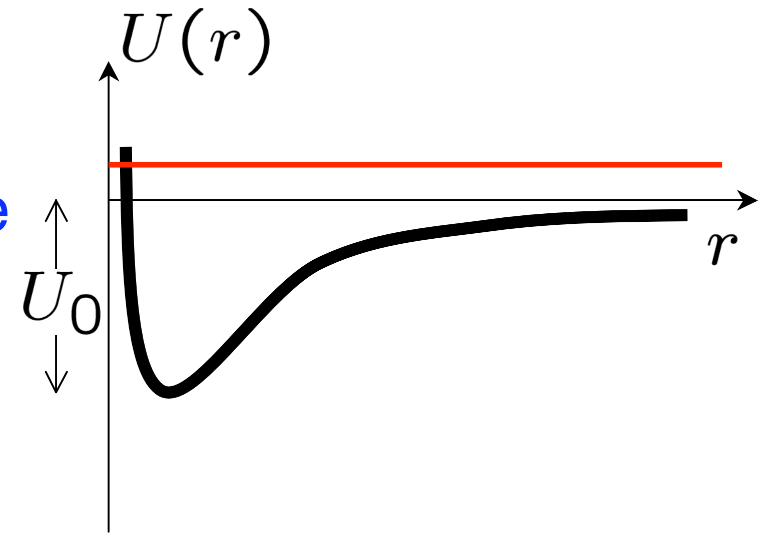
$$u_0(r) = \begin{cases} c_1(r-a) & r > R_e \\ c_2 \sin(k_0 r) & r < R_e \end{cases}$$



In general

1) The sign of a depends on the energy of the highest bound state

2) If there are no bound states a<0 (attractive interaction)



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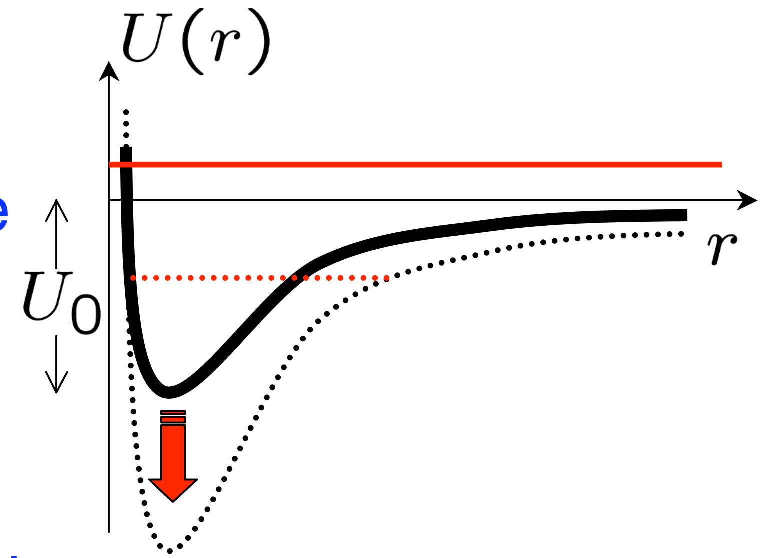
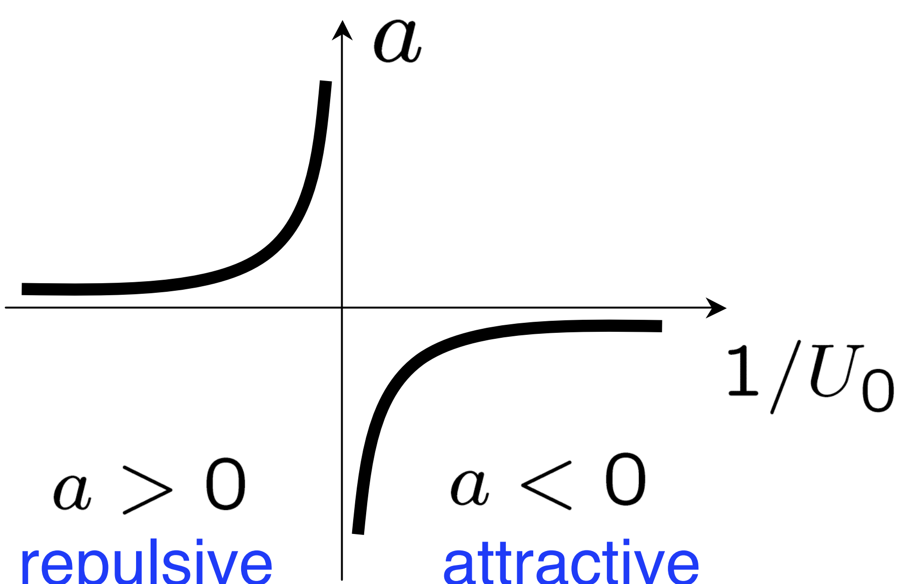
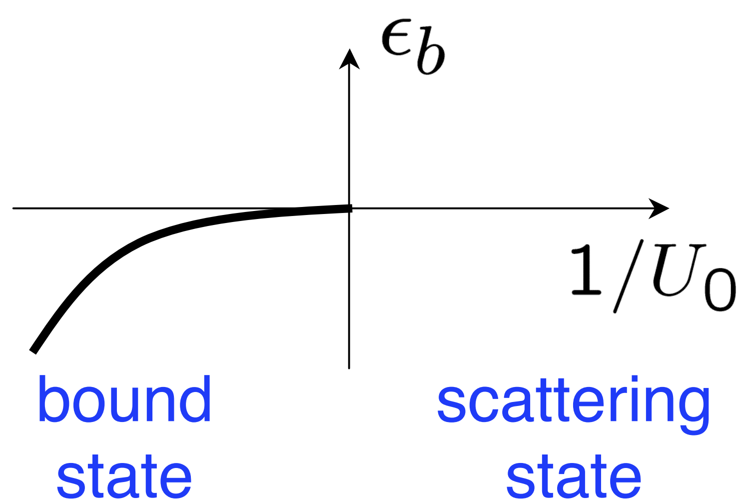


2) If there are no bound states a<0 (attractive interaction)

3) Every time a bound state is formed:

