Course in 3 lectures on

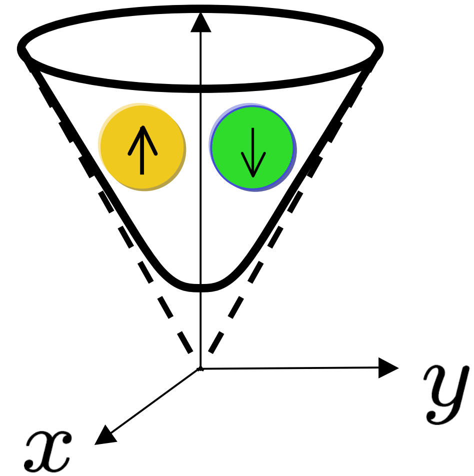
Superfluidity in Ultracold Fermi Gases

F.M. Marchetti

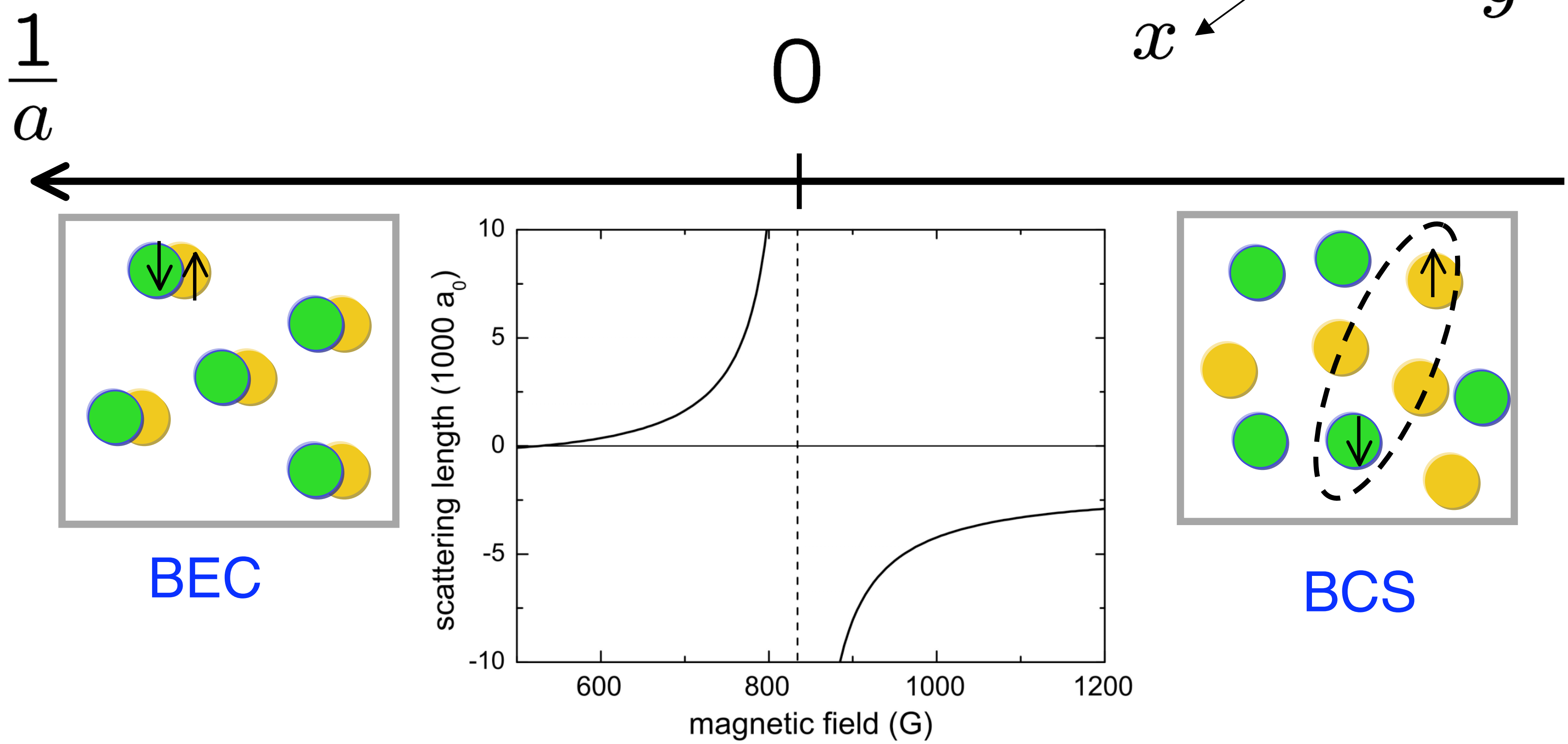
Physics by the Lake, Ambleside, 11, 12, 13 September 2007

During lecture II. …

1) Tuneable interaction strength in two-component Fermi mixtures

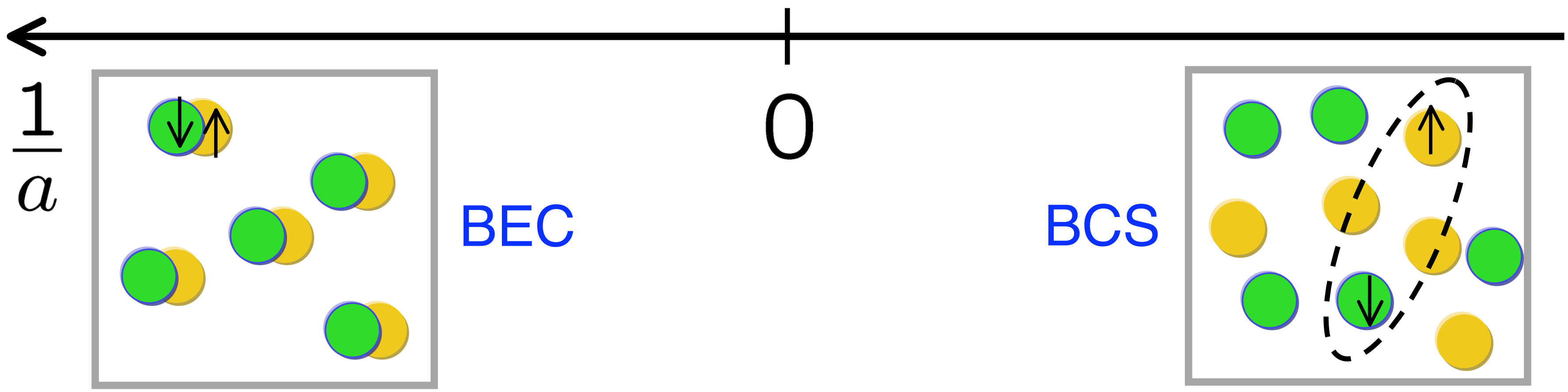


2) BEC-BCS crossover

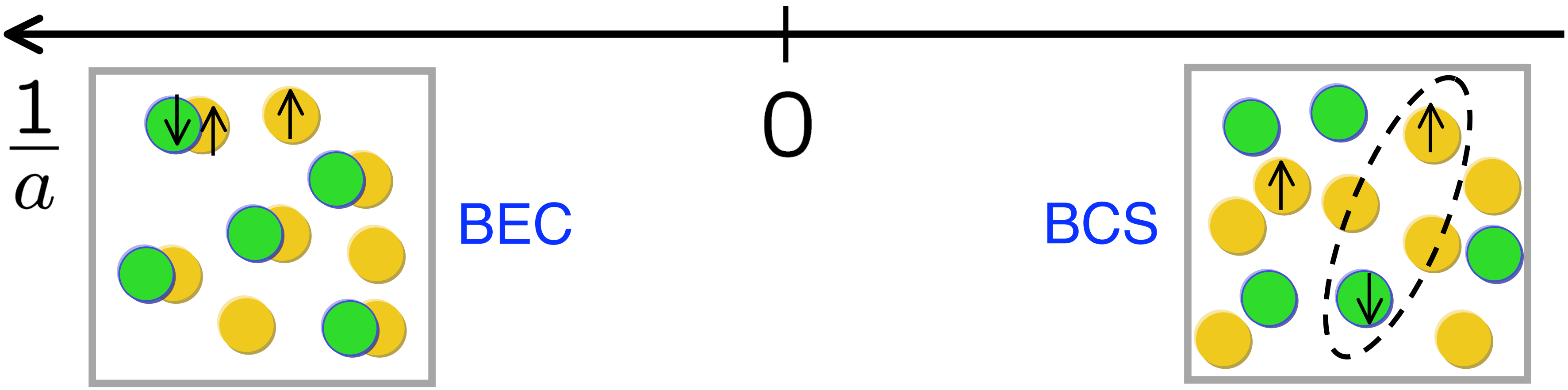


III. Polarised Fermi gases

Balanced populations



Population imbalance



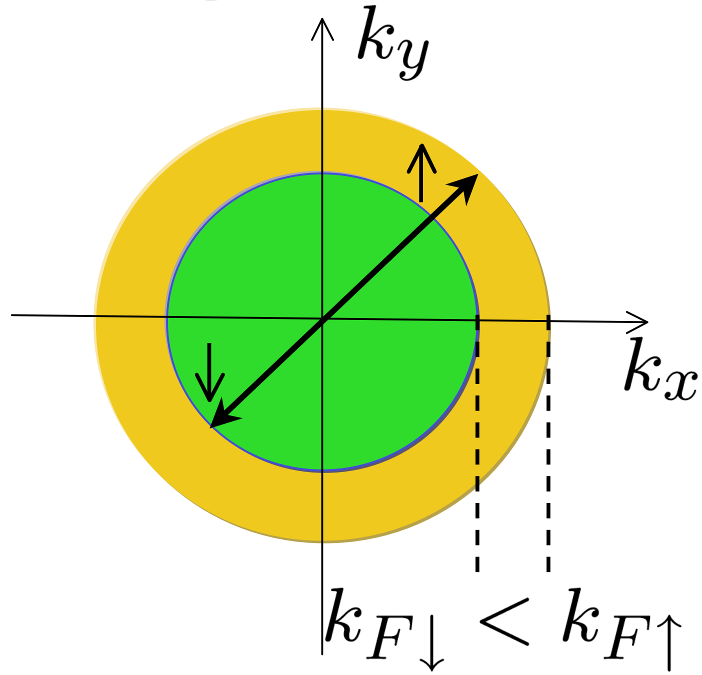
- Control over the atom number in different spin species

Can superfluidity persist in presence of
a population imbalance?

The many-body version of Gunnar dilemma

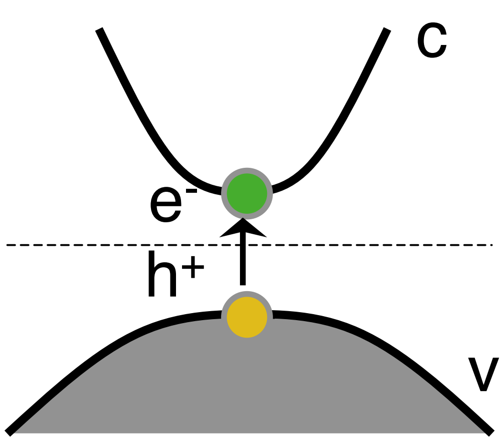
Why interesting?

