Exercises week 4

November 2, 2017

Please submit your work before the following class (name the script file as `your_name_problems-week4.m`) at the following e-mail address `francesca.marchetti@uam.es`.

1 Exercise:

Evaluate both numerically and analytically the area enclosed between the two curves $f_1(x) = x^2 - x - 2$ and $f_2(x) = |x|$ (i.e., above $f_1(x)$ and below $f_2(x)$) and compare the results.

2 Exercise:

Demonstrate both numerically and analytically that, for a generic natural number n,

$$F(n) = \int_0^\infty dx x^n e^{-x} = n! \; .$$

Evaluate the integral numerically for a real number n and plot $F(n)$ for $n \in [0, 5]$.

3 Exercise:

Find the crossing points between a circle centered in $(x_0, y_0) = (2, -3)$ and radius $R = 5$ and the exponential function $f(x) = e^{-x/2} - 4$ (answer: $x_1 \simeq -2.42$ and $x_2 \simeq 6.91$). Plot both functions and mark the crossing points you have found numerically.

4 Exercise: Maxwell-Boltzmann distribution

The Maxwell-Boltzmann distribution describes the velocity (v) distribution of a classical gas of atoms with mass m at thermal equilibrium at a given temperature T:

$$f(v) = 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-\frac{mv^2}{2k_B T}} \ . \tag{1}$$

In SI units the Boltzmann constant is $k_B = 1.38 \times 10^{-23}$ J/K and let's consider the specific case of Argon atoms, whose mass is $m = 67 \times 10^{-27}$ kg.

1. Numerically evaluate the maximum of this distribution `fmax` for a temperature $T = 300$ K and its corresponding velocity `vmax`; use the loop `for` and compare your result with the built-in operation `max`, taking into account the syntax `[fmax,imax]=max(f)`, where `imax` is the index of the maximum value of the vector `f`;

2. Assuming the system is kept at a pressure low enough to always remain in its gaseous phase in the temperature range $T \in [1, 1000]$ K, plot in two separate subplots `fmax` and `vmax` as a function of temperature.

5 Exercise: Minimal distance between two circles

Consider the following two circles,

$$(x - x_0)^2 + (y - y_0)^2 = r^2 \ , \tag{2}$$

one with center $(x_{01}, y_{01}) = (-3, -0.5)$ and radius $r_1 = 1$ and the other with $(x_{02}, y_{02}) = (3, 2)$ and $r_2 = 2.5$. Numerically evaluate the minimal distance between the two circles, compare the result you get with the exact one, $\sqrt{(x_{02} - x_{01})^2 + (y_{02} - y_{01})^2} - (r_1 + r_2)$, and represent graphically your result by plotting the two circles and the segment representing the minimal distance.

6 Exercise:

Write a scripts which plots the following function of two variables $f(x, y) = (x^2 + y^2) - (x^2 + y^2)^2/2$ in the interval $x, y \in [-1, 1]$ using the commands `meshgrid` and `mesh(x,y,f)`. Evaluate the maximum value of the function $f(x, y)$, by making use of a loop `for` and the condition `if`. Compare the result you find this way with the one found with the command `max`, and check they match with the exact answer — N.B. For a matrix A, `max(A)` returns a row vector containing the maximum element from each column. When you have doubts about a command, type `help command`, e.g. in this case `help max`.

7 Exercise:

Consider the energy potential $U(x, y) = -(x^2 + y^2) + (x^2 + y^2)^2/2$, plot the potential using the commands `meshgrid` and `mesh(x,y,f)`, evaluate the force field $(F_x, F_y) = -(\partial_x U, \partial_y U)$ and plot it with the command `quiver`. Describe the motion of a particle under such a vector force field. Use first your routine to evaluate the partial derivatives $\partial_x U$ and $\partial_y U$, and then use the build-in Matlab routine `[Fx, Fy] = gradient(U)` to calculate the gradient of a given scalar field `U`.

8 Exercise:

Plot the potential for a system of two point charges in a 2D plane, one with charge $Q = -1$ positioned at $\mathbf{r}_0 = (1, 1.5)$ and another of charge $Q = +1$ positioned at $\mathbf{r}_0 = (-1, -1.5)$ — the electric potential for a system of point charges is the sum of the individual potentials.

9 Exercise:

Write a script which plots the electric field $\mathbf{E}$ of two point charges $Q_1 = 1$ located at $\mathbf{r}_{10} = (1, 0)$ and $Q_2 = -1$ located at $\mathbf{r}_{20} = (-1, 0)$ (dipole).