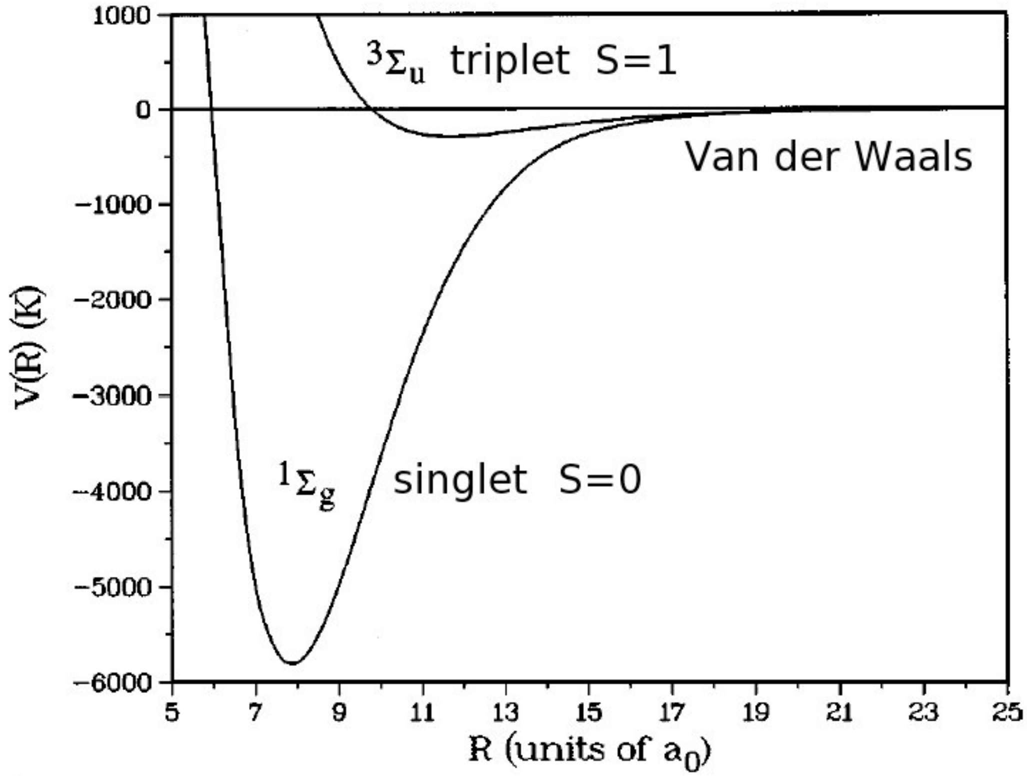
$$\epsilon_b = -\frac{1}{2m_r a^2}$$





But a scattering potential cannot be changed externally…

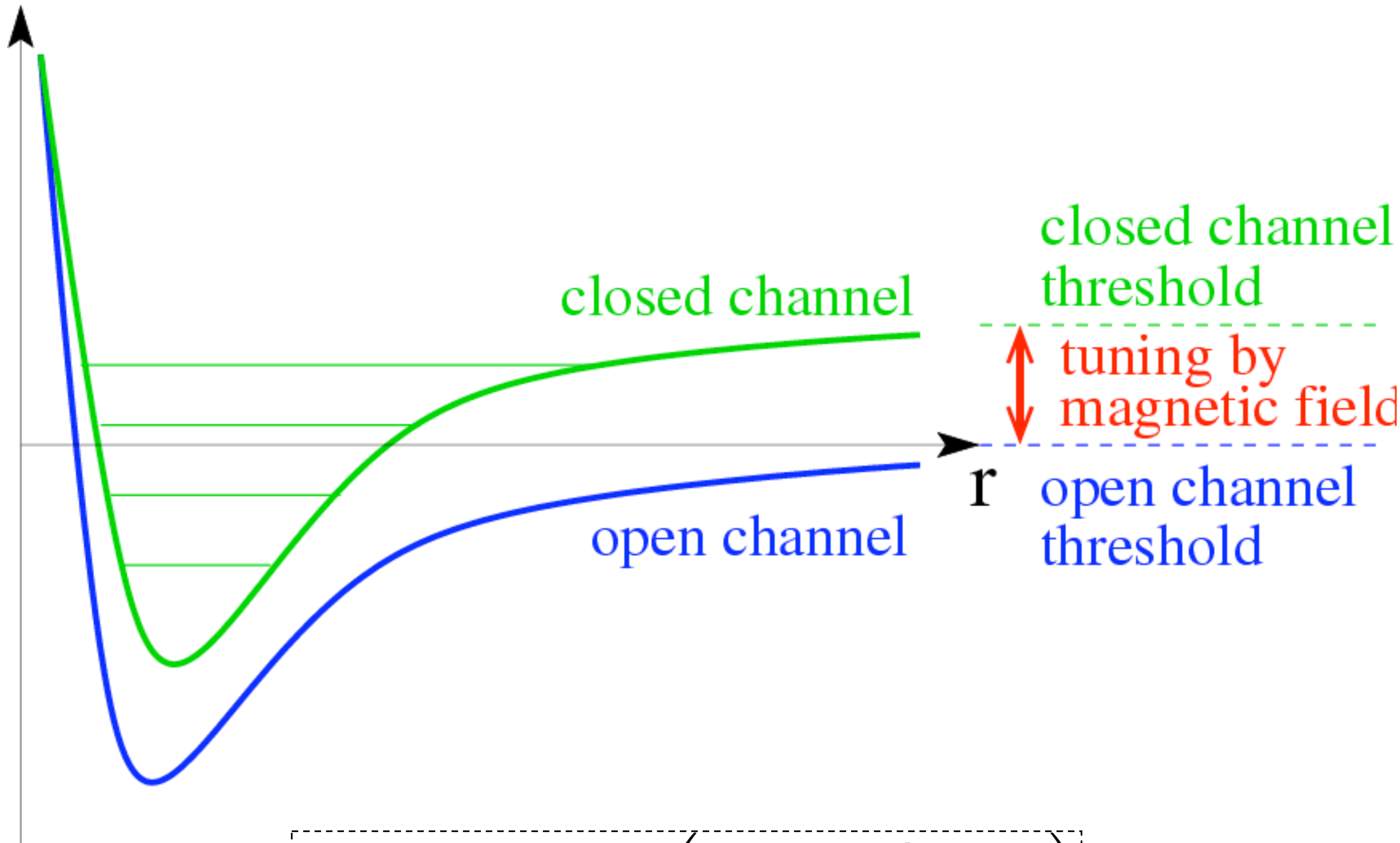
Interactions between atoms



- At short distances, the electronic spin of the two atoms can flip $\uparrow\downarrow \rightarrow \downarrow\uparrow$ and change the initial hyperfine states

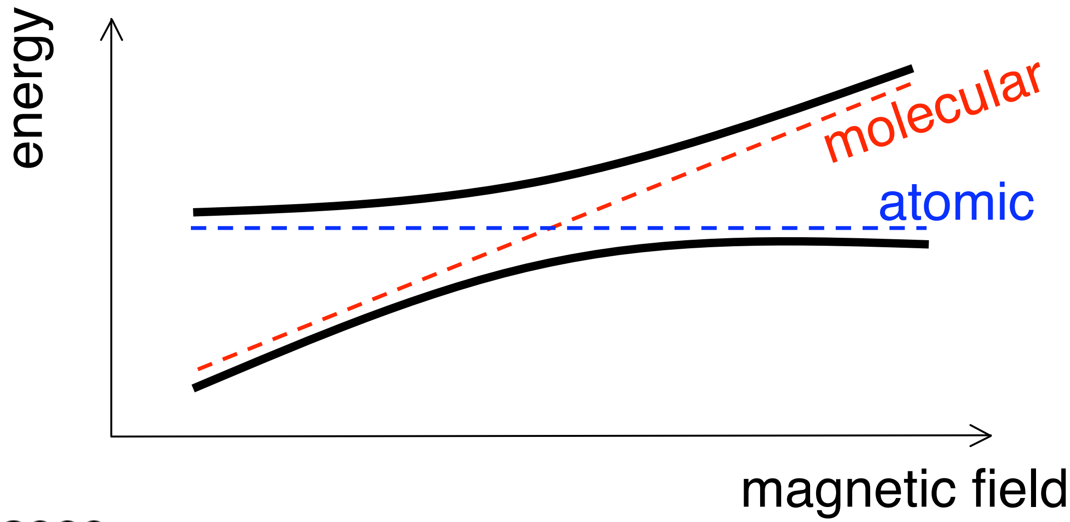
$$\hat{H}_{\text{int}} = \frac{U_s + 3U_t}{4} + (U_t - U_s)\hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2$$

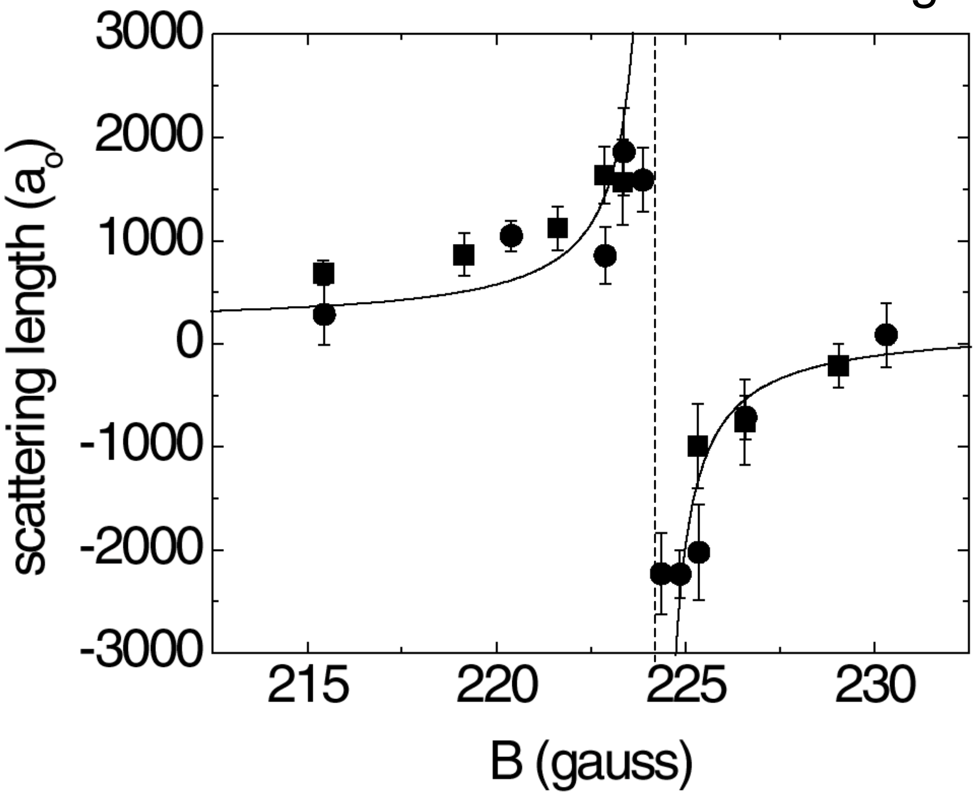
Feshbach resonances



$$a(B) = a_{\mathrm{bg}} \left(1 - \frac{\Delta B}{B - B_0} \right)$$

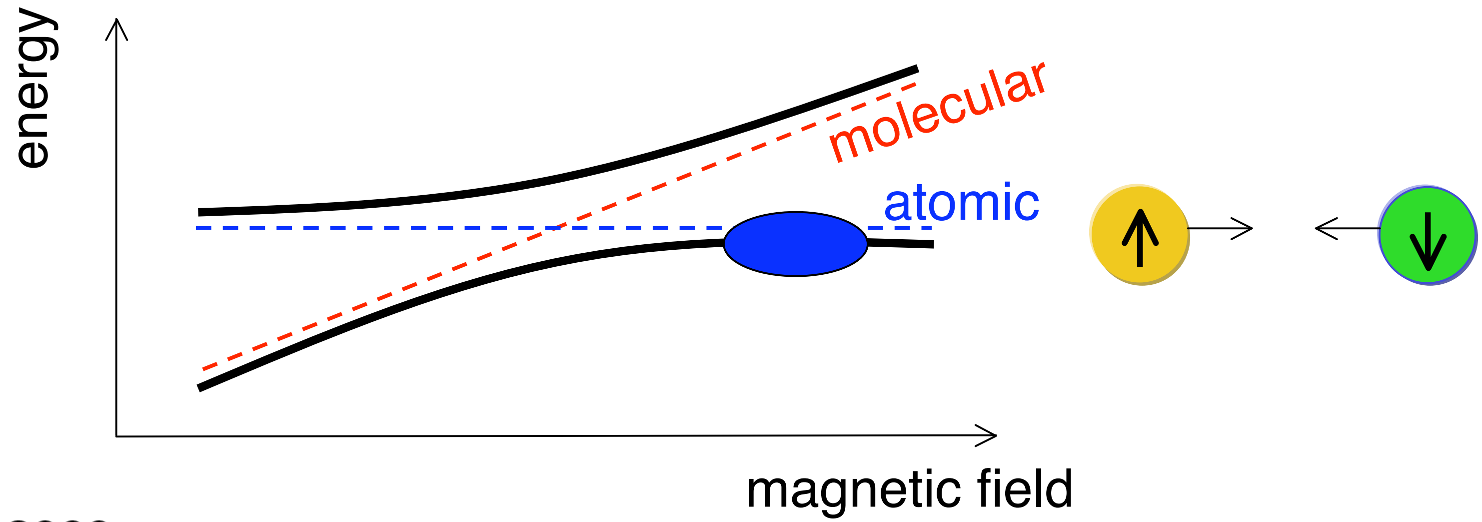
Feshbach resonances

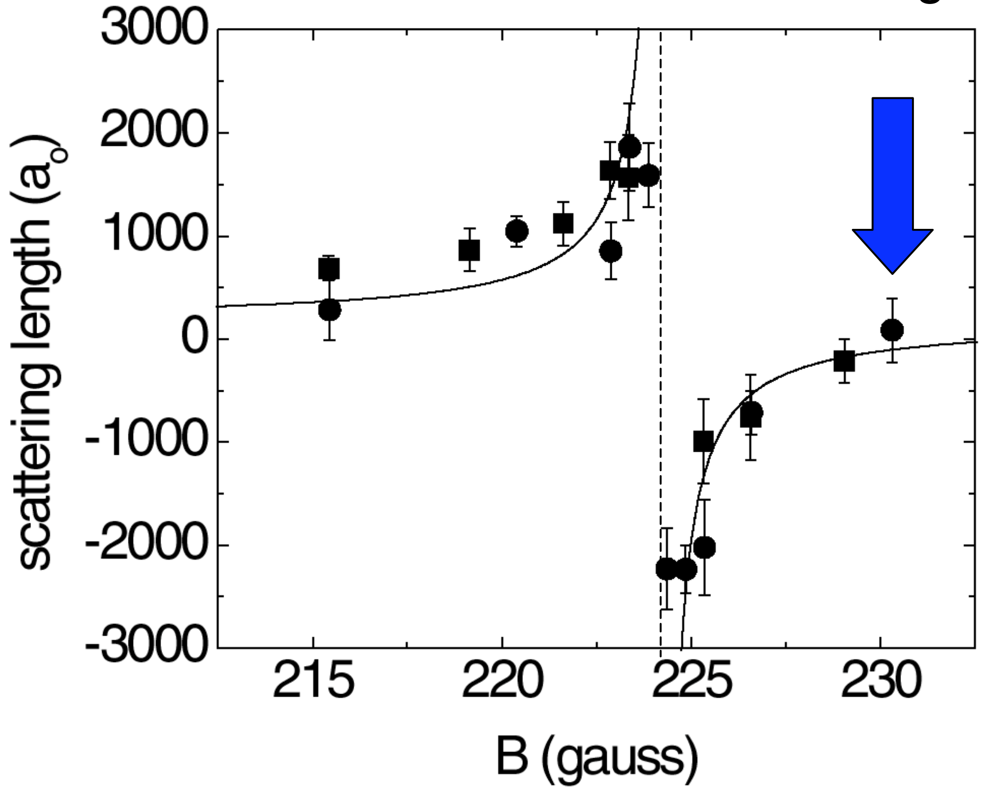




[C. A. Regal *et al.*, *Nature* **424**, 47 (2003)]