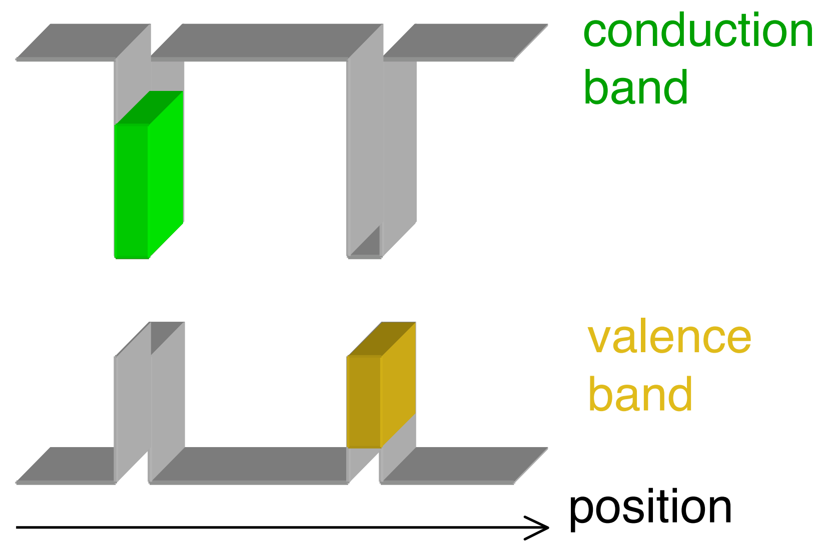
- Magnetised superconductors (Zeeman)



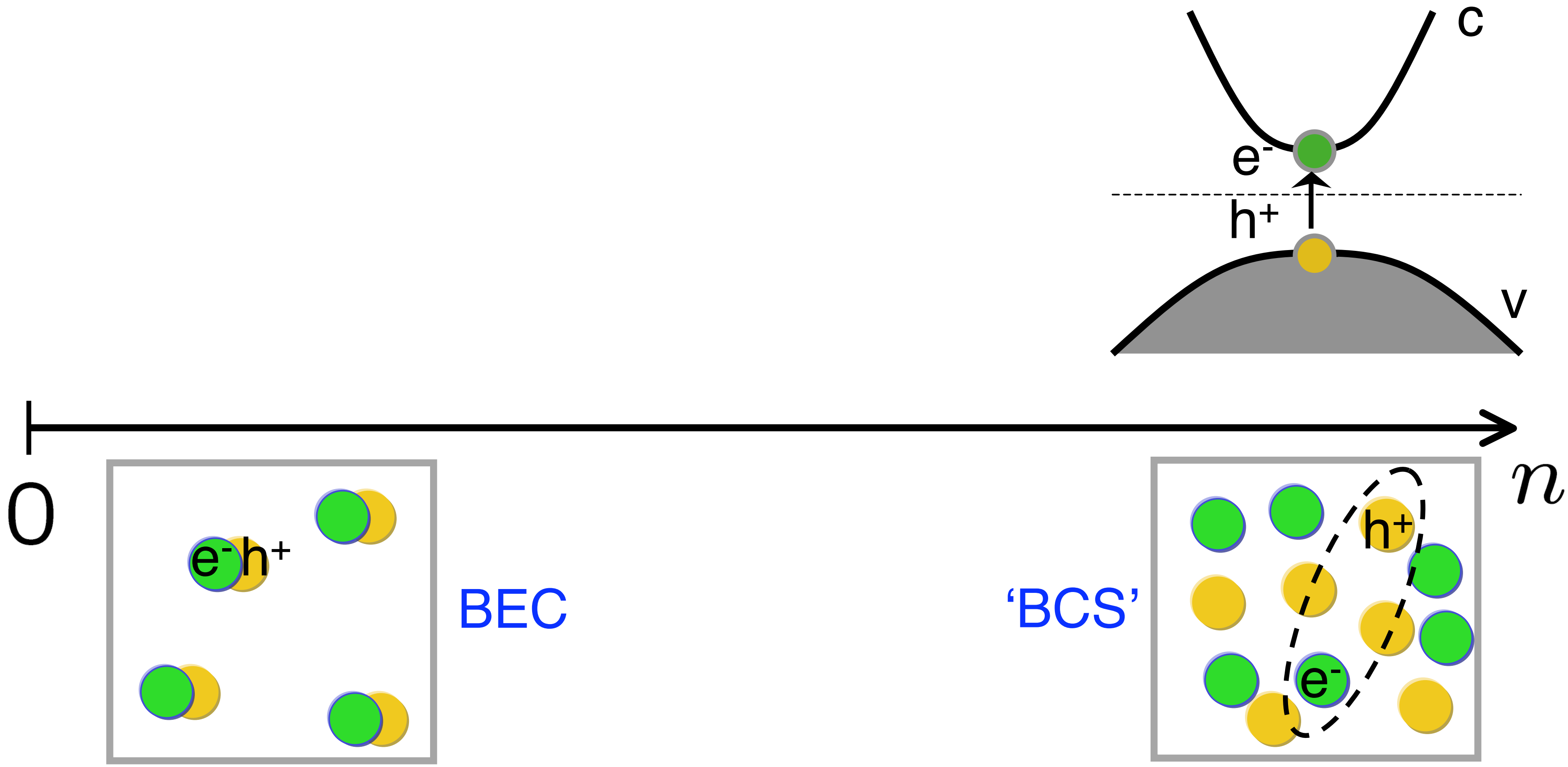
- Quantum Chromodynamics (and neutron stars)

- Electron-hole bilayers





BEC-BCS crossover in electron-hole systems!



BEC of excitons!

exciton insulator

[L.V. Keldysh & Yu V. Kopaev, *Sov. Phys. Solid State* **6**, 2219 (1965)]

The model for polarised Fermi gases

$$\hat{\mathcal{H}} - \sum_{\sigma=\uparrow,\downarrow} \mu_\sigma \hat{N}_\sigma$$
$$= \sum_{\mathbf{k},\sigma=\uparrow,\downarrow} (\epsilon_{\mathbf{k}} - \mu_\sigma) c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{g}{V} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} c^\dagger_{\mathbf{k}+\mathbf{q}/2\uparrow} c^\dagger_{-\mathbf{k}+\mathbf{q}/2\downarrow} c_{-\mathbf{k}'+\mathbf{q}/2\downarrow} c_{\mathbf{k}'+\mathbf{q}/2\uparrow}$$

- Allow for different densities of atoms in the two spin states

$$\begin{cases} \hat{n}_\uparrow = \dfrac{1}{V} \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} c_{\mathbf{k}\uparrow} \\ \hat{n}_\downarrow = \dfrac{1}{V} \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\downarrow} c_{\mathbf{k}\downarrow} \end{cases}$$

- Averaged chemical potential & ‘Zeeman’ term
  (total density & population imbalance [or ‘magnetisation’])

$$\mu = (\mu_\uparrow + \mu_\downarrow)/2 \qquad \hat{n} = \hat{n}_\uparrow + \hat{n}_\downarrow$$
$$h = (\mu_\uparrow - \mu_\downarrow)/2 \qquad \hat{m} = \hat{n}_\uparrow - \hat{n}_\downarrow$$

Mean-field & ground state

$$\hat{H} - \sum_{\sigma=\uparrow,\downarrow} \mu_\sigma \hat{N}_\sigma$$
$$\simeq \sum_{\mathbf{k}} \begin{pmatrix} c^\dagger_{\mathbf{k}\uparrow} & c_{-\mathbf{k}\downarrow} \end{pmatrix} \begin{pmatrix} \xi_{\mathbf{k}} - h & -\Delta \\ -\Delta & -\xi_{\mathbf{k}} - h \end{pmatrix} \begin{pmatrix} c_{\mathbf{k}\uparrow} \\ c^\dagger_{-\mathbf{k}\downarrow} \end{pmatrix} + \sum_{\mathbf{k}} (\xi_{\mathbf{k}} + h) - \frac{\Delta^2}{g} V$$

- No change in the symmetry of the Hamiltonian!

$$\hat{H} - \sum_{\sigma} \mu_\sigma \hat{N}_\sigma \simeq \sum_{\mathbf{k},\sigma=\uparrow,\downarrow} E_{\mathbf{k}\sigma} \gamma^\dagger_{\mathbf{k}\sigma} \gamma_{\mathbf{k}\sigma} + \sum_{\mathbf{k}} (\xi_{\mathbf{k}} - E_{\mathbf{k}}) - \frac{\Delta^2}{g} V$$

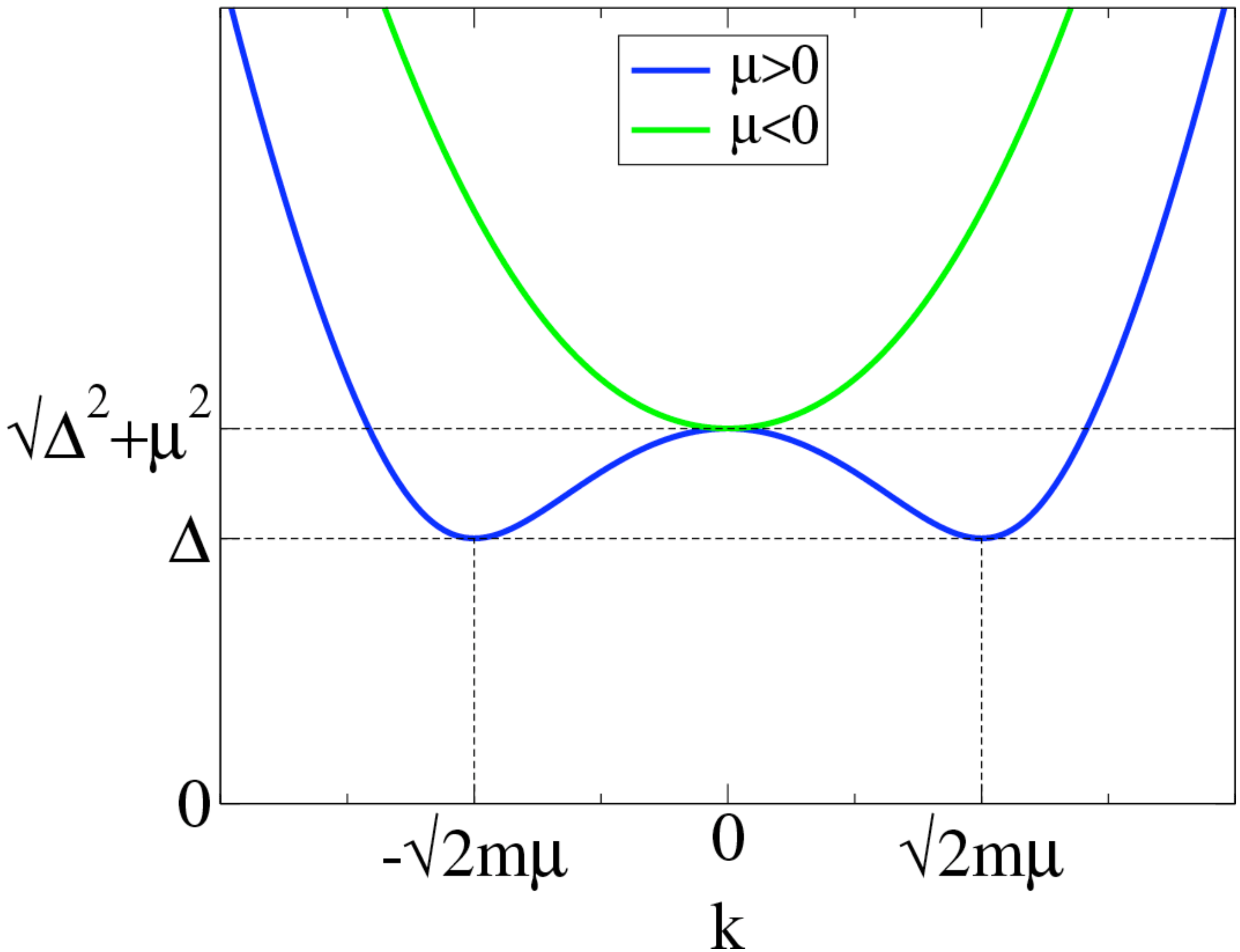
$$E_{\mathbf{k}\sigma} = E_{\mathbf{k}} \mp h = \sqrt{\xi_{\mathbf{k}}^2 + \Delta^2} \mp h$$

…i.e., same ground state $|\psi\rangle = \prod_{\mathbf{k}} \left(\cos\theta_{\mathbf{k}} + \sin\theta_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} \right) |0\rangle$

but different way to fill the states up!

