Exercises week 7

November 30, 2017

Please submit your work before the following class (name the script file as `your_name_problems-week7.m`) at the following e-mail address `francesca.marchetti@uam.es`.

1 Exercise:

Solve both numerically and analytically the following system of two coupled linear first order differential equations

$$\frac{dx_1(t)}{dt} = x_1(t) - x_2(t) + 7 \qquad x_1(t=0) = 3$$
$$\frac{dx_2(t)}{dt} = -2x_1(t) + x_2(t) + 2 \qquad x_2(t=0) = -1 \; ,$$

and compare the numerical results with the exact solution by plotting both solutions in an interval $t \in [0, 1]$.

Hints to solve Exercise 1 In order to solve the system analytically, remember that:

- you can rewrite the equation in the following vector form:

$$\frac{d\mathbf{x}(t)}{dt} = A\mathbf{x}(t) + \mathbf{b} \qquad \mathbf{x}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$$
$$A = \begin{pmatrix} 1 & -1 \\ -2 & 1 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} 7 \\ 2 \end{pmatrix} \; ;$$

- the critical points $\mathbf{x}_0$ can be found by solving the following system of linear equations

$$A\mathbf{x}_0 + \mathbf{b} = 0 \; ;$$

- the eigenvalues of A, $\lambda_{1,2}$, determine the stability of the critical points; the eigenvalues and eigenvectors $\mathbf{u}_{1,2}$ of A allow to write the general solution of the differential equation as

$$\mathbf{x}(t) = a\mathbf{u}_1 e^{\lambda_1 t} + b\mathbf{u}_2 e^{\lambda_2 t} + \mathbf{x}_0 \; ,$$

  where the constants a and b can be found by solving the system

$$\mathbf{x}(t=0) = a\mathbf{u}_1 + b\mathbf{u}_2 + \mathbf{x}_0 = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \; .$$

2 Exercise:

Solve both numerically and analytically the following system of three coupled linear first order differential equations

$$\frac{d\mathbf{x}(t)}{dt} = A\mathbf{x}(t) + \mathbf{b} \qquad \mathbf{x}(t=0) = \begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ -3 \end{pmatrix}$$

$$A = \begin{pmatrix} -1 & -1 & -2 \\ -2 & 1/2 & -5 \\ -3 & 4 & -8 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} .$$

Find the critical points and their stability. Compare the numerical results with the exact solution by plotting both solutions in an interval $t \in [0, 20]$.

3 Exercise:

Numerically solve the following equations describing two competing populations

$$\frac{dx(t)}{dt} = 60x - 4x^2 - 3xy$$
$$\frac{dy(t)}{dt} = 42y - 2y^2 - 3xy$$

for different initial conditions x_0 and y_0. Evaluate the critical points and their stability. Linearise close to one of the stable critical points, evaluate analytically the solutions and plot them together with the numerical ones for the full problem.