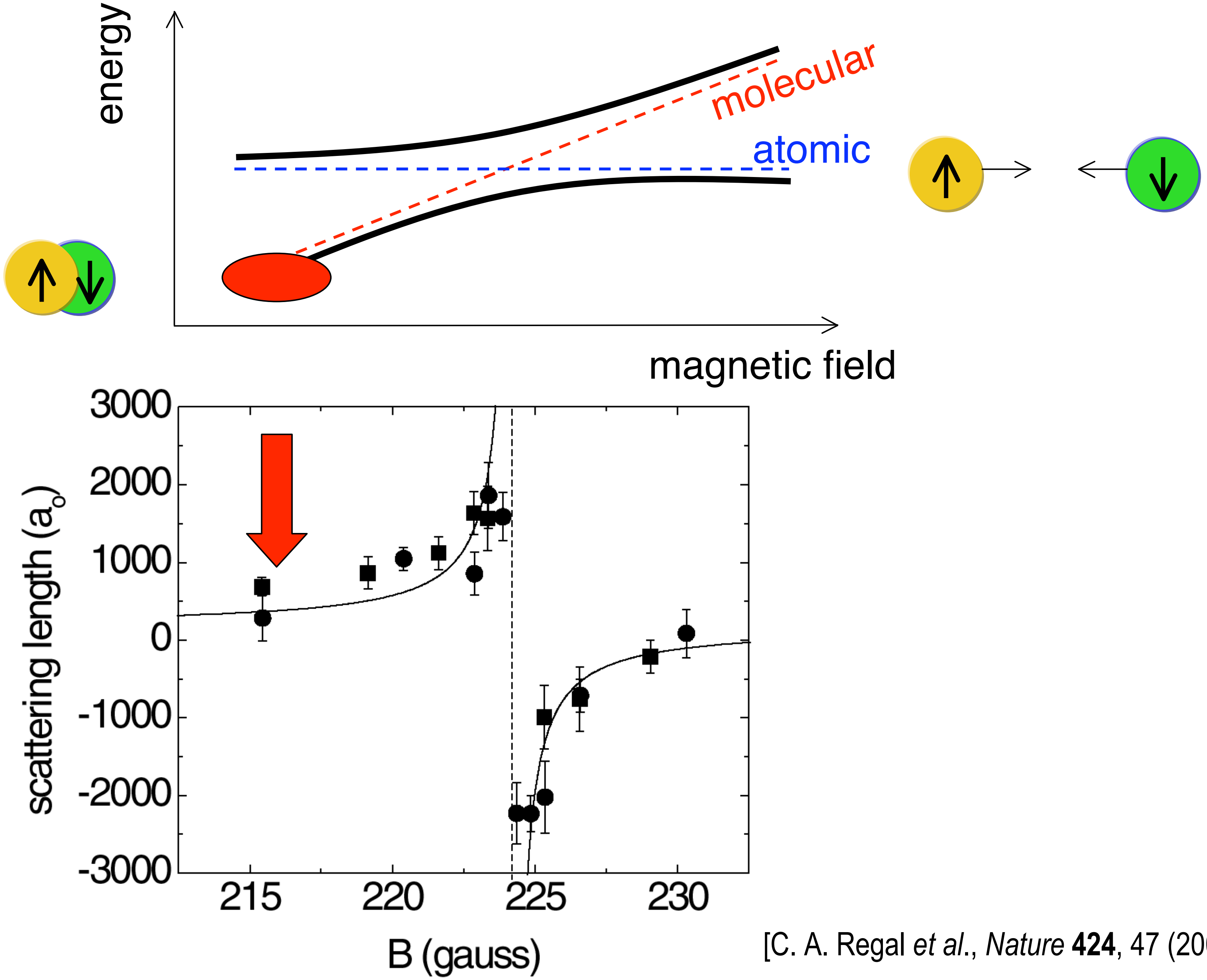
Feshbach resonances





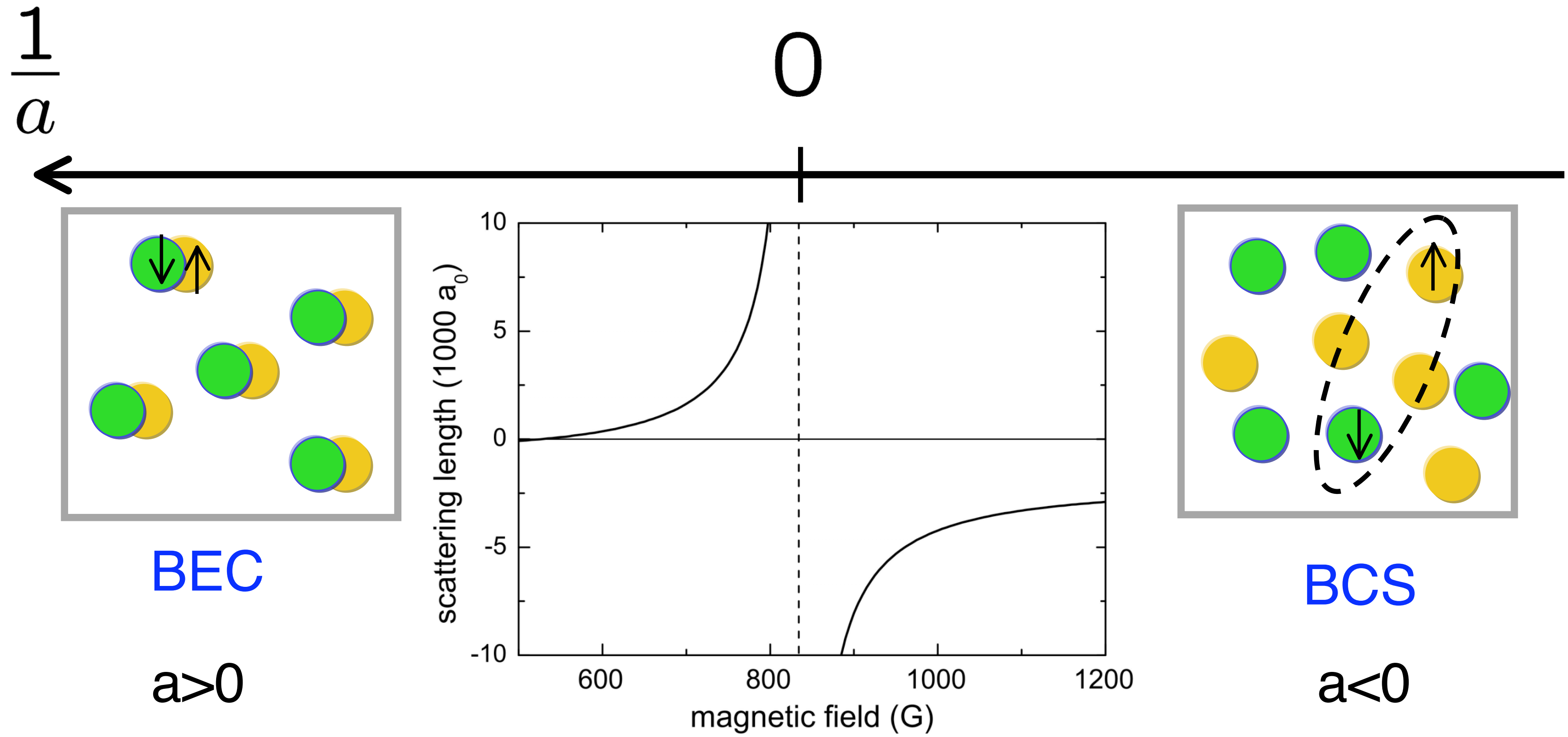
[C. A. Regal *et al.*, *Nature* **424**, 47 (2003)]

Feshbach resonances



[C. A. Regal *et al.*, *Nature* **424**, 47 (2003)]

BEC-BCS crossover



- weakly repulsive diatomic molecules

- weakly attractive fermionic atoms

At T=0, described by the same ground state

T=0 mean-field theory

$$\hat{\mathcal{H}} = \sum_{\mathbf{k},\sigma=\uparrow,\downarrow} \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{1}{V} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} U_{\mathbf{k},\mathbf{k}'} c^\dagger_{\mathbf{k}+\mathbf{q}/2\uparrow} c^\dagger_{-\mathbf{k}+\mathbf{q}/2\downarrow} c_{-\mathbf{k}'+\mathbf{q}/2\downarrow} c_{\mathbf{k}'+\mathbf{q}/2\uparrow}$$

- Fix the total number of atoms (grand-canonical): N.B. $\mu \neq \varepsilon_F$

$$\hat{N} = \sum_{\mathbf{k},\sigma=\uparrow,\downarrow} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} \qquad \boxed{\hat{\mathcal{H}} - \mu \hat{N}}$$

- The BCS ground state can also describe the BEC limit

$$|\psi\rangle = \prod_{\mathbf{k}} \left(\cos\theta_{\mathbf{k}} + \sin\theta_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} \right) |0\rangle$$