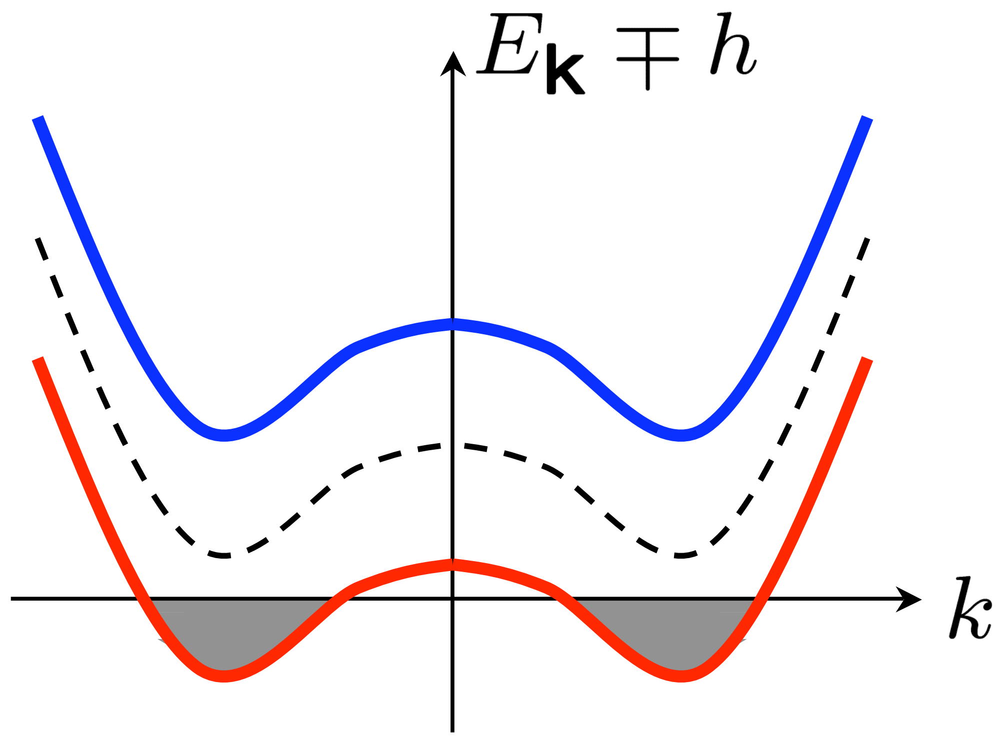
Remember spectrum of excitations when h=0

$$E_{\mathrm{gap}} = \min_{\mathbf{k}} \sqrt{(\epsilon_{\mathbf{k}} - \mu)^2 + \Delta^2} = \begin{cases} \Delta & \mu > 0 \\ \sqrt{\Delta^2 + \mu^2} & \mu < 0 \end{cases}$$



Mean-field & ground state

- E.g., on the BCS side…



…one start depleting paired states when

$$h = \min_{\mathbf{k}} E_{\mathbf{k}} = \Delta \ll \varepsilon_F$$

- We can expect the BCS superfluid phase to be very fragile!

- On the BEC side

$$h = \min_{\mathbf{k}} E_{\mathbf{k}} = \sqrt{\Delta^2 + \mu^2} \sim \mu \gg \varepsilon_F$$

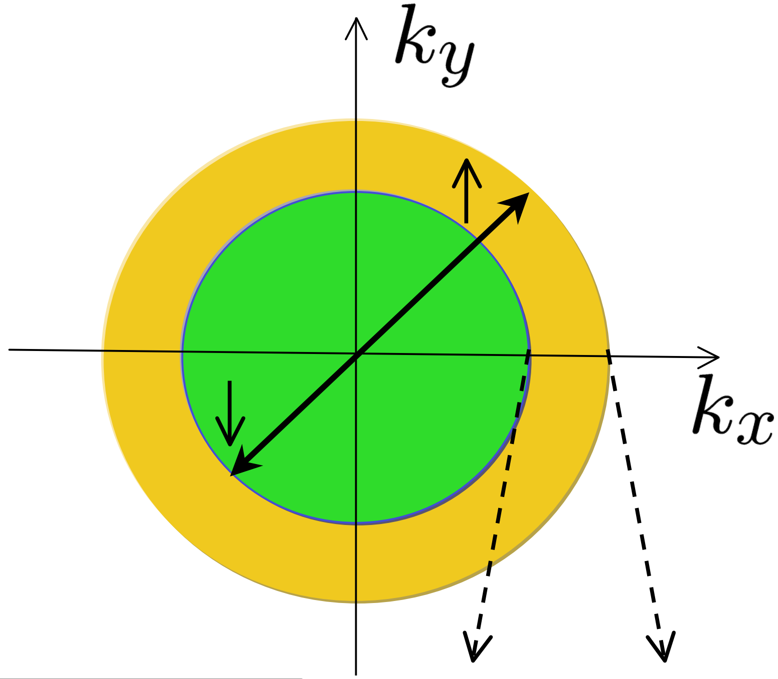
requires much higher h (much more robust superfluid phase to imbalance!)

Why is h called Zeeman term?

- Mean-field Hamiltonian

$$\hat{H} - \sum_{\sigma=\uparrow,\downarrow} \mu_\sigma \hat{N}_\sigma$$

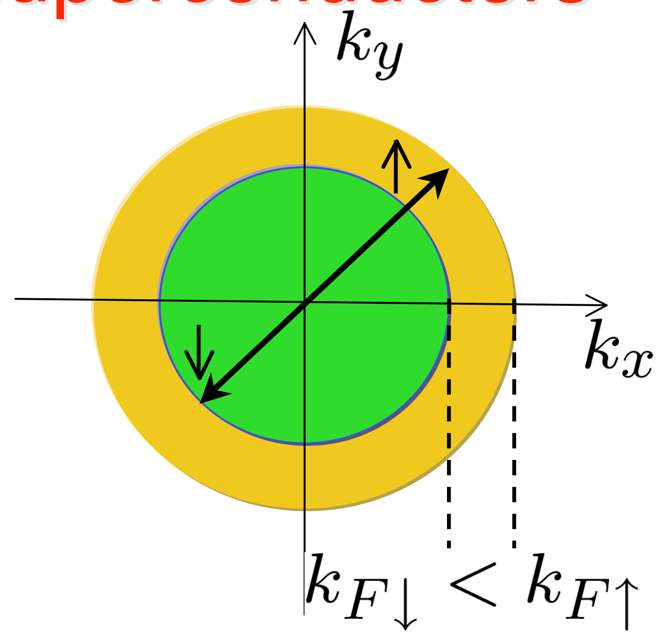
$$\simeq \sum_{\mathbf{k}} \begin{pmatrix} c^\dagger_{\mathbf{k}\uparrow} & c_{-\mathbf{k}\downarrow} \end{pmatrix} \begin{pmatrix} \xi_{\mathbf{k}} - h & -\Delta \\ -\Delta & -\xi_{\mathbf{k}} - h \end{pmatrix} \begin{pmatrix} c_{\mathbf{k}\uparrow} \\ c^\dagger_{-\mathbf{k}\downarrow} \end{pmatrix} + \sum_{\mathbf{k}} (\xi_{\mathbf{k}} + h) - \frac{\Delta^2}{g} V$$



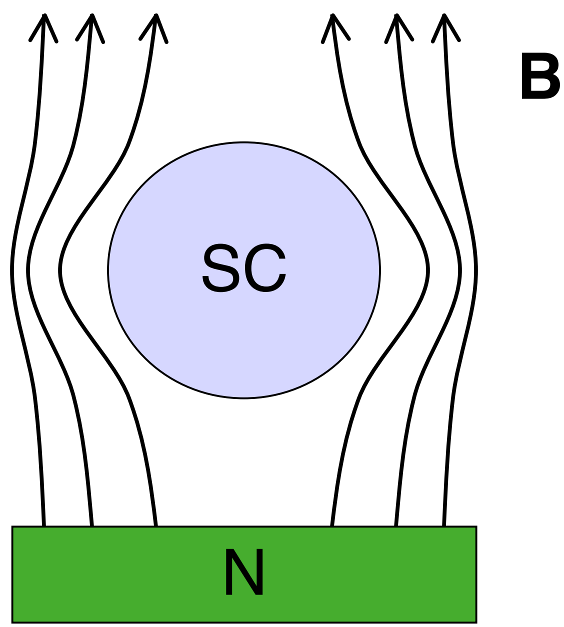
$$\sqrt{2m(\varepsilon_F - h)} = k_{F\downarrow} < k_{F\uparrow} = \sqrt{2m(\varepsilon_F + h)}$$

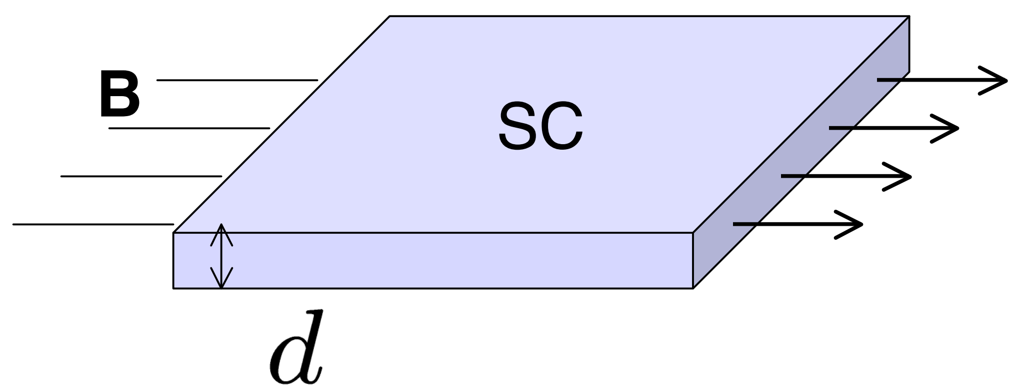
Analogy with magnetised superconductors

- A population imbalance like a Zeeman term in a superconductor



- Neglect the orbital effect?





$$\begin{cases} B_c \simeq \Phi_0/(\xi d) \\ B_Z = \text{const} \end{cases}$$

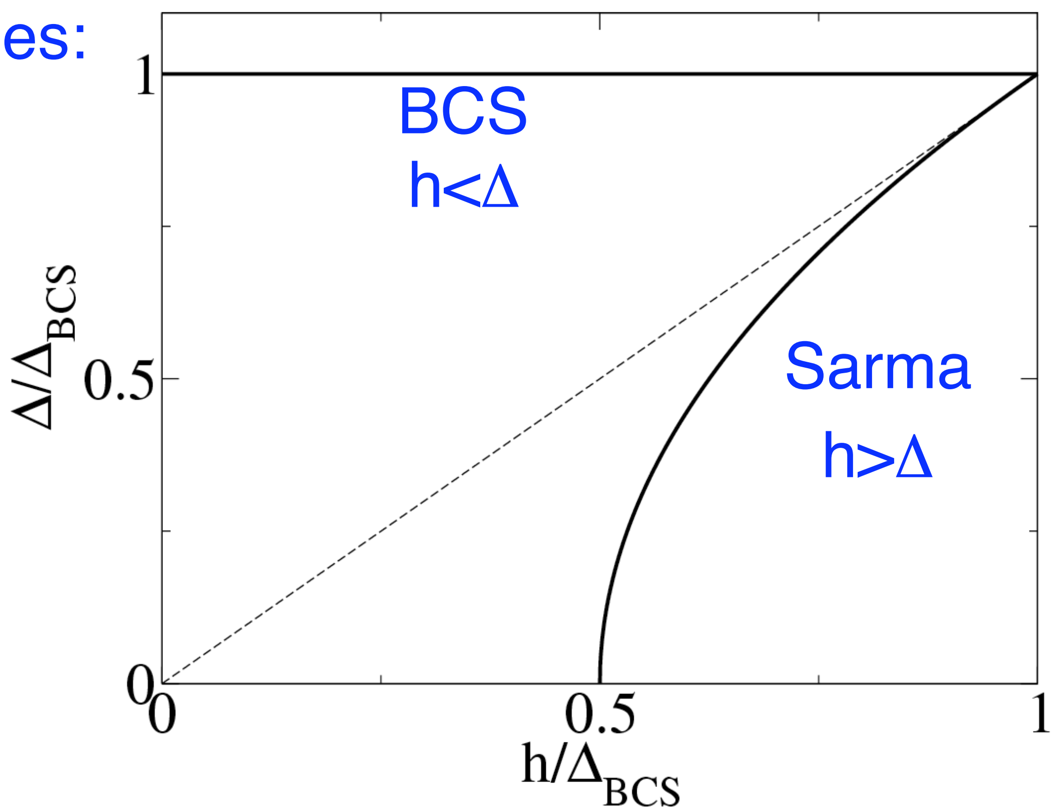
$$d < d_c$$

T=0 magnetised superconductors (BCS side)

- Gap equation (problem 4)

$$\Delta = \lambda \mathcal{N}(\varepsilon_F)\Delta \int_{\sqrt{\max[0,h^2-\Delta^2]}}^{\omega_D} \frac{d\xi}{\sqrt{\xi^2+\Delta^2}} \quad (1)$$

