Exercises week 6

November 23, 2017

Please submit your work before the following class (name the script file as `your_name_problems-week6.m`) at the following e-mail address `francesca.marchetti@uam.es`.

1 Exercise:

Solve both numerically and analytically the following first order differential equation

$$\frac{dx(t)}{dt} = [3x(t) + 2]e^{-2t} \qquad\qquad x(t_0 = 0) = x0 \ ,$$

for four different initial conditions, $x_0 = 2.1$, $x_0 = 3.2$, $x_0 = -2$, and $x_0 = -4$ and compare the numerical results with the exact solution by plotting them in an interval $t \in [0, 4]$. Find an expression of the asymptotic limit $\lim_{t\to\infty} x(t)$ and compare it with what you find numerically.

2 Exercise:

Show that the solution of the following first order differential equation

$$3x(t)\frac{dx(t)}{dt} + tx^2(t) = 2t \qquad\qquad x(t_0 = 0) = x0 \ ,$$

is given by

$$x(t) = \sqrt{2 + e^{-t^2/3}(-2 + x_0^2)} \ .$$

Solve numerically the equation in the interval $t \in [0, 7]$ for the following different initial conditions, $x_0 = 0$, $x_0 = \sqrt{2}$, $x_0 = 2$, and $x_0 = 3$ and compare the numerical results with the exact solution.

3 Exercise: Planar Pendulum

Solve numerically the planar pendulum

$$\frac{d^2\theta}{d\tilde{t}^2} = -\sin\theta \; . \tag{1}$$

with initial condition $\theta_0 = 30^\circ$ and $v_0 = 0$ in the interval $\tilde{t} \in [0, 12\pi]$ by making use of both the Euler method and the 2^{nd}–order Runge–Kutta algorithm.

Questions:

1. Plot the numerical solution. Determine for which value of the integration steps N the solution starts not to visibly change any longer;
2. for this value of N, compare by plotting the numerical solution with the exact solution $\theta(\tilde{t}) = \theta_0 \cos(\tilde{t})$ valid for small angles only. Comment the result you get;
3. change the initial condition θ_0: for which value of θ_0 the two curves are approximatively equal? Why?
4. plot the solution in the phase space θ, $d\theta/d\tilde{t}$.