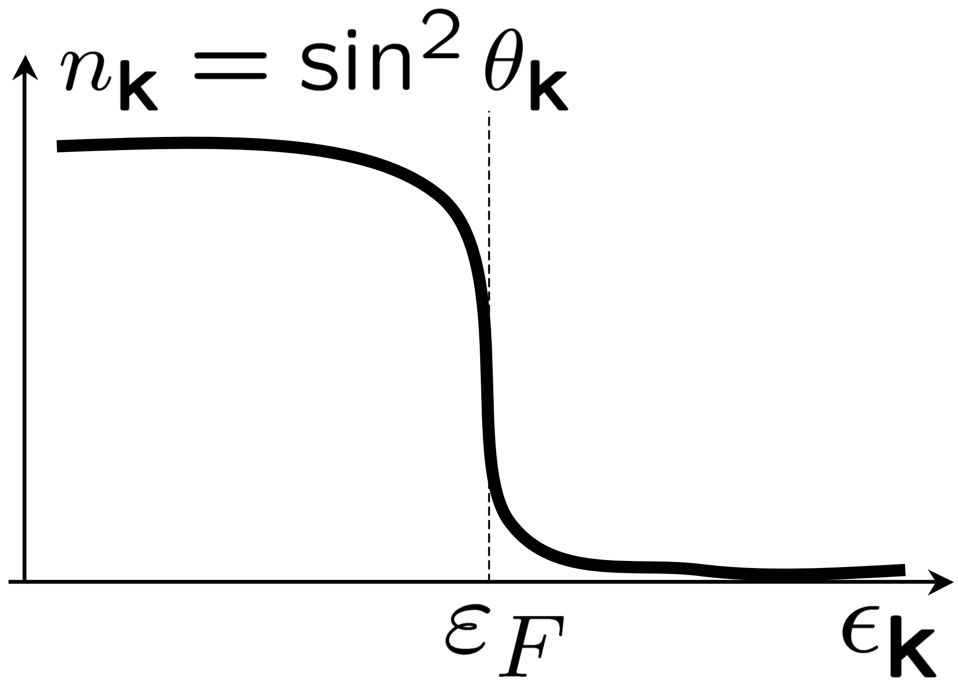
- Order parameter $\Delta_{\mathbf{k}} = -\frac{1}{V} \sum_{\mathbf{k}'} U_{\mathbf{k},\mathbf{k}'} \langle \psi | c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow} | \psi \rangle$

- Gap & number equation (now solve simultaneously as $\mu \neq \varepsilon_F$)

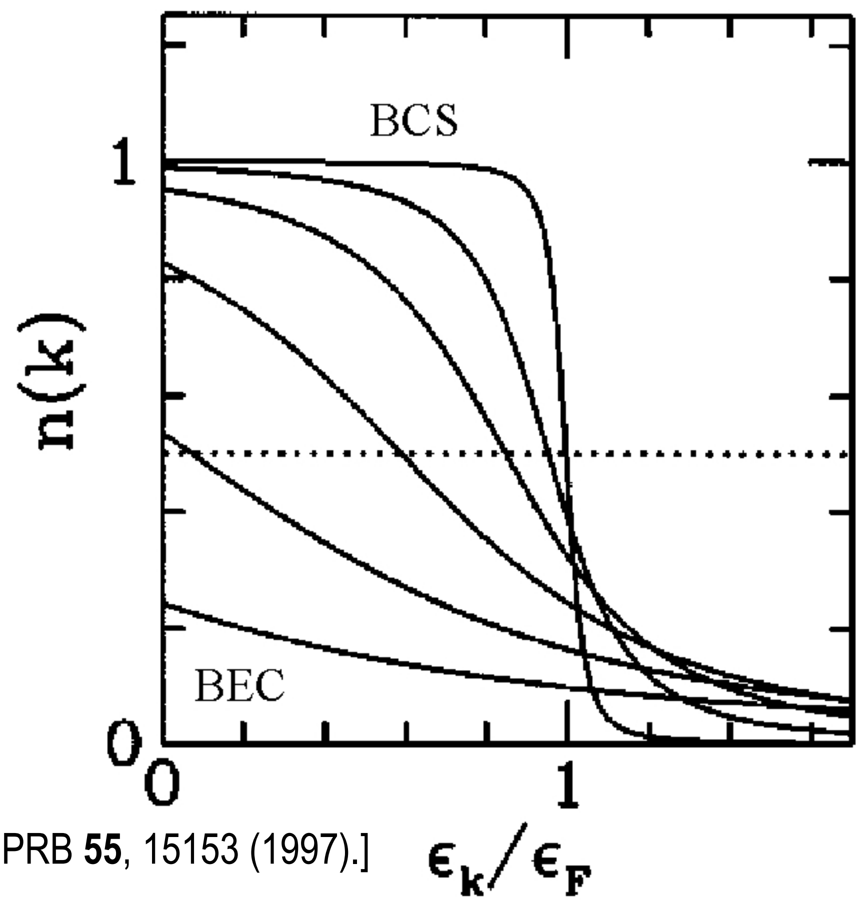
$$\Delta_{\mathbf{k}} = -\frac{1}{V} \sum_{\mathbf{k}'} U_{\mathbf{k},\mathbf{k}'} \frac{\Delta_{\mathbf{k}'}}{2E_{\mathbf{k}'}} \qquad n = \frac{1}{V} \sum_{\mathbf{k}} \left(1 - \frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}} \right)$$

T=0 mean-field theory

- We remember that in the BCS limit



…now however



[J.R. Engelbrecht *et al.*, PRB **55**, 15153 (1997).]

T=0 mean-field theory

- Contact interaction

$$\hat{\mathcal{H}} = \sum_{\mathbf{k},\sigma=\uparrow,\downarrow} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{g}{V} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} c^{\dagger}_{\mathbf{k}+\mathbf{q}/2\uparrow} c^{\dagger}_{-\mathbf{k}+\mathbf{q}/2\downarrow} c_{-\mathbf{k}'+\mathbf{q}/2\downarrow} c_{\mathbf{k}'+\mathbf{q}/2\uparrow}$$

- Introduce the scattering length (T-matrix: see App. B)

$$\frac{m}{4\pi a} = \frac{1}{g} + \frac{1}{V} \sum_{\mathbf{k}}^{k_0=1/R_e} \frac{1}{2\epsilon_{\mathbf{k}}}$$

- Now no (log) divergence in the gap equation

$$\frac{m}{4\pi a} = \frac{1}{V} \sum_{\mathbf{k}}^{k_0=1/R_e\to\infty} \left(\frac{1}{2\epsilon_{\mathbf{k}}} - \frac{1}{2E_{\mathbf{k}}} \right)$$

- Solved simultaneously with the number equation

$$n = \frac{1}{V} \sum_{\mathbf{k}} \left(1 - \frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}} \right)$$

BCS limit

- Problem 3

$$\frac{1}{k_F a} \to -\infty$$

$$\varepsilon_F \simeq \mu$$ from number equation

$$\Delta \ll \varepsilon_F$$

$$\Delta \simeq \varepsilon_F \exp\left(-\frac{\pi}{2|a|k_F}\right)$$ from gap equation

BEC limit

- Problem 3

$$\frac{1}{k_F a} \to +\infty$$

$$\varepsilon_F \ll \Delta \ll |\mu|$$

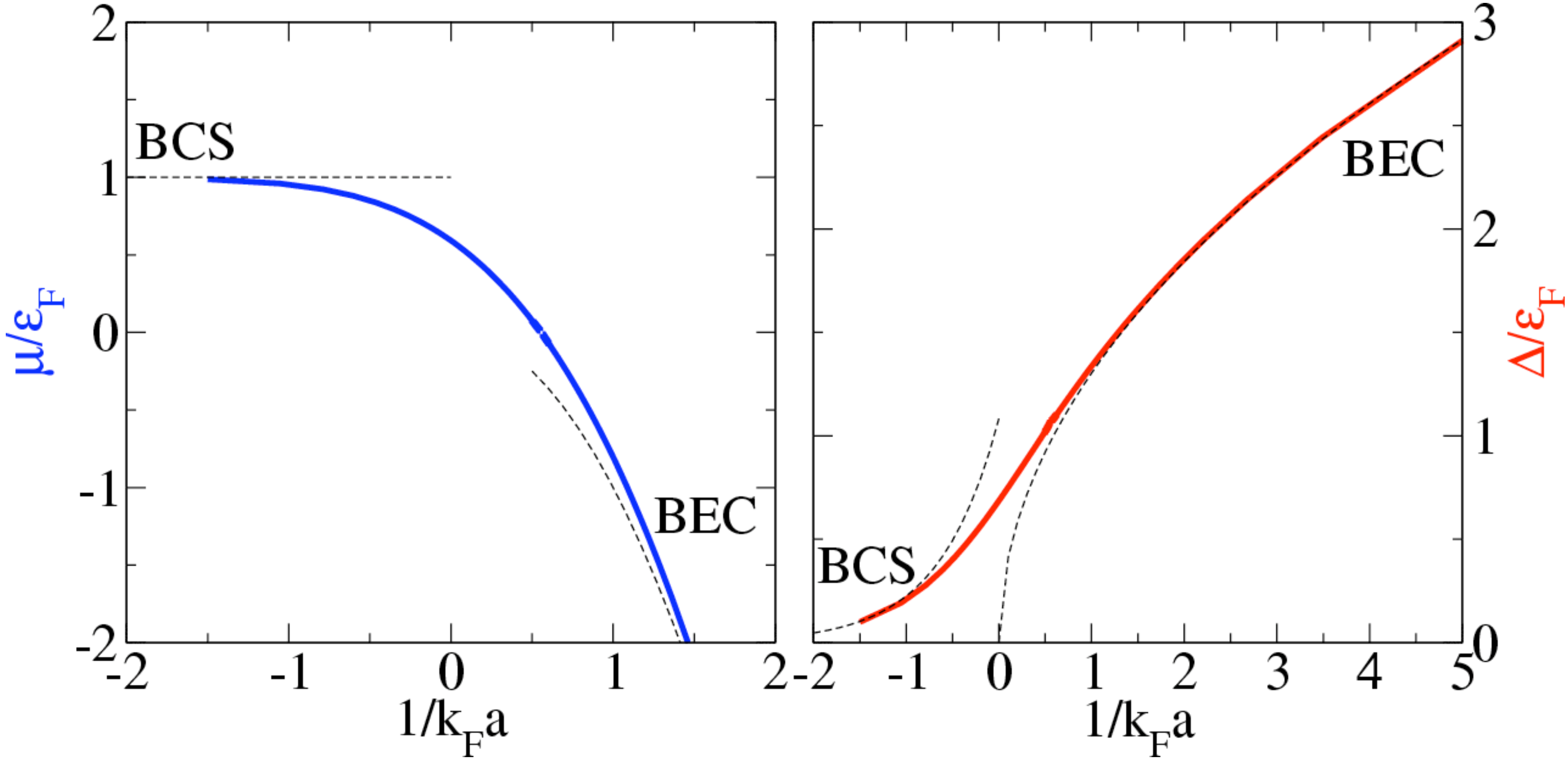
$$\Delta \simeq \sqrt{\frac{16}{3\pi}}\varepsilon_F \sqrt{\frac{1}{k_F a}}$$