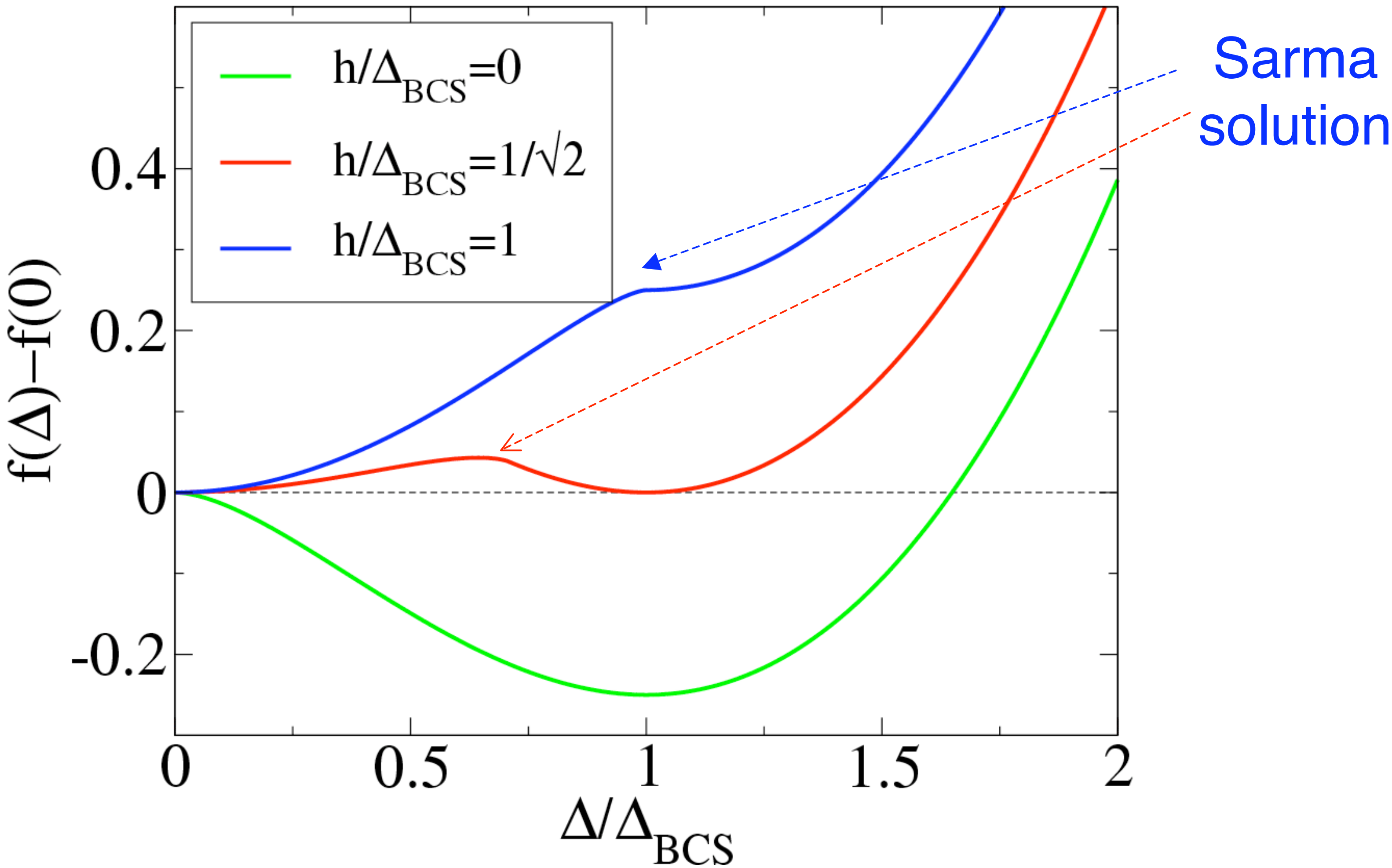
- Two branches:



Which solution shall we choose when $h/\Delta_{BCS}>0.5$?

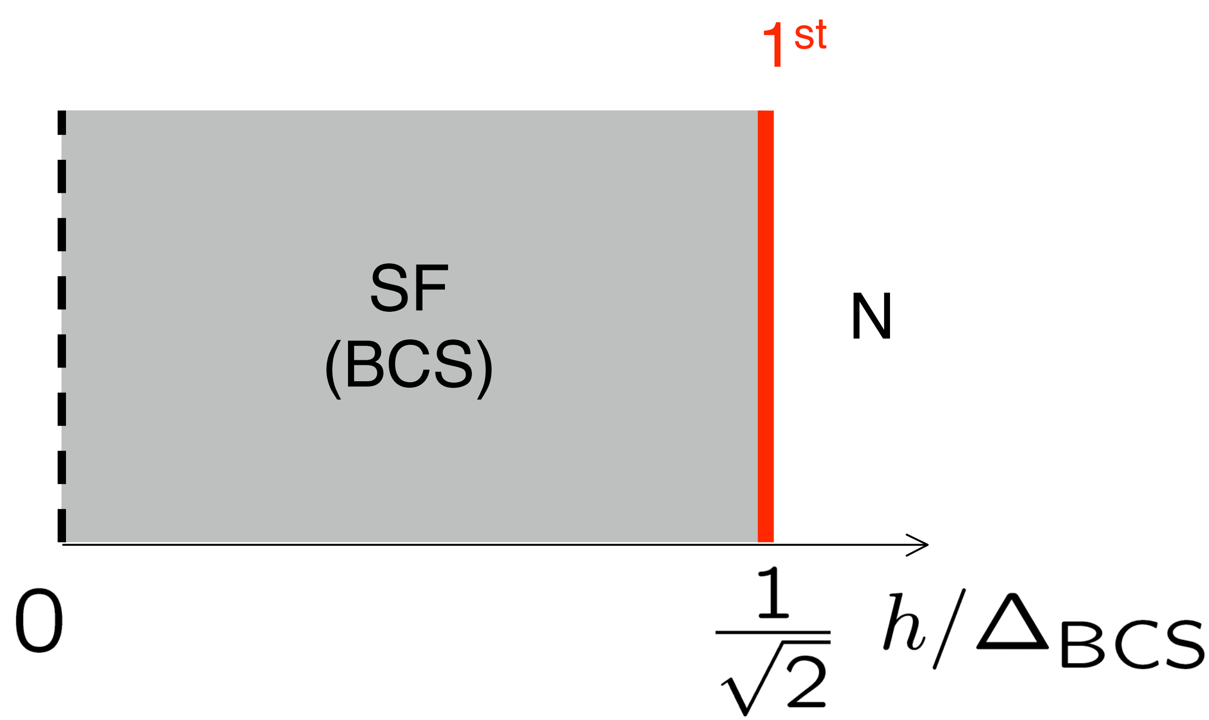
1^{st} order phase transition

- Free energy $$f(\Delta; h, \mu \simeq \varepsilon_F) = \langle \psi | \hat{H} - \sum_\sigma \mu_\sigma \hat{N}_\sigma | \psi \rangle$$



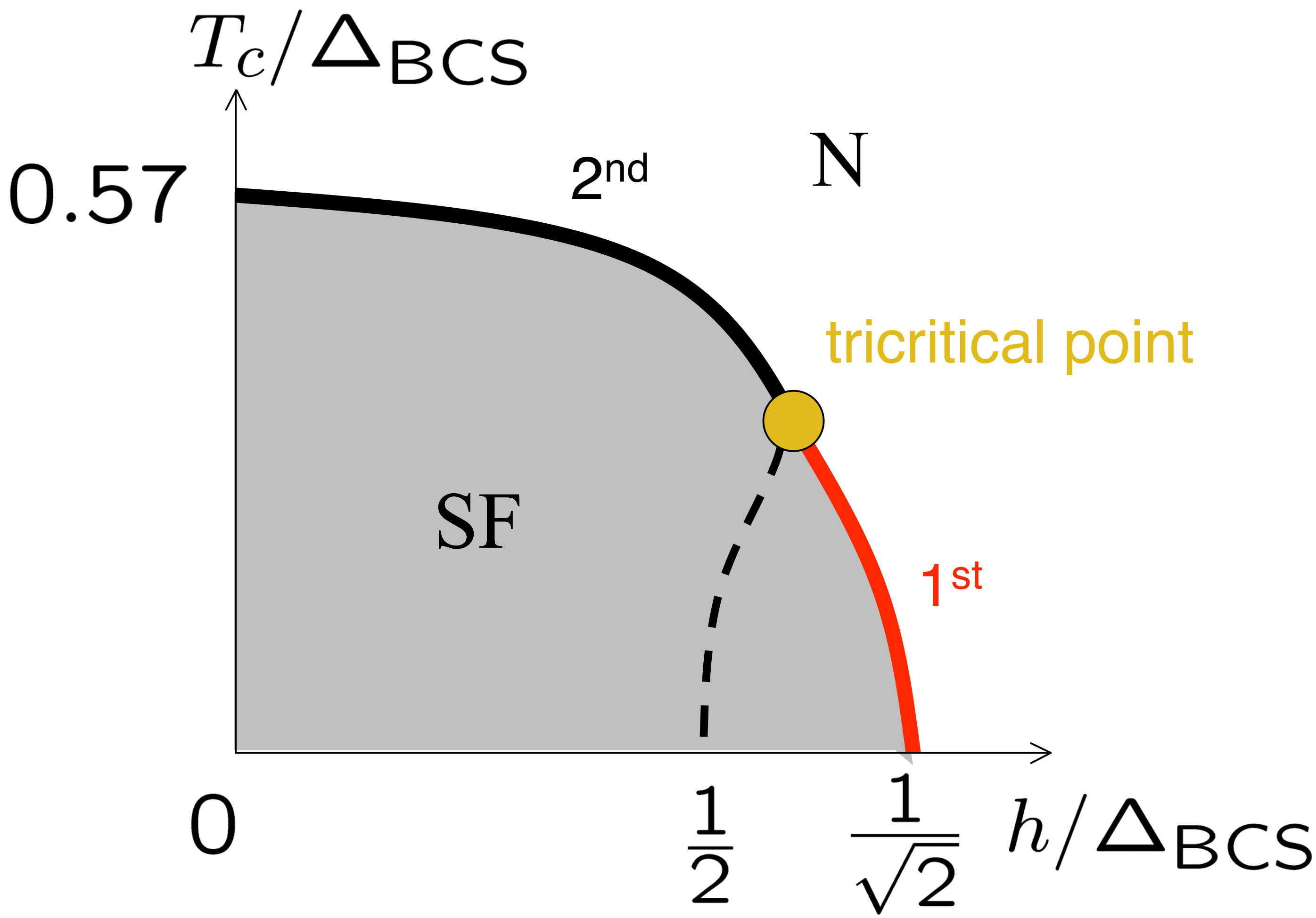
Zero temperature phase diagram

[G. Sarma, J. Phys. Chem. Solids **24** 1029 (1963)]



