Polarised Fermi Superfluids

Phase Diagram & Dynamics

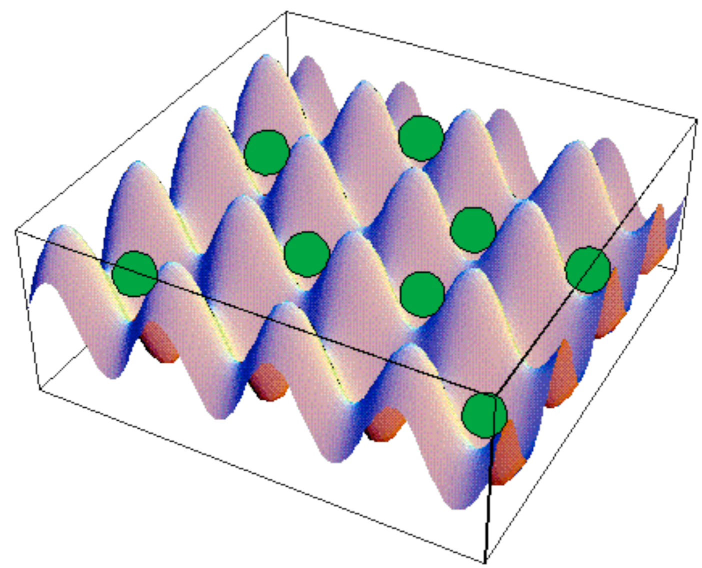
F.M. Marchetti

Workshop on BdG & GP equations, Manchester, 29 September 2007

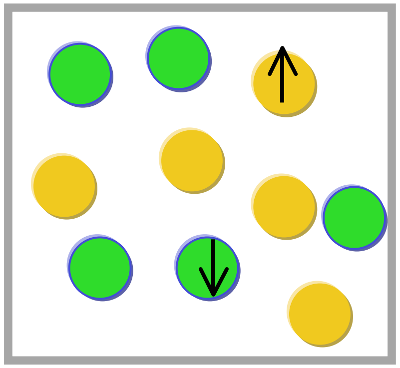
Why atomic gases?

Search for novel phases of quantum coherent matter

- Tuning the interaction strength
- Mixtures of different statistics
- Optical lattices
- 1D, 2D
- …



Fermi superfluids



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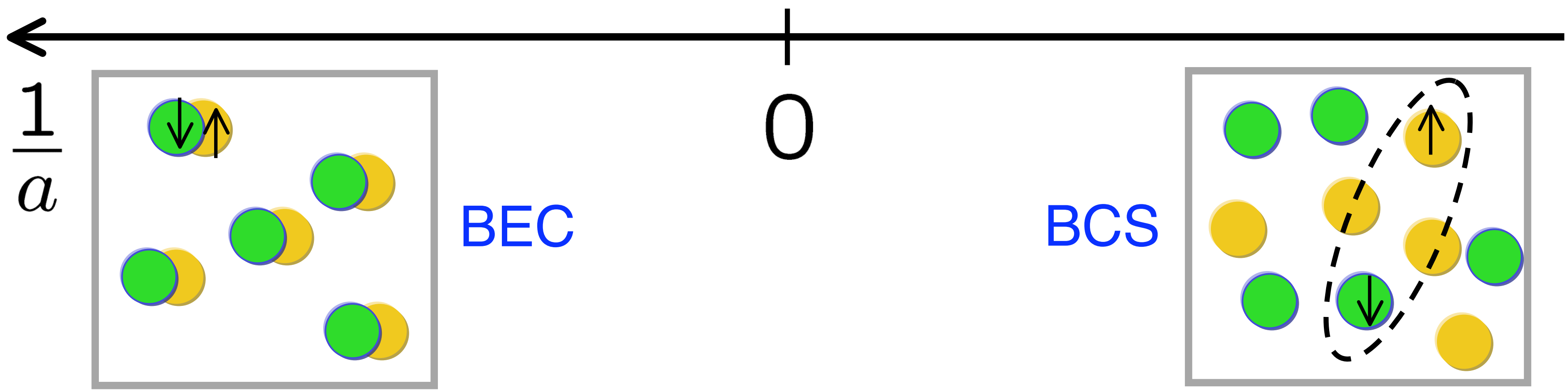
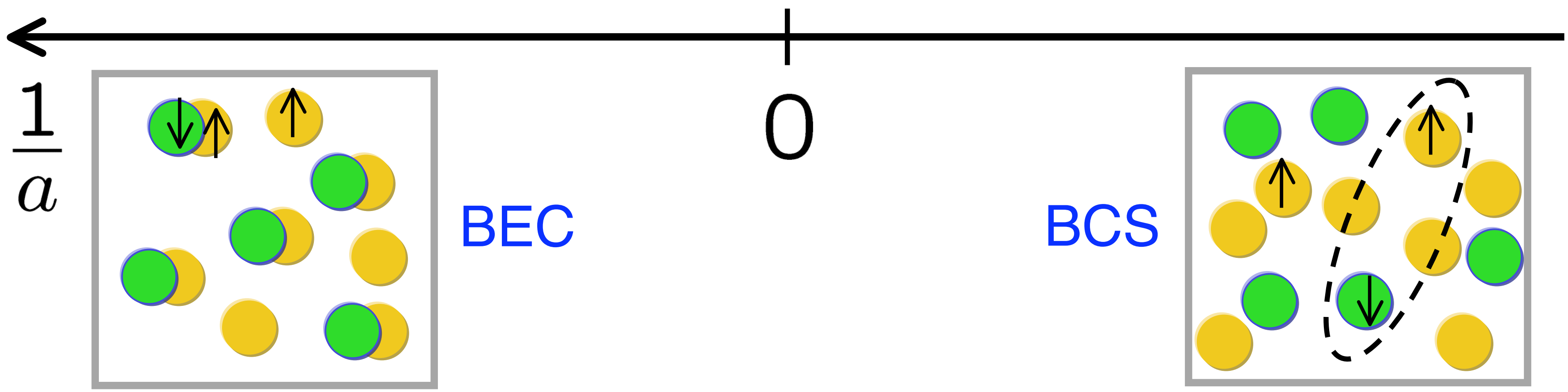