from number equation

$$\mu \simeq \frac{\epsilon_b}{2} = -\frac{1}{2ma^2}$$

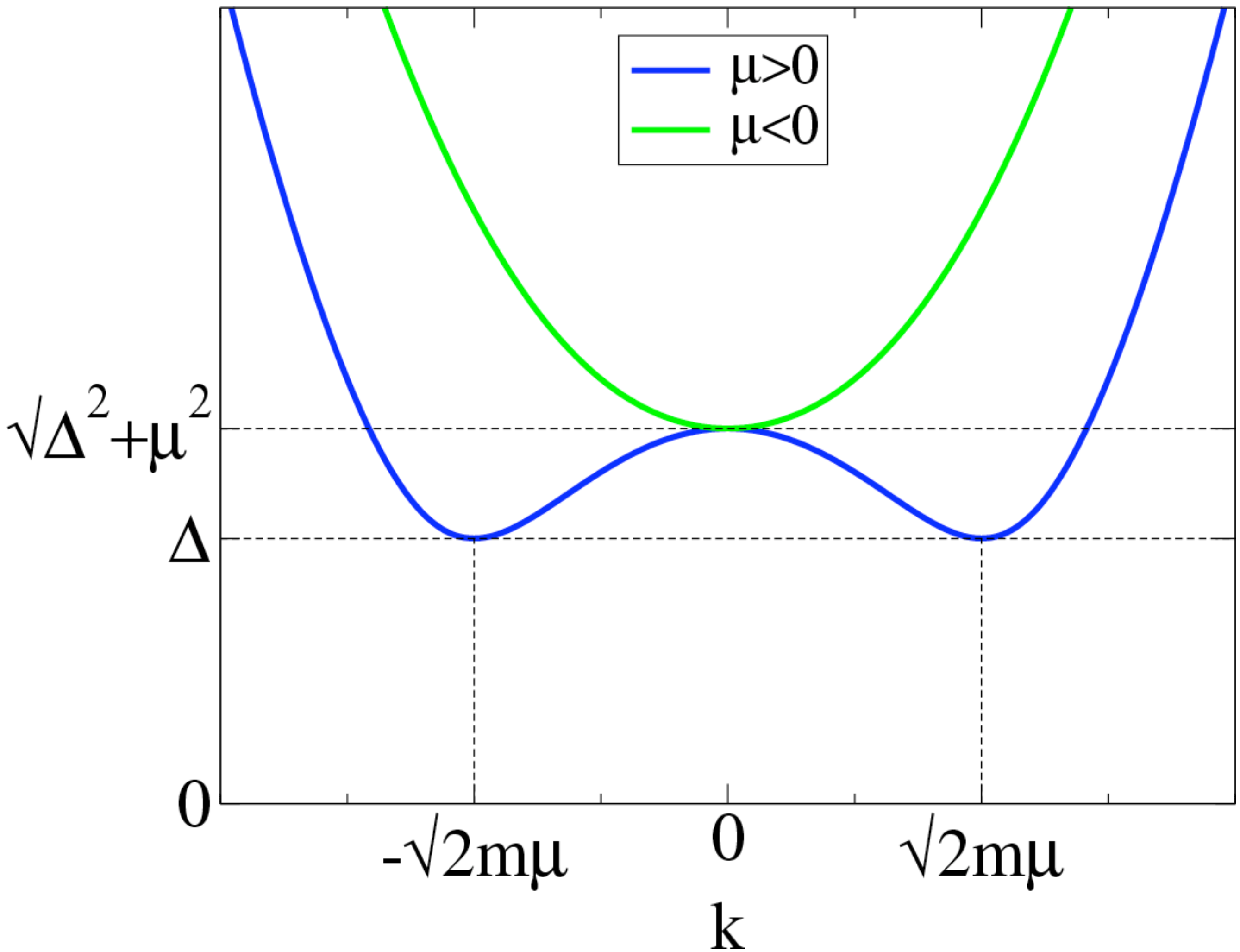
from gap equation

Crossover



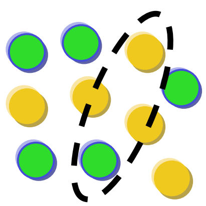
Spectrum of excitations

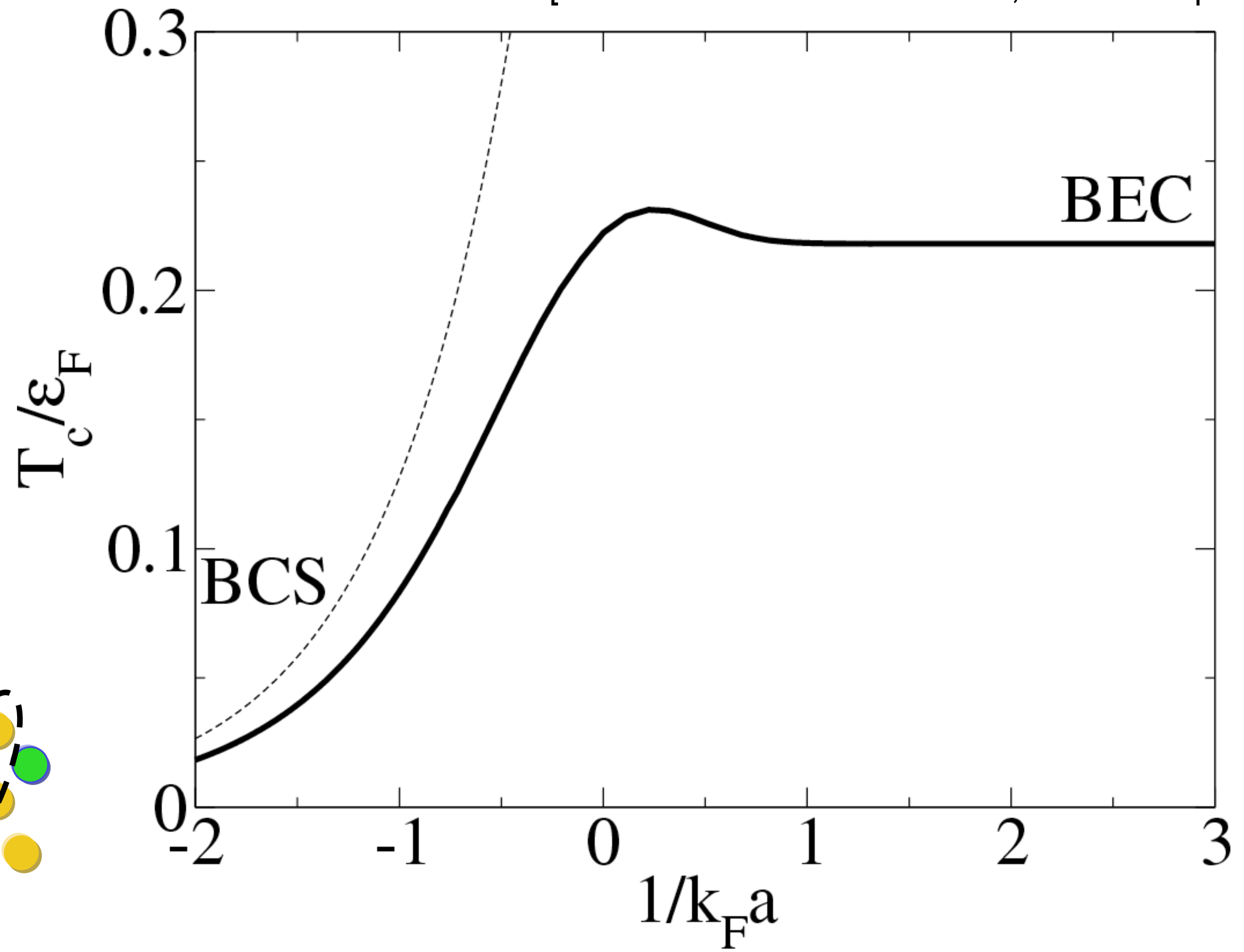
$$E_{\mathrm{gap}} = \min_{\mathbf{k}} \sqrt{(\epsilon_{\mathbf{k}} - \mu)^2 + \Delta^2} = \begin{cases} \Delta & \mu > 0 \\ \sqrt{\Delta^2 + \mu^2} & \mu < 0 \end{cases}$$

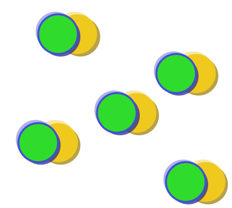


Finite T

[P. Nozieres & S. Schmitt-Rink, J. Low temp. Phys. **59**, 195 (1985)]







BCS: pairing instability

$$\Delta_{\mathrm{BCS}} \sim k_B T_{\mathrm{BCS}} \ll \varepsilon_F$$

BEC: condensate forms out of preformed molecules

$$T_{\mathrm{BEC}} \sim T_F \ll T_{\mathrm{diss}}$$

Finite T

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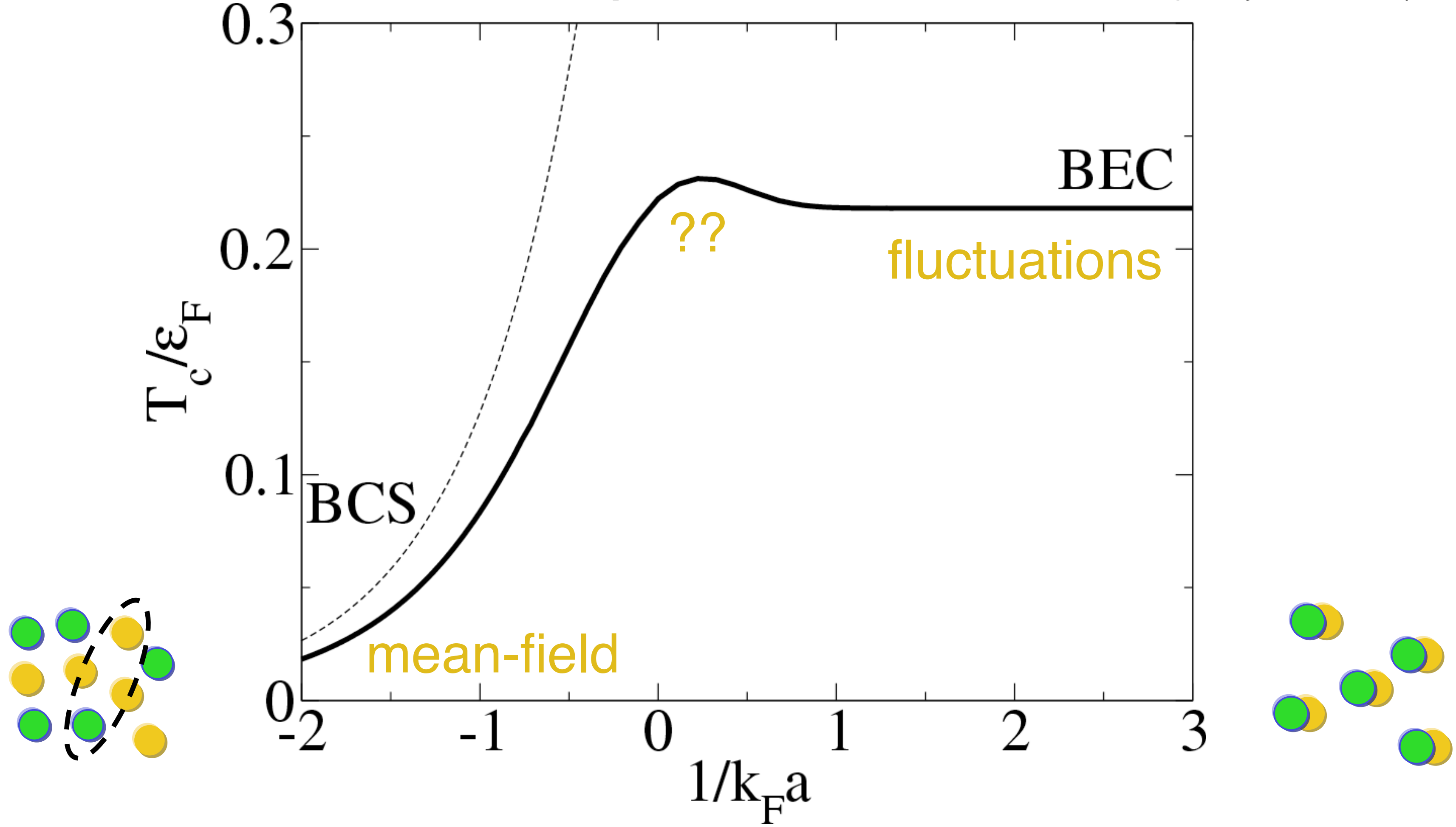