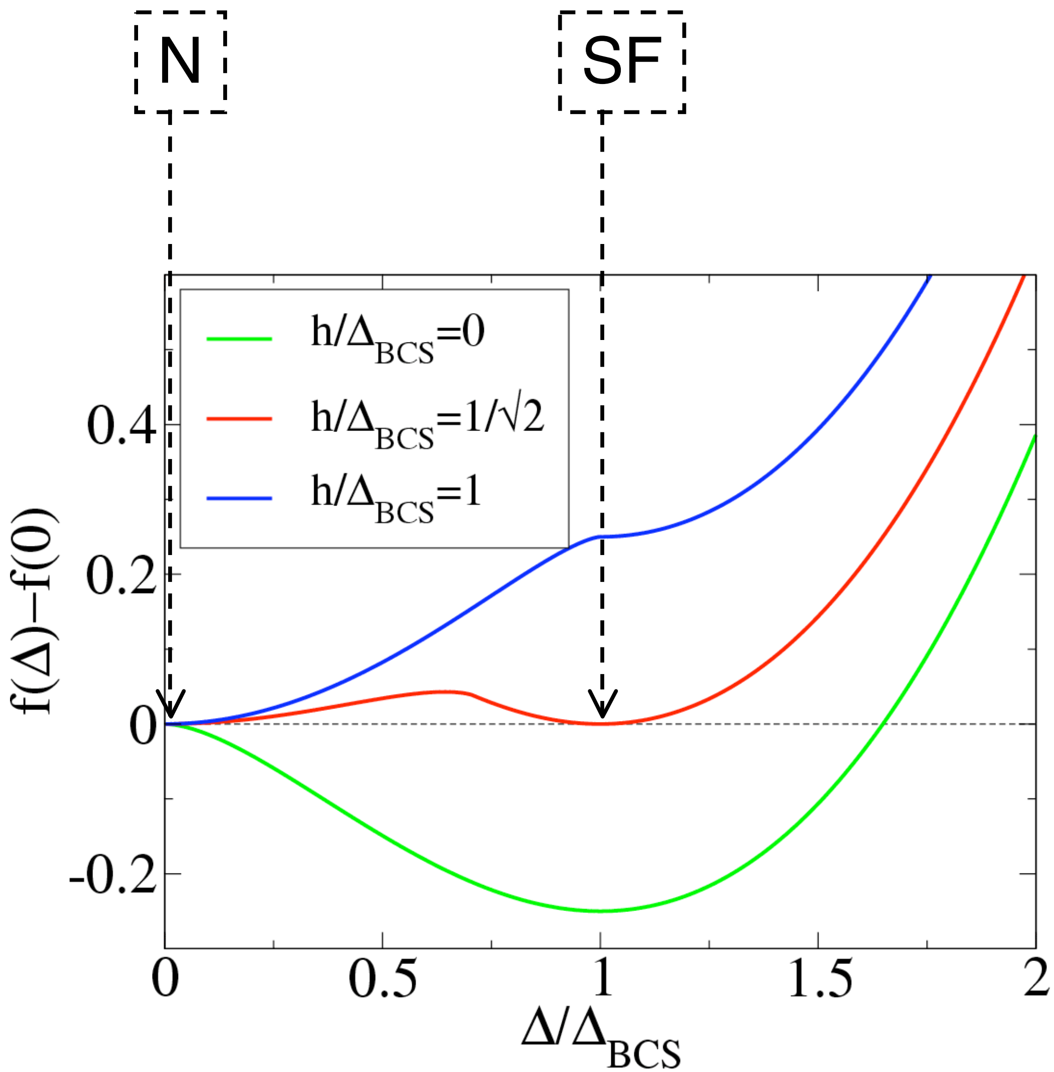
Finite temperature phase diagram

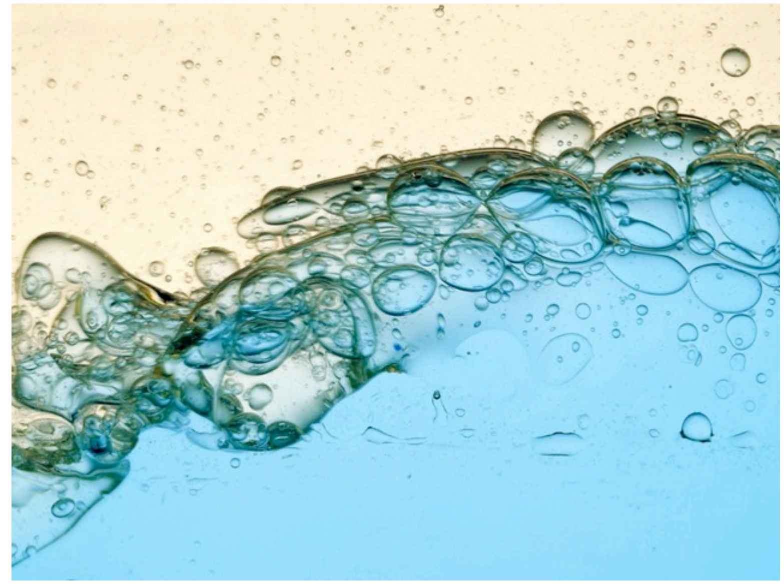
[G. Sarma, J. Phys. Chem. Solids **24** 1029 (1963)]



What this implies for the polarised Fermi gas?

1^{st} order phase transition

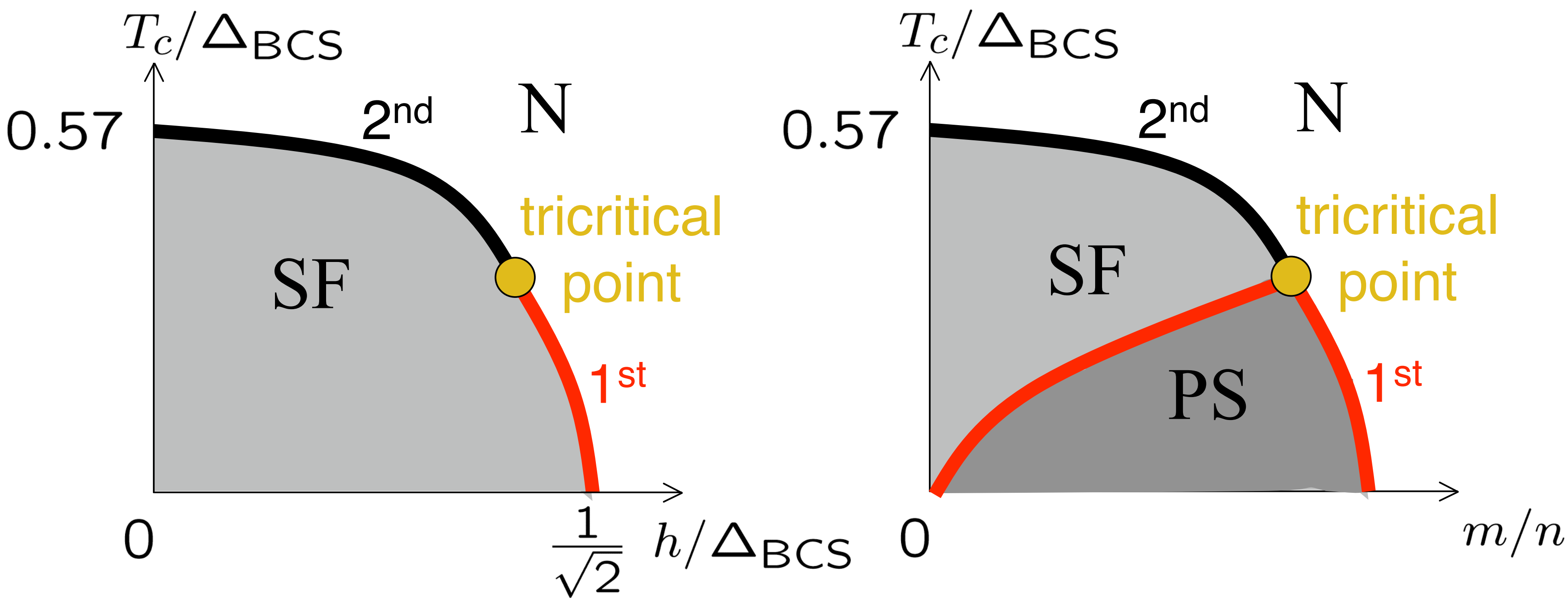




(oil&water)

phase separation in density space!

