Exercises week 8

December 14, 2017

Please submit your work after the Xmas break (name the script file as `your_name_problems-week8.m`) at the following e-mail address `francesca.marchetti@uam.es`.

1 Suggestions for a possible project

The cycloidal pendulum (brachistochrone problem)

A simple pendulum is not in general isochronous, i.e., the period of oscillations depends on their amplitude. However, if a simple pendulum is suspended from the cusp of an inverted cycloid, such that the “string” is constrained between the adjacent arcs of the cycloid, and the pendulum’s length is equal to that of half the arc length of the cycloid (i.e., twice the diameter of the generating circle), the bob of the pendulum also traces a cycloid path. Such a cycloidal pendulum is isochronous, regardless of amplitude. Study this problem and compare the numerical solutions of the equations of motion with the analytical ones.

The double pendulum

A double pendulum is a pendulum (point mass suspended from a string of negligible mass) with another pendulum attached to its end. Despite being a simple physical system, the dynamics of the double pendulum is rich and can exhibit chaotic behaviour and great sensitivity to initial conditions.

The butterfly effect

The butterfly effect in chaos theory refers to sensitive dependence on initial conditions that certain deterministic non-linear systems have. Here, a small change in one state of a deterministic non-linear system can result in large differences in a later state. In this project you can numerically simulate this effect by solving the Lorenz equations., i.e. a system of ordinary differential equations that describe the convection rolls of air in the Earths atmosphere. Such a system is notable for having chaotic solutions for certain parameter values and initial conditions. In particular, the Lorenz attractor is a set of chaotic solutions of the Lorenz system which, when plotted, resemble a butterfly or figure eight.

The swinging spring

The swinging spring or elastic pendulum is a simple mechanical system that exhibits complex dynamics. It consists of a heavy mass suspended from a fixed point by a light spring which can stretch but not bend, moving under gravity.

The swinging Atwood's machine

The swinging Atwood's machine is a mechanism that resembles a simple Atwood's machine except that one of the masses is allowed to swing in a two-dimensional plane, producing a dynamical system that is chaotic for some system parameters and initial conditions.

Epidemic modelling

The SIR model is a model describing three classes of individuals, the individuals not yet infected with the disease (or those susceptible to the disease), the individuals who have been infected with the disease and are capable of spreading the disease to those in the susceptible category, and individuals who have been infected and then removed from the disease, either due to immunization or due to death. Those in this category are not able to be infected again or to transmit the infection to others. Use this model to study the transmission of communicable disease through individuals.

The solar system

Consider the equations of motion describing the movement of planets (say the inner ones, Mercury, Venus, the Earth and Mars) orbiting around the Sun. Note that because of the conservation of angular momentum the motion of all bodies is in a plane and hence you can treat it as a two-dimensional problem. Numerically simulate the problem finding the orbits, confirming energy conservation and Kepler's laws.

A model about another system that interests you

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