BCS: pairing instability

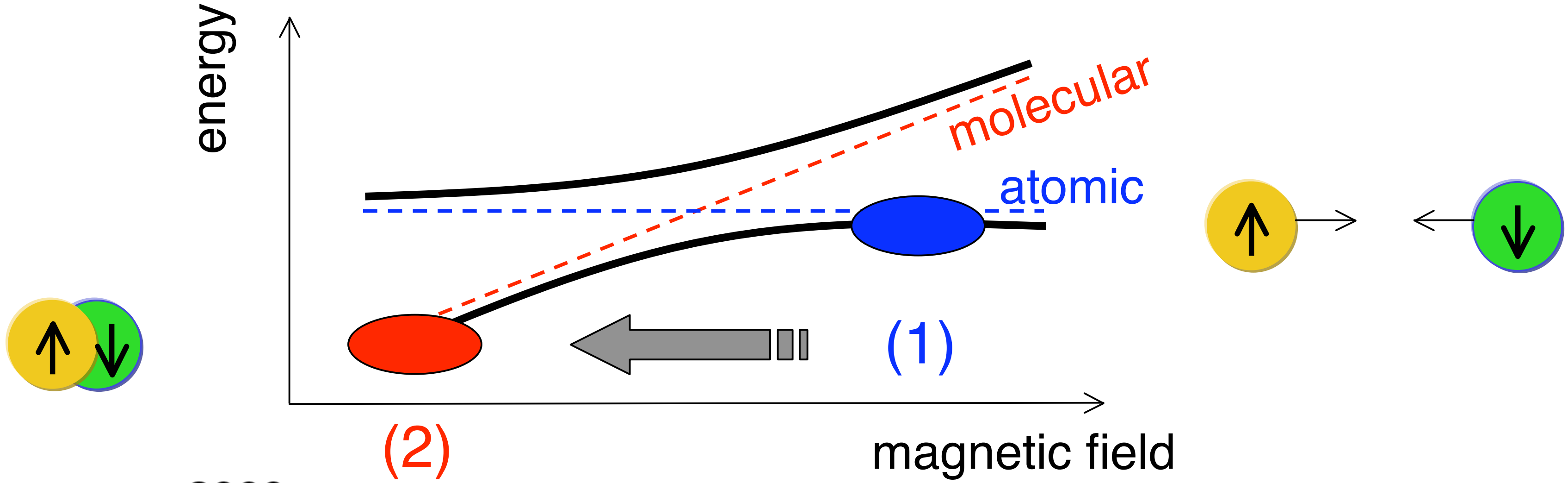
$$\Delta_{\text{BCS}} \sim k_B T_{\text{BCS}} \ll \varepsilon_F$$

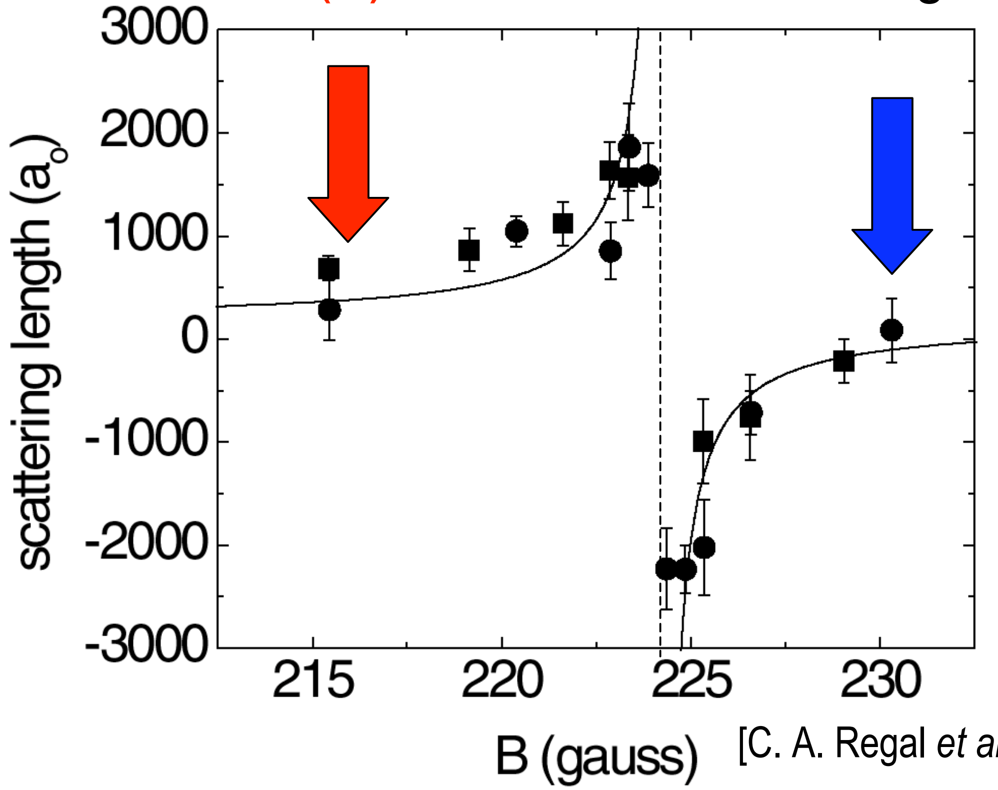
BEC: condensate forms out of preformed molecules

$$T_{\text{BEC}} \sim T_F \ll T_{\text{diss}}$$

What is measured in experiments?

Molecule formation





A. Very fast sweep: nothing happens

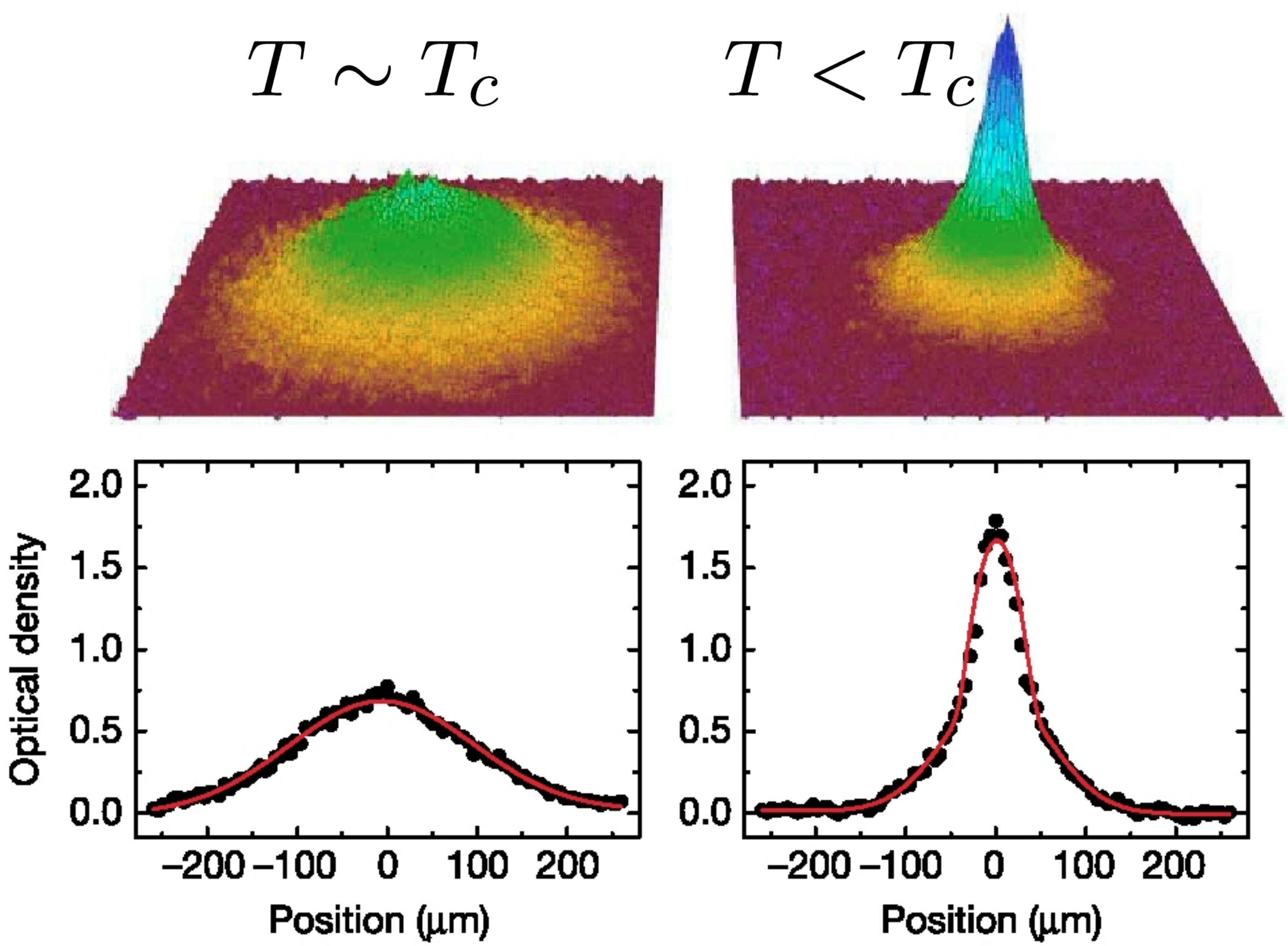
B. Slow sweep: creation of molecules

C. Very slow sweep: creation of a BEC of molecules

[C. A. Regal *et al.*, *Nature* **424**, 47 (2003)]