Phase diagram in density space

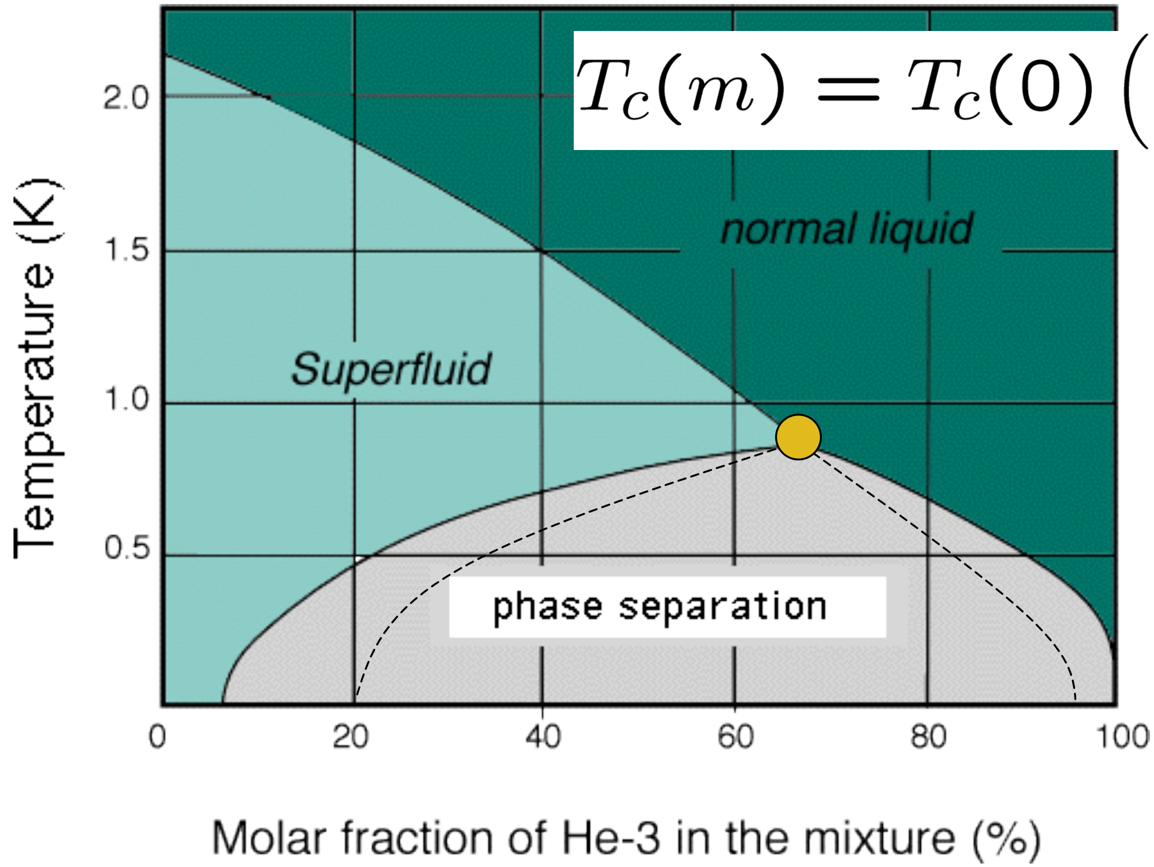


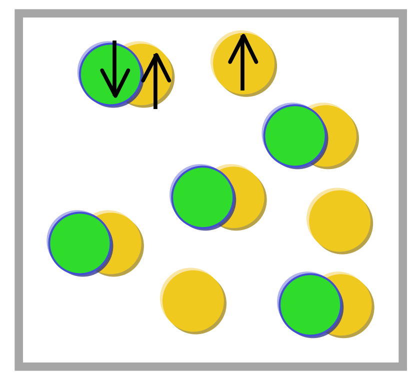
population imbalance

$$m = n_\uparrow - n_\downarrow$$

Analogy with ^{3}He-^{4}He mixtures

$$T_c(m) = T_c(0)\left(1 - \frac{m}{n}\right)^{\alpha}$$



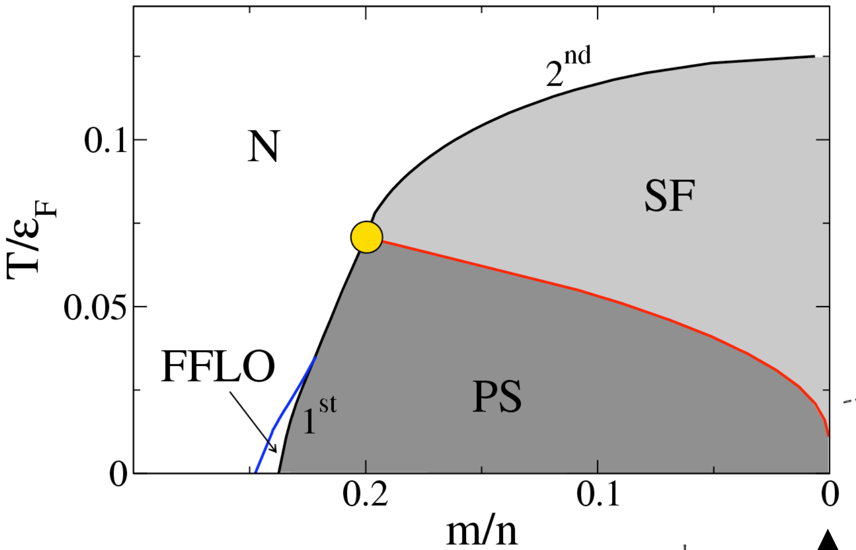


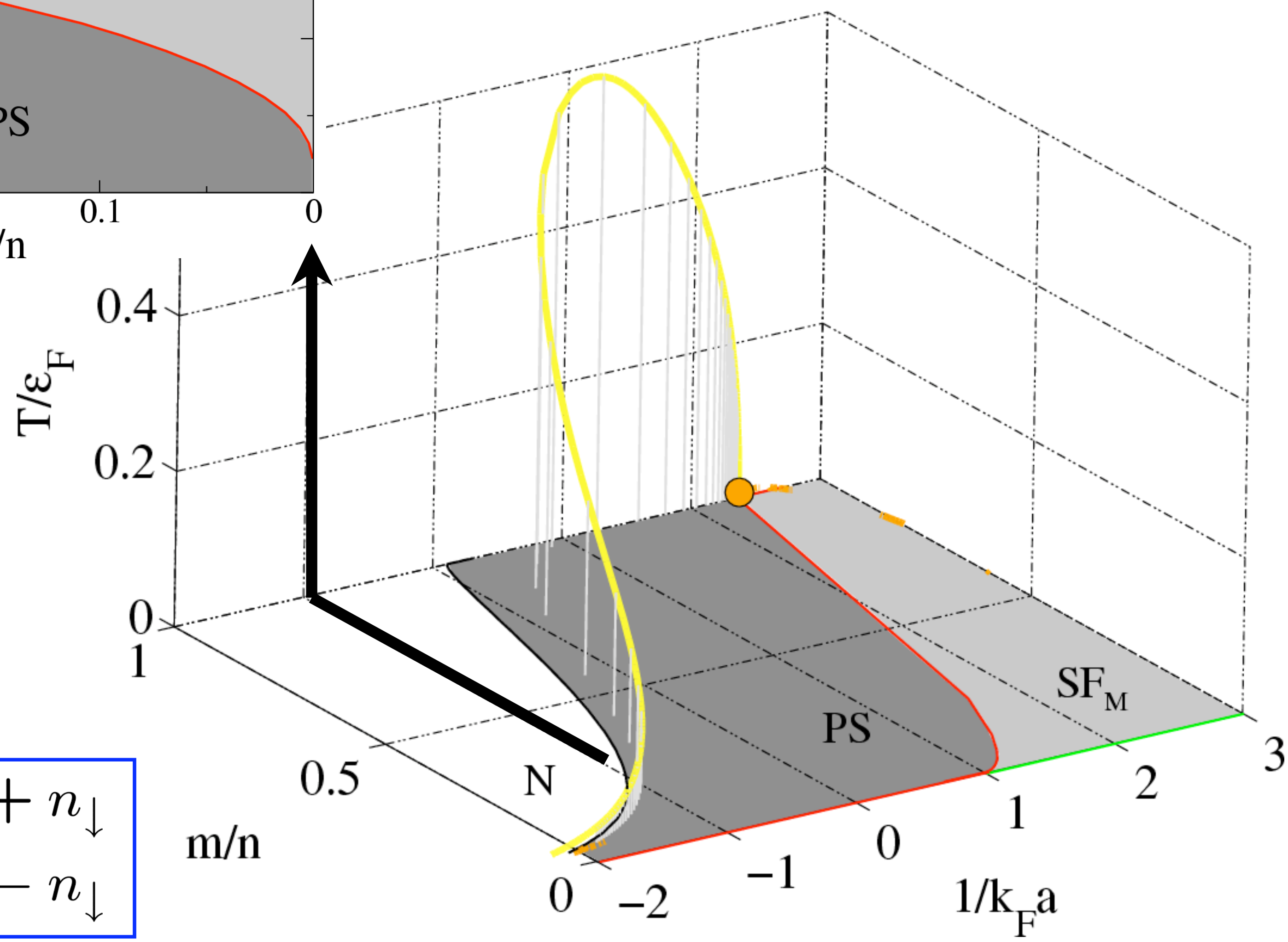
- But ^{3}He-^{4}He is a Bose-Fermi mixture!
  … and the polarised Fermi gas is a Bose-Fermi mixture on the BEC side of the resonance

Expect the same structure on the BEC side?

Phase diagram across the resonance

[M. Parish, F.M. Marchetti *et al.*, *Nature Physics* **3**, 124 (2007)]

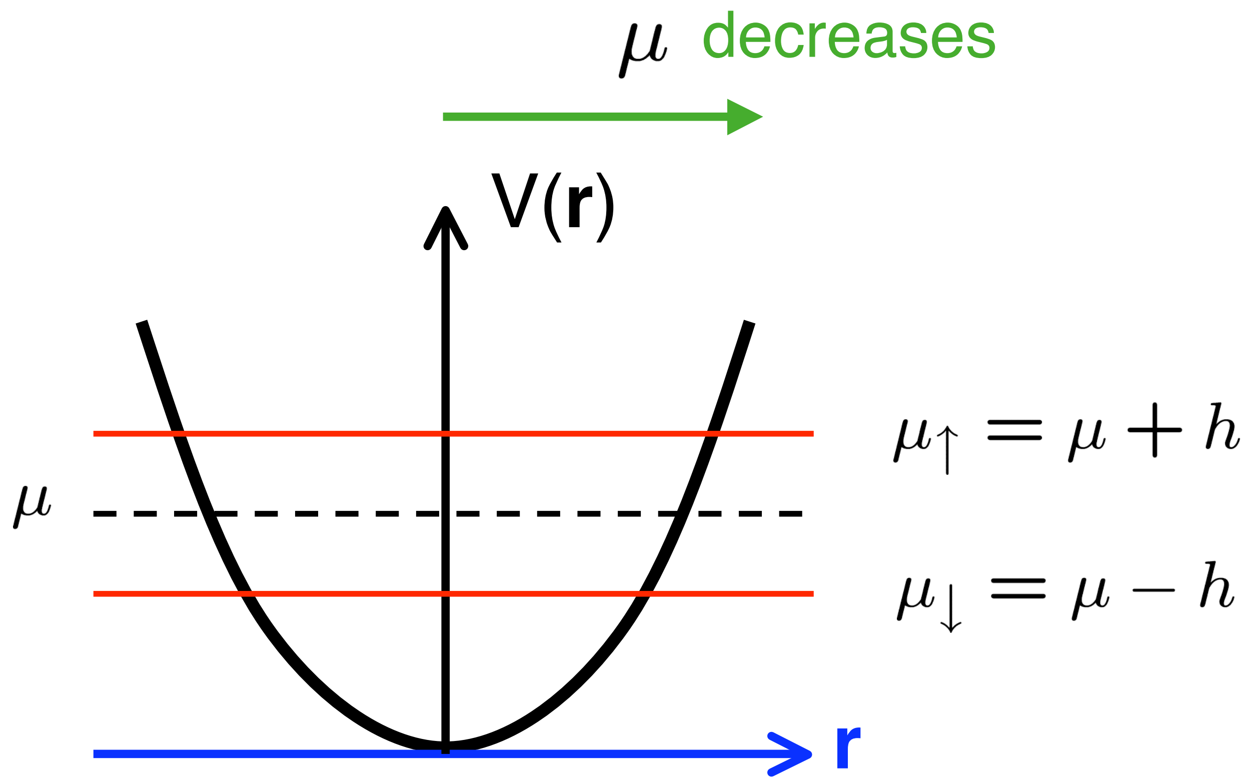




$$n = n_\uparrow + n_\downarrow$$
$$m = n_\uparrow - n_\downarrow$$

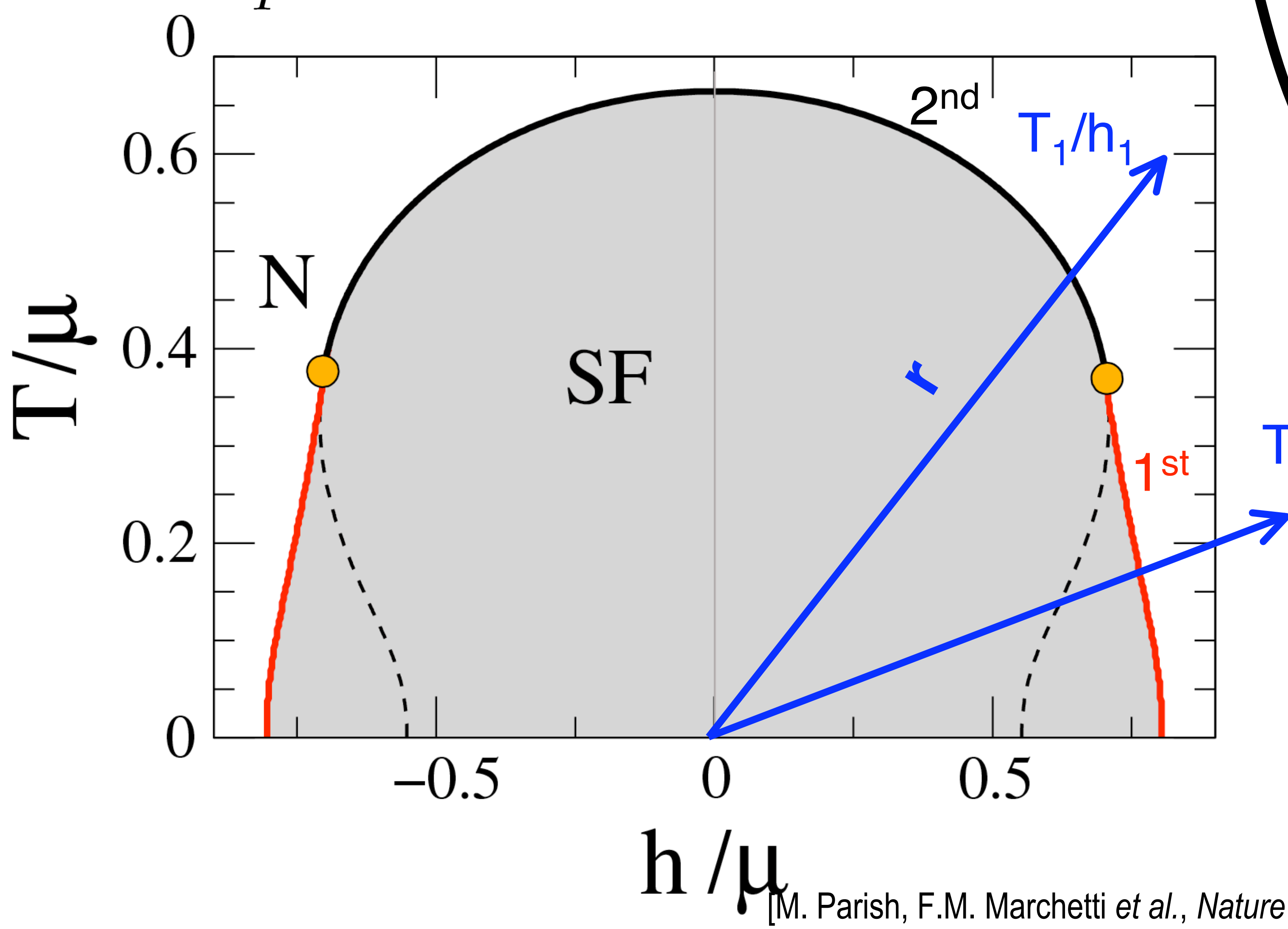
Trapped Fermi Gases

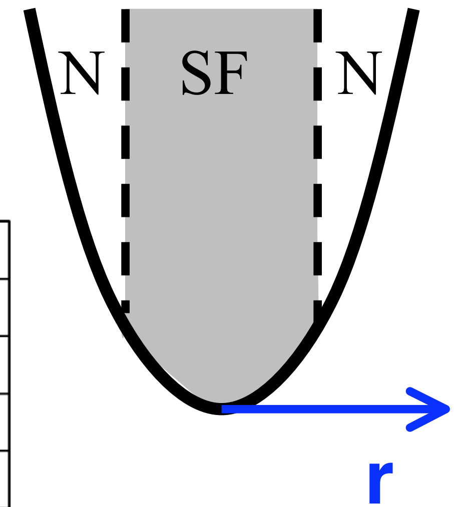
- LDA $\mu_{\uparrow,\downarrow}(\mathbf{r}) = \mu_{\uparrow,\downarrow} - V(\mathbf{r})$



Phase Diagram for Trapped Gases

$$\frac{1}{k_F a} = 0$$

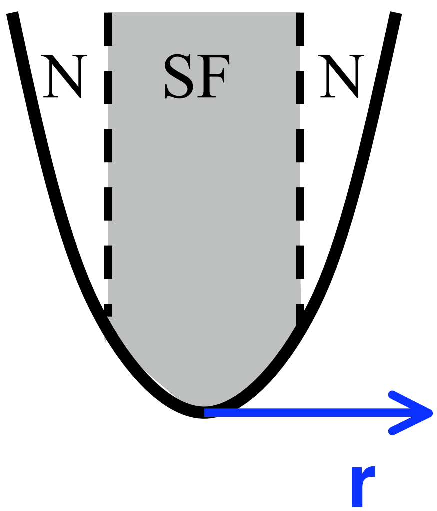




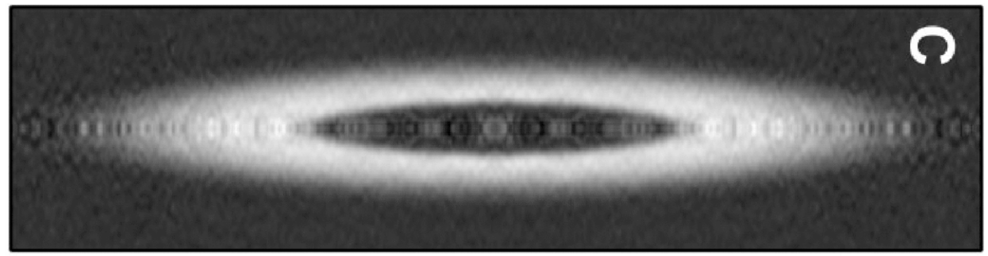
[M. Parish, F.M. Marchetti *et al.*, *Nature Physics* **3**, 124 (2007)]

