

Exercises week 1

September 28, 2017

Please submit your work before the following class (name the script file as `your_name_problems-week1.m`) at the following e-mail address `francesca.marchetti@uam.es`.

1 Exercise:

Consider the following matrices

$$A = \begin{pmatrix} 4 & 2 & 1 \\ 5 & 9 & 12 \end{pmatrix} \quad B = \begin{pmatrix} 4 & 2 & -7 \\ 9 & 2 & 0 \end{pmatrix} \quad C = \begin{pmatrix} 2 & 5 \\ -3 & 2 \\ 5 & -9 \end{pmatrix}.$$

1. Consider the following operations $A \cdot B$, $A \cdot C$, $A \cdot C'$, $B \cdot C$, $B \cdot C'$, $A \cdot C$, $C \cdot A$, $A \cdot B$, $A \cdot B'$, and determine which of these operations is valid and explain the result;
2. explain what is the difference between the operation \cdot and \cdot' . When can you use one and when the other?
3. evaluate $A \wedge 2$ and explain the result; why you cannot consider $A \wedge 2'$?

2 Exercise:

Define two row vectors a and b of 4 elements each: a has the first even numbers (2,4,6,8) and b the first odd numbers in reverse order (7,5,3,1) — use a different definition than the trivial one!

1. Find two equivalent ways to define the vector dot product between the two vectors ($\sum_{i=1}^4 a(i)b(i)$);
2. Find two equivalent ways to define the modulus of each vector;
3. Evaluate the angle between a and b in radians and degrees;
4. Describe which kind of matrix/vector one gets by considering $a \cdot b'$, $a' \cdot b$, $a \cdot b$, $(b \cdot a)'$.

3 Exercise:

Plot the following functions in the required intervals:

$$\begin{aligned} f_1(x) &= x^2 + x - 4 & x &\in [-3, 2] \\ f_2(x) &= e^{x^2} - 3x^2 & x &\in [-1.5, 1.5] \\ f_3(x) &= \ln(x + 3) - x^2 & x &\in [-1, 3]. \end{aligned}$$

For each function and interval find the *local* extrema (minima and maxima), as well as the *global* ones, and compare the results you get with the ones you obtain analytically.

4 Exercise:

It is in general convenient to plot power-laws ($f(x) = x^n$) in logarithmic scale, as in the following example

```
x=logspace(log10(1), log10(1000), 100)
plot(log10(x), log10(x.^3), 'o')
loglog(x,x.^3,'+')
```

1. What is the difference between `plot()` and `loglog()`?
2. Can you determine the power-law exponent from the plots in logarithmic scale?
3. What happens if you use `linspace()` rather than `logspace()`?

5 Exercise:

The number e can be equivalently defined as

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = \sum_{n=0}^{\infty} \frac{1}{n!}.$$

Find an estimate of e by using both the definitions given above and compare them with the built-in value of Matlab (or `exp(1)`) — for summing the vector components you can use the command `sum` or you can find an equivalent way of doing it by using the multiplication operation between vectors.

6 Exercise:

Write a script that evaluates the factorial $n!$ of a given natural number n and compare the results with the built-in function `factorial(n)` — remember that the factorial is defined as $n! = n(n-1)(n-2)\dots 2*1$; Hint: store the result in a variable `f` that needs to be initialised to `f=1` prior to the loop.