BEC of diatomic molecules

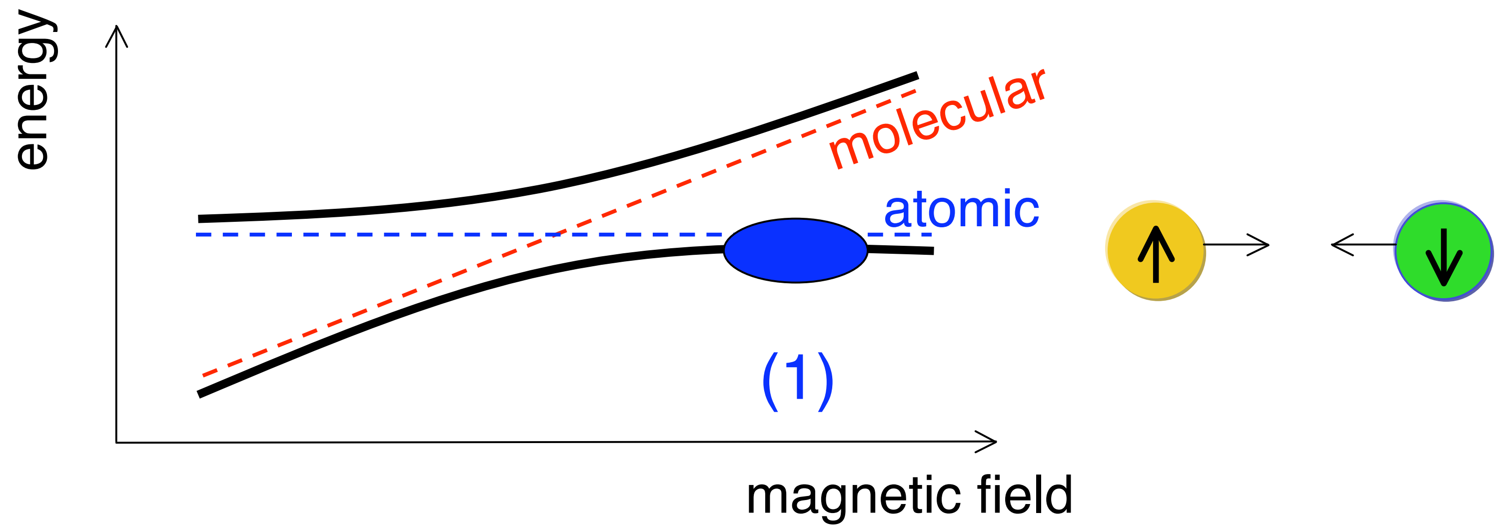
- Bimodal distribution for the molecular cloud



[M. Greiner *et al.*, *Nature* **426**, 537 (2004)]

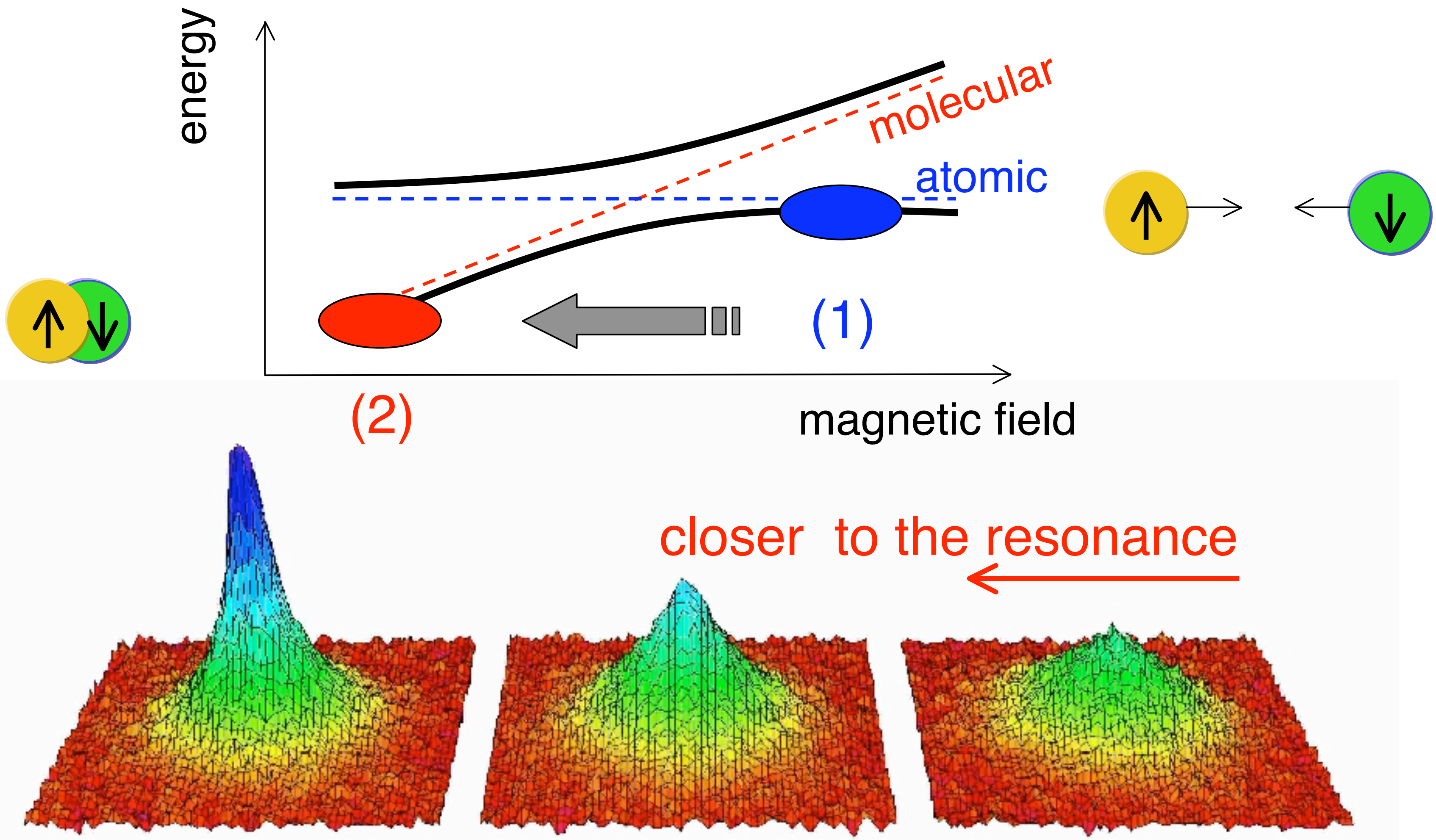
Condensation on the BCS side

- You would not see much from the density distribution!!



Condensation on the BCS side

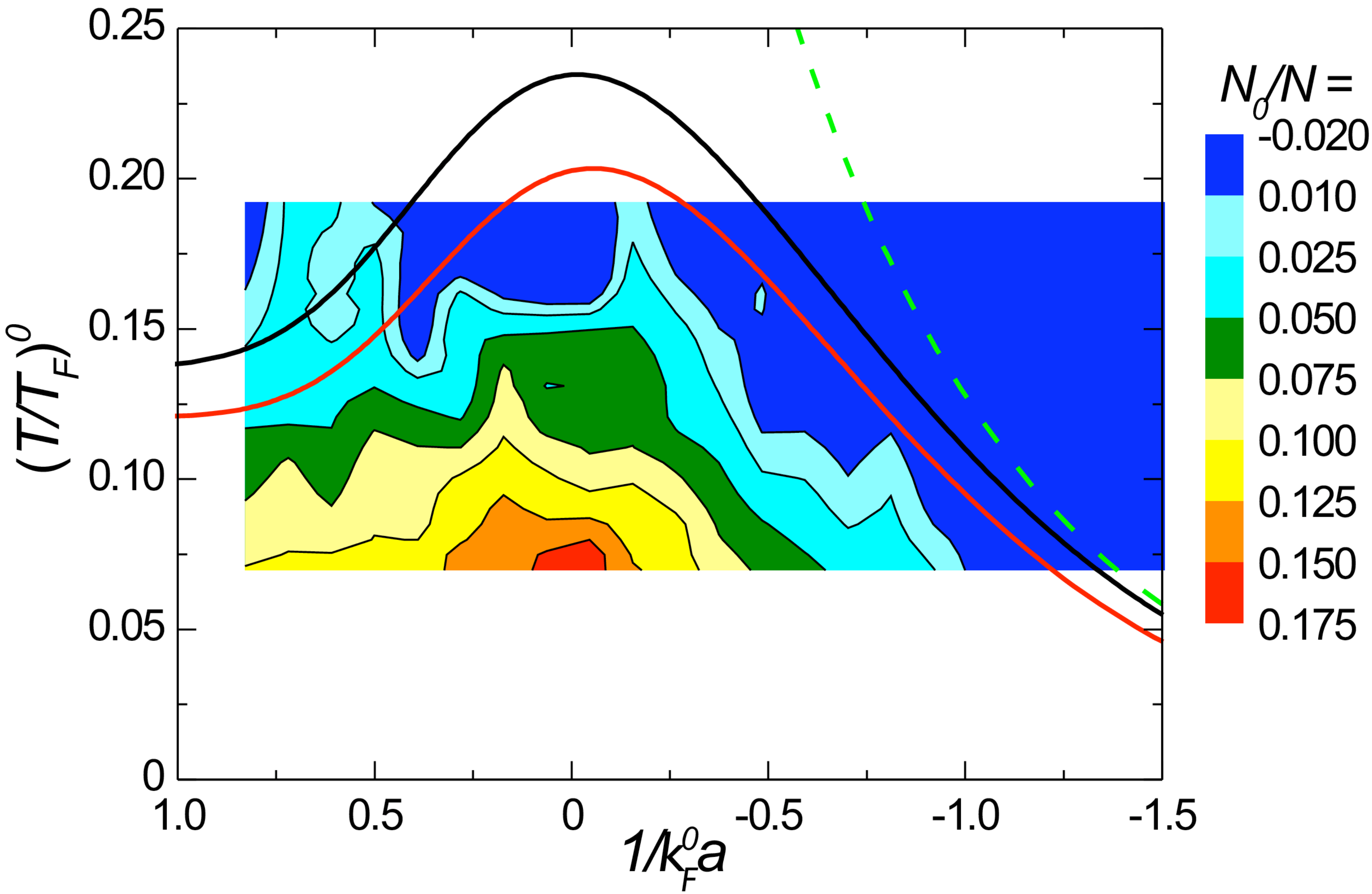
- BCS of Fermi pairs: probe the condensate by pair-wise projection into molecules