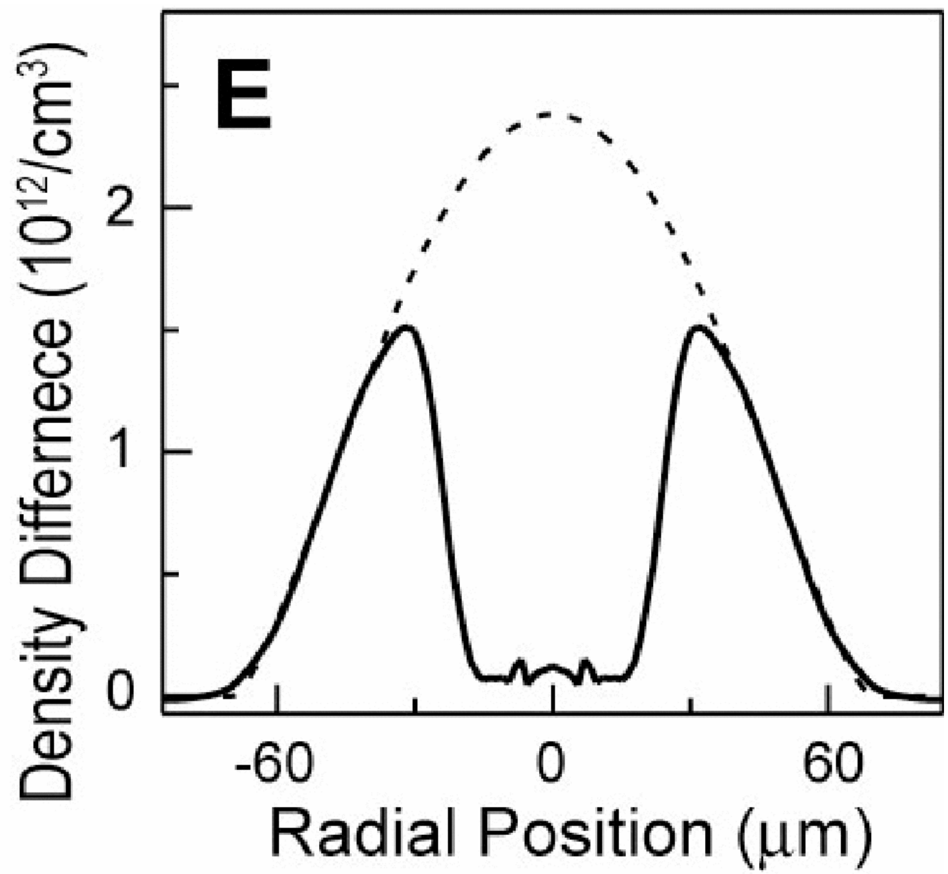
Experiments on Imbalanced Fermi Clouds

- In-situ imaging of phase separation (3D density distribution $n_{\uparrow,\downarrow}(\mathbf{r})$)



$n_{\uparrow}(\mathbf{r}) - n_{\downarrow}(\mathbf{r})$



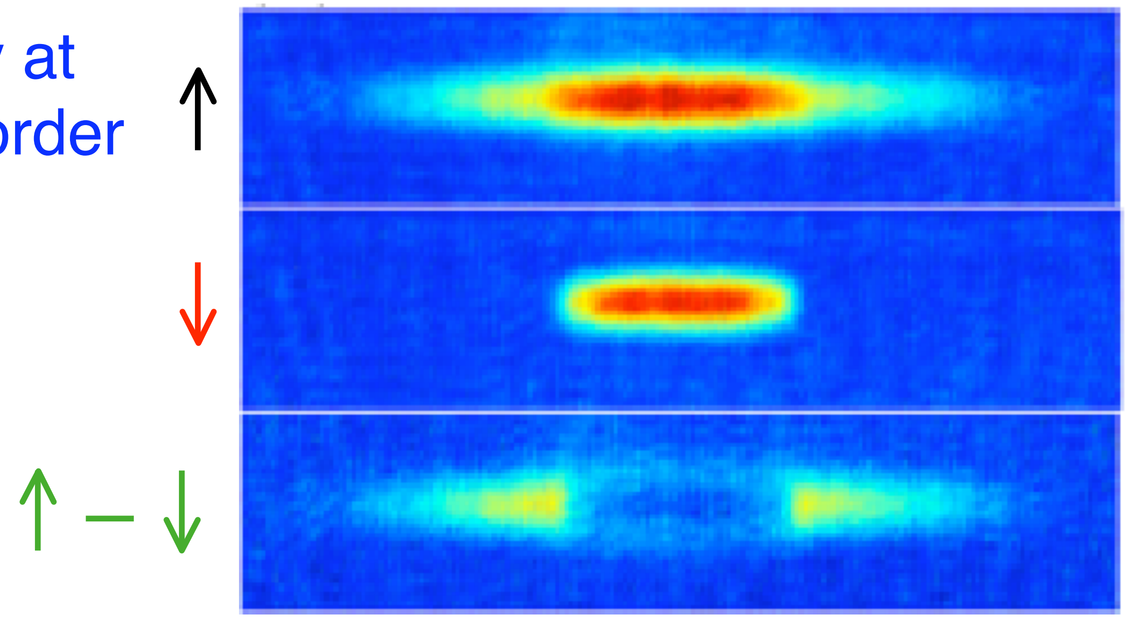


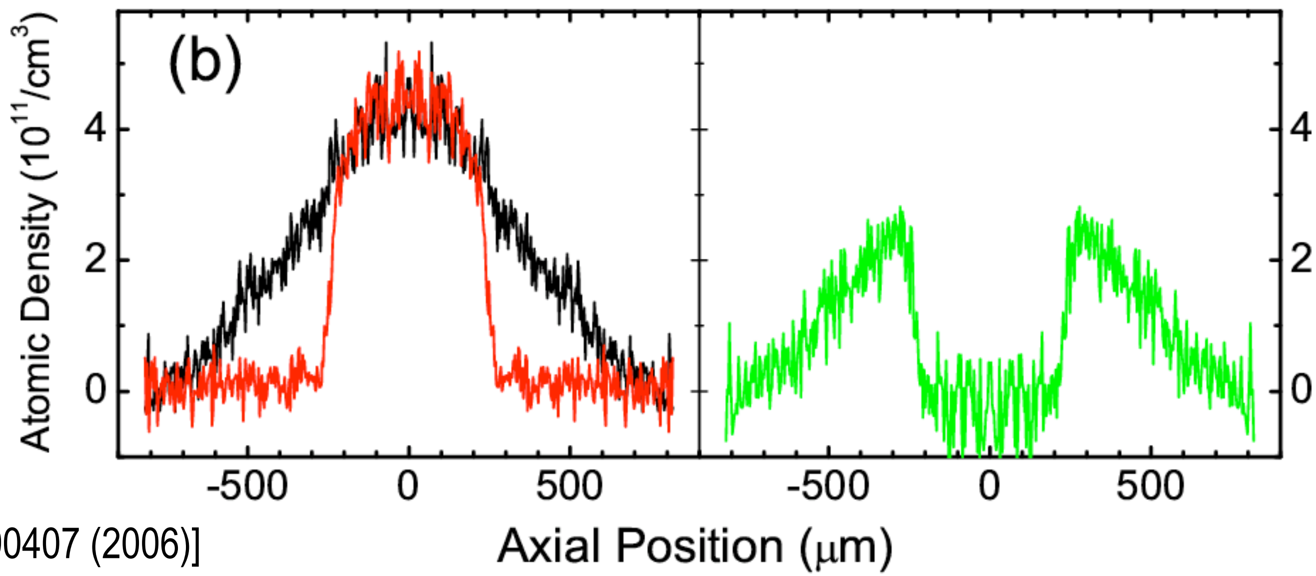
[Y. Shin *et al.*, PRL **97**, 030401 (2006)]

Experiments on Imbalanced Fermi Clouds

- Sharp phase boundary at low temperatures (1st order transition)

$T < 0.05\ T_F$
$m/n = 0.35$

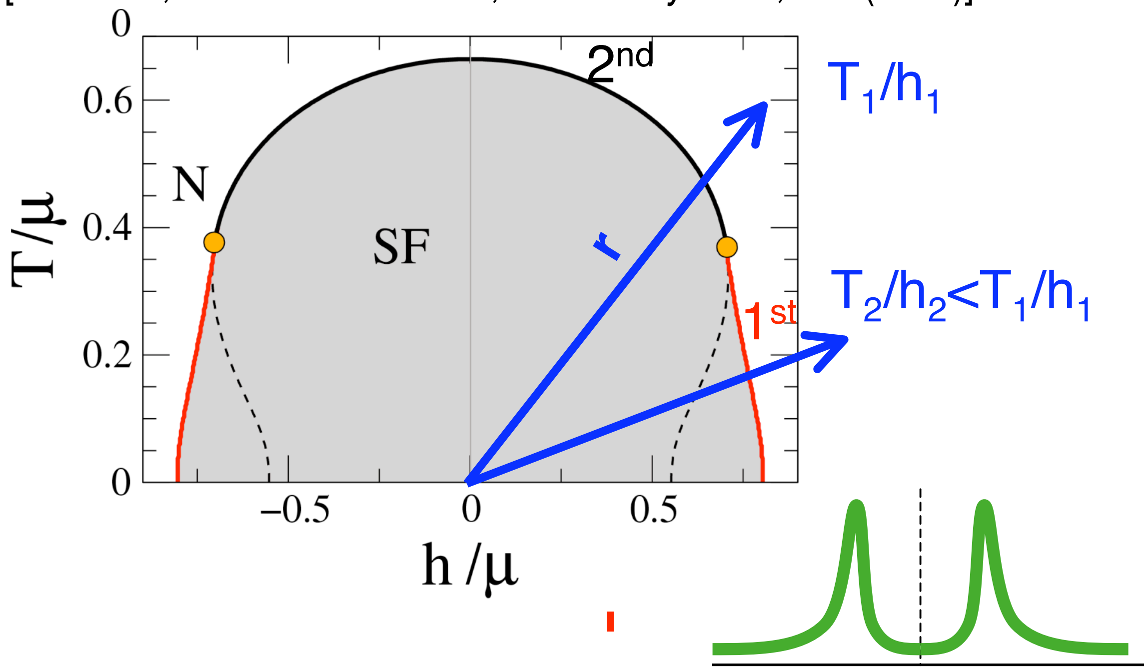




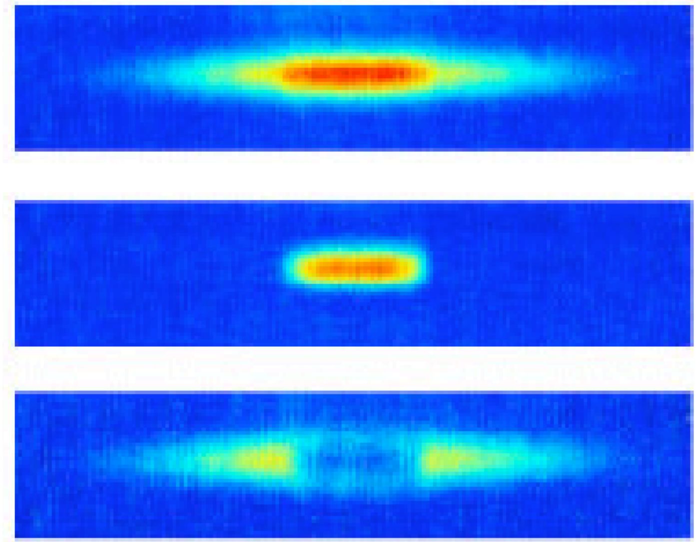
[G. B. Partridge *et al.*, PRL **97**, 190407 (2006)]