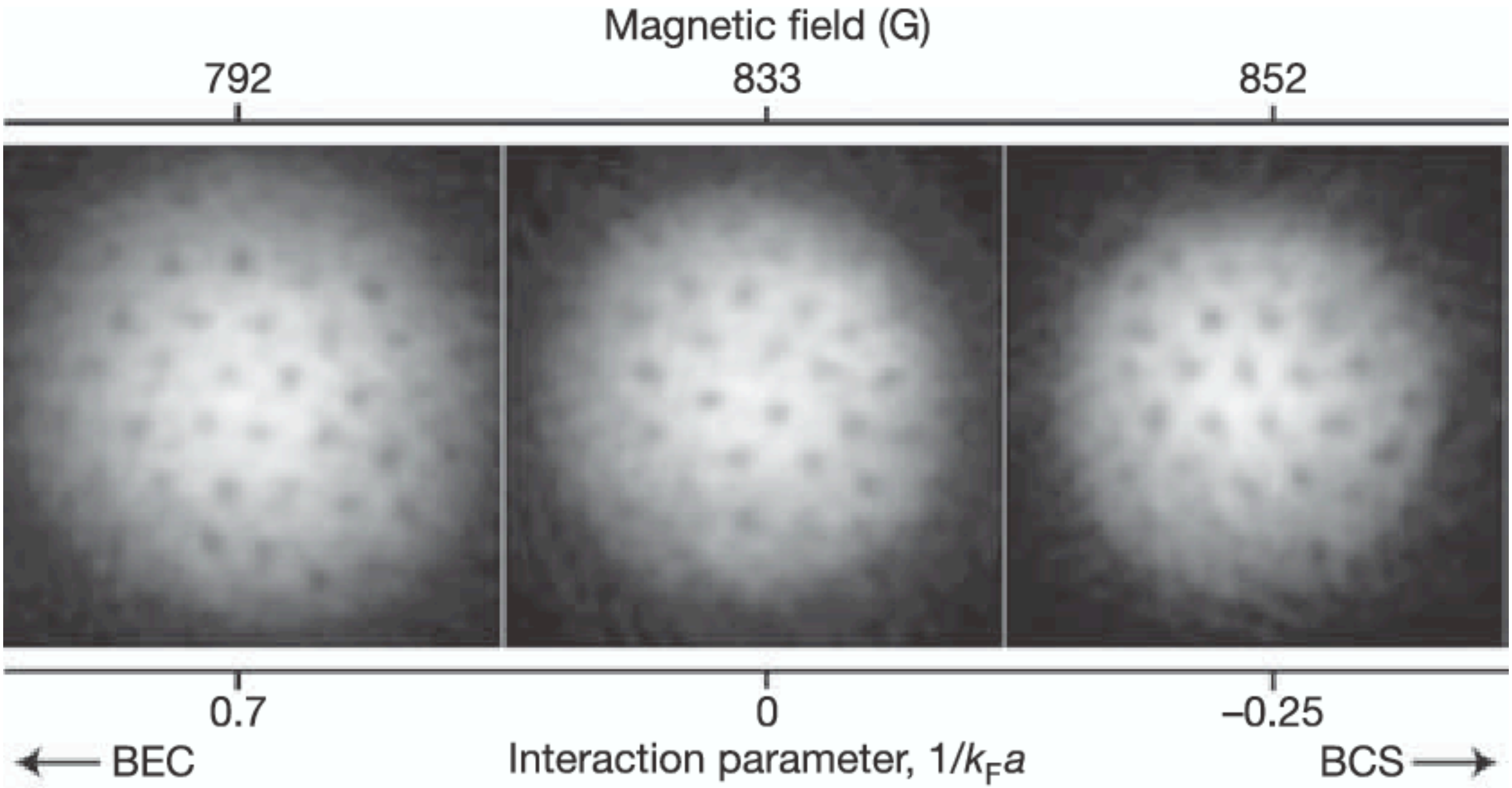
[C. A. Regal *et al.*, PRL **92**, 040403 (2004)]

BEC-BCS Crossover in Experiments

[Q.J. Chen *et al.*, PRA **73**, 041601 (2006)]

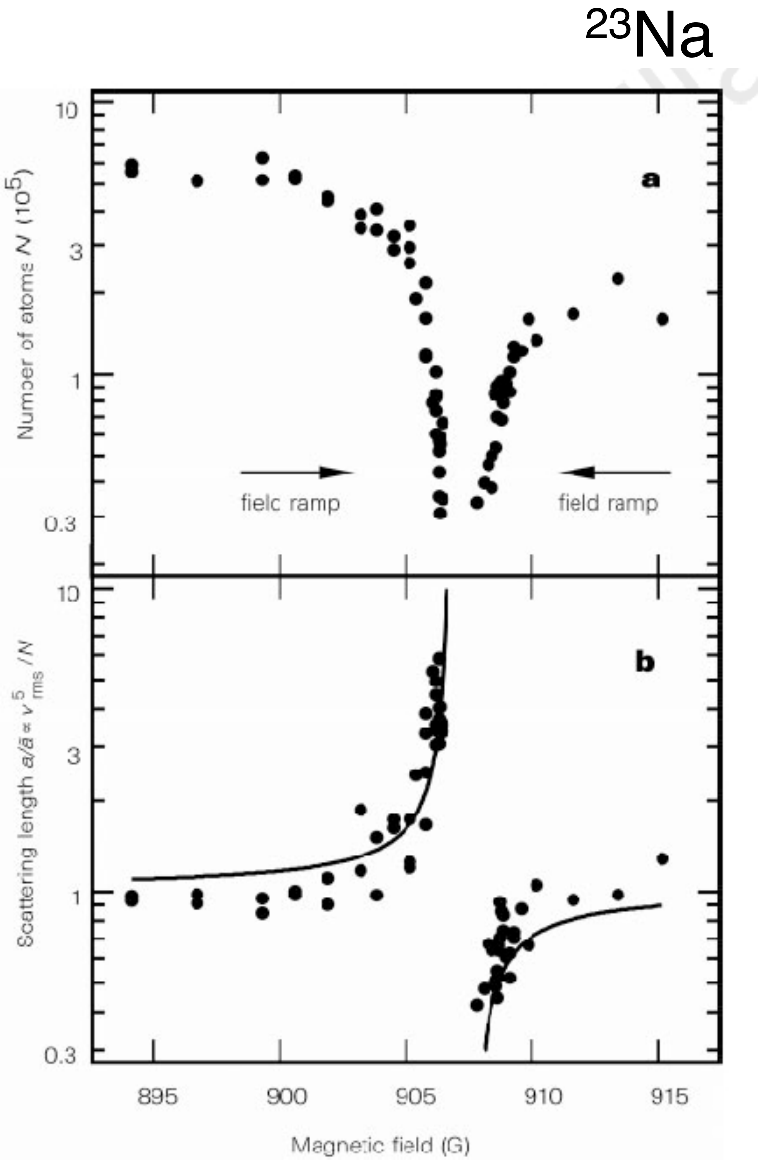


Superfluidity across the resonance

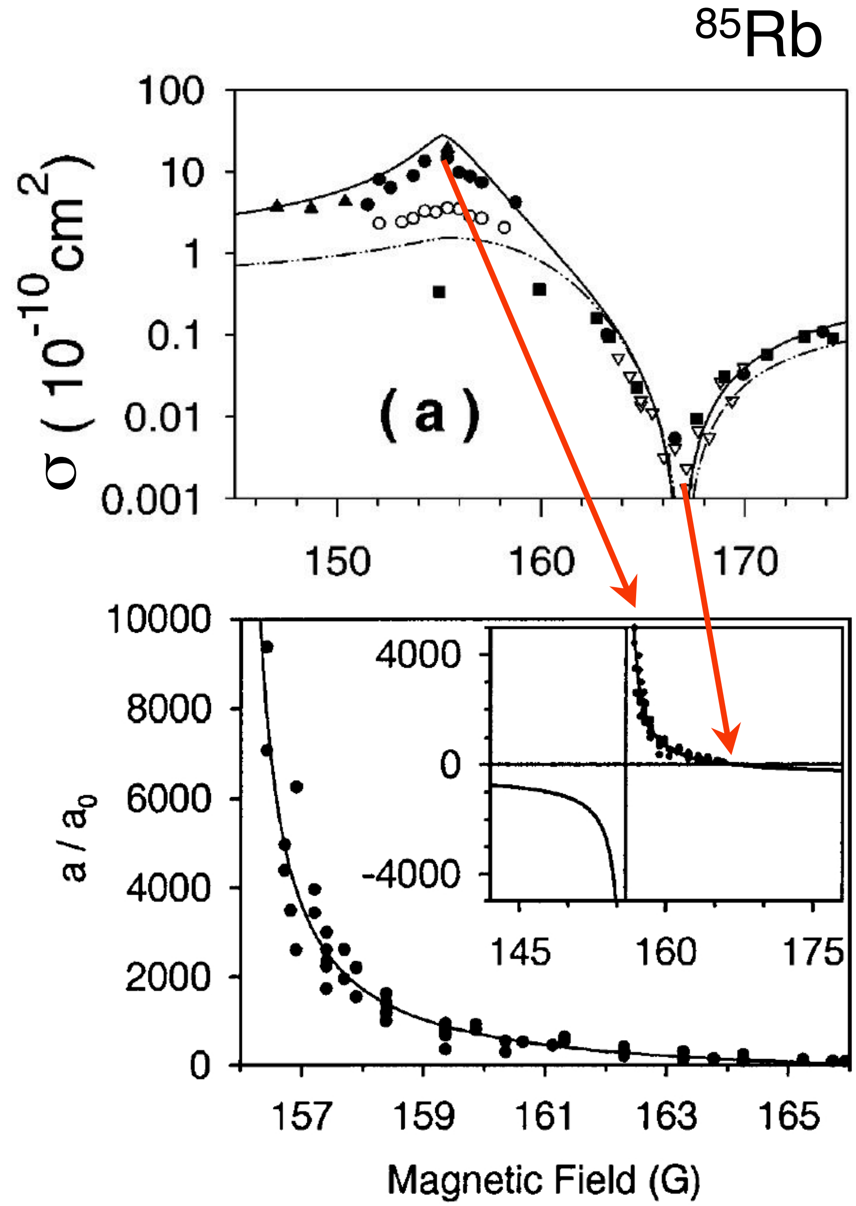


[M.W. Zwierlein *et al.*, Nature **435**, 1047 (200)]]

Feshbach resonances



[S. Inouye *et al*., Nature **392**, 151 (1998)]



[J. L. Roberts *et al*., PRL **81**, 5109 (1998)]
[S. L. Cornish *et al*., PRL **85**, 1795 (2000)]