Temperature Dependence of Phase Separation

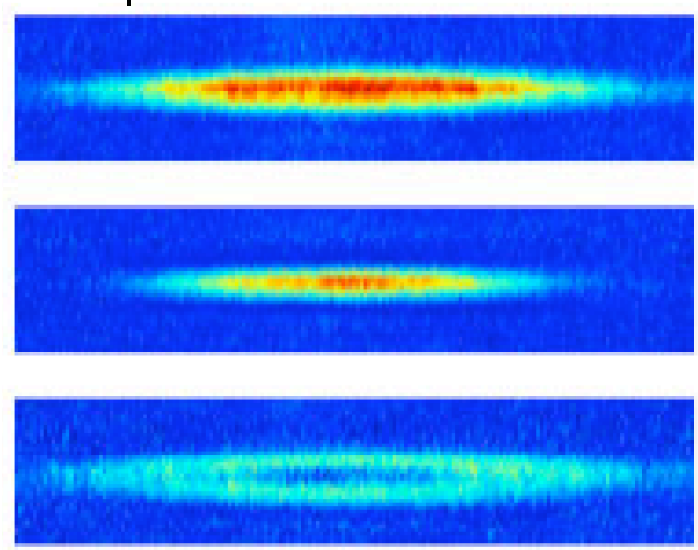
[M. Parish, F.M. Marchetti *et al.*, *Nature Physics* **3**, 124 (2007)]



$T/T_F < 0.05$

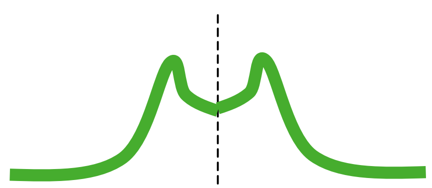


$T/T_F = 0.2$



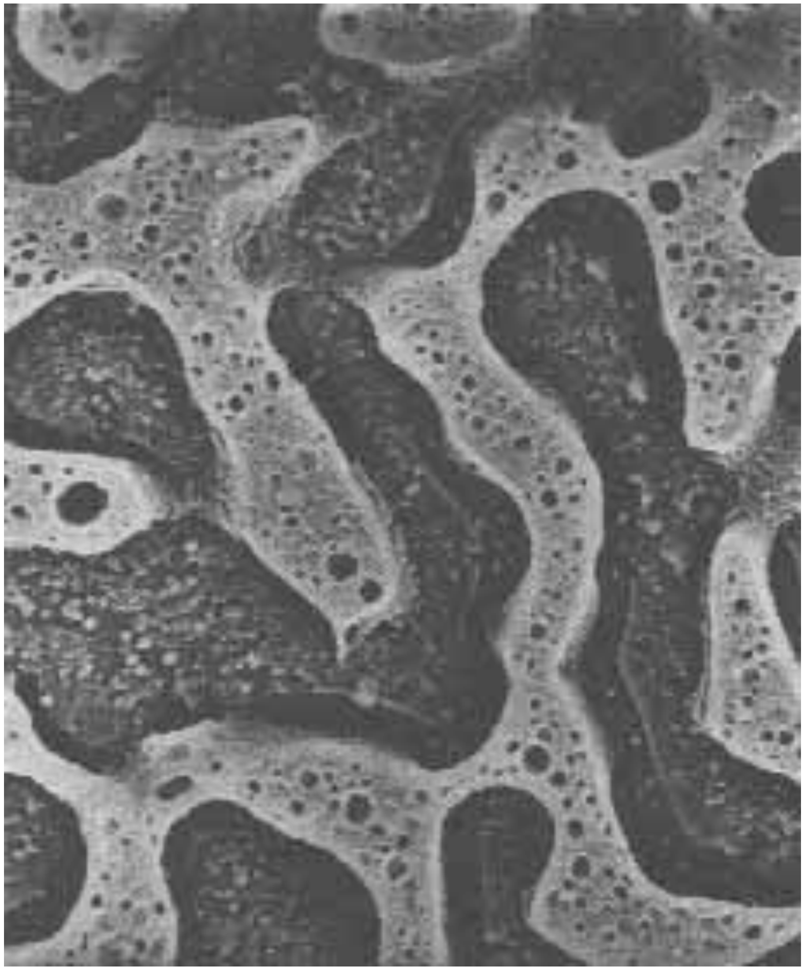
[G. B. Partridge *et al.*, PRL **97**, 190407 (2006)]

2nd



Phase separation: spinodal decomposition?

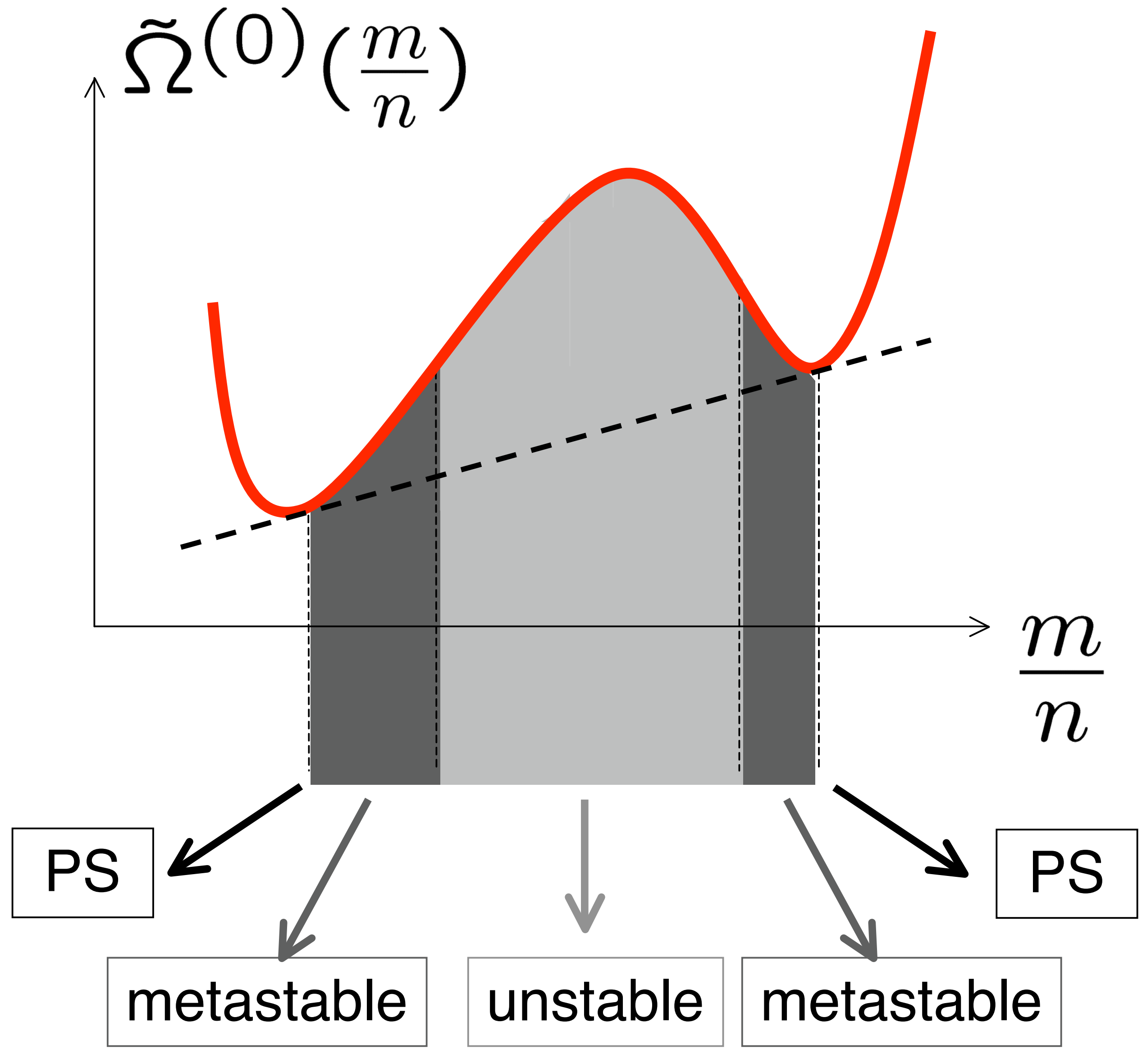
… from Mike Evans lectures!
Temperature quenches in, e.g., polymers in solutions, homogenised colloids, …



[Courtesy of Nigel Clarke, Polymer IRC]

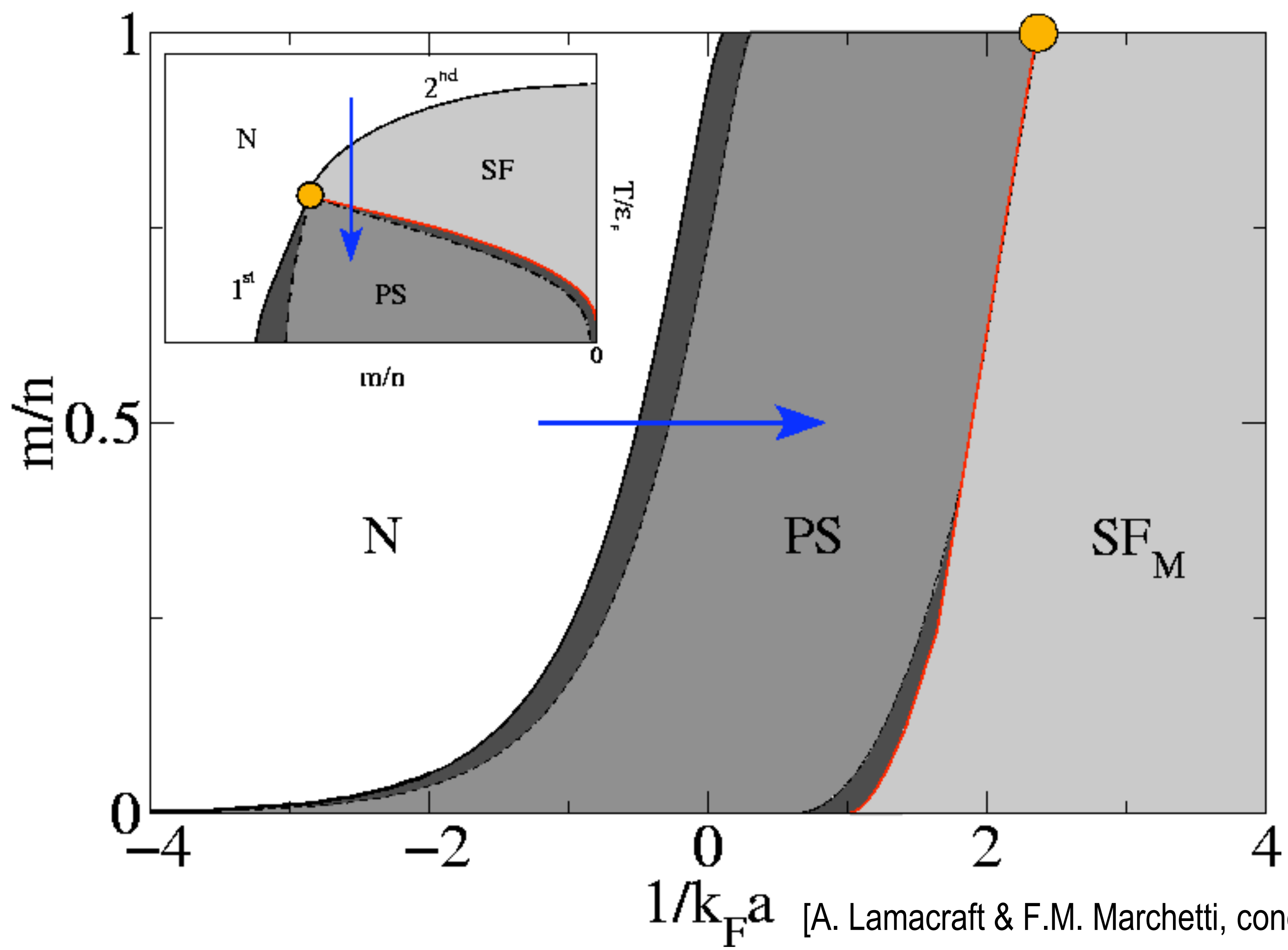
…and also many examples in solid state

Spinodal Region



Dynamics of Phase Separation

- Spinodal: phase separation <u>starts</u> via a linear instability



[A. Lamacraft & F.M. Marchetti, cond-mat/0701692]