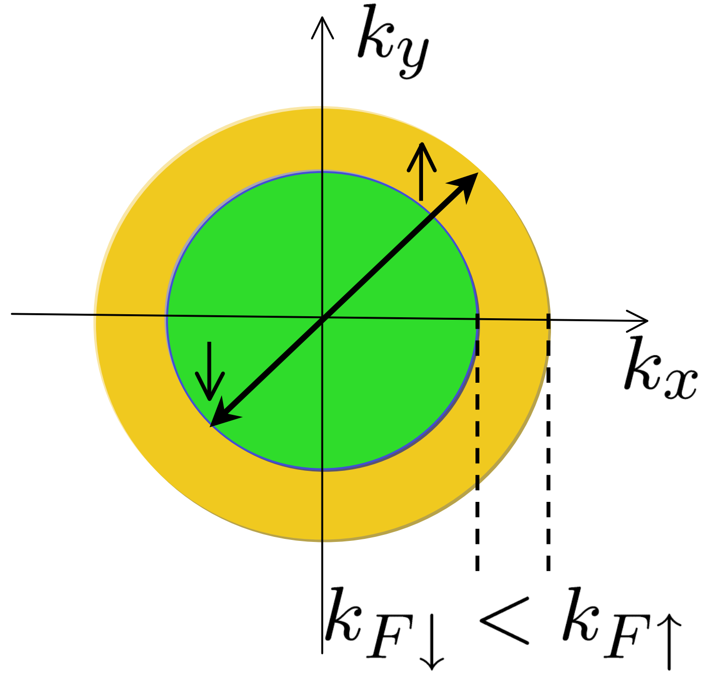
Polarised Fermi superfluids



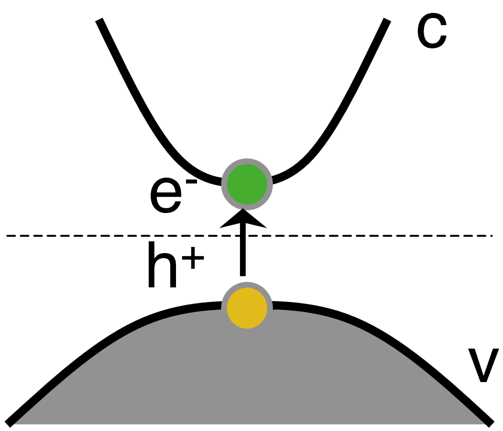
Can superfluidity persist in presence of
a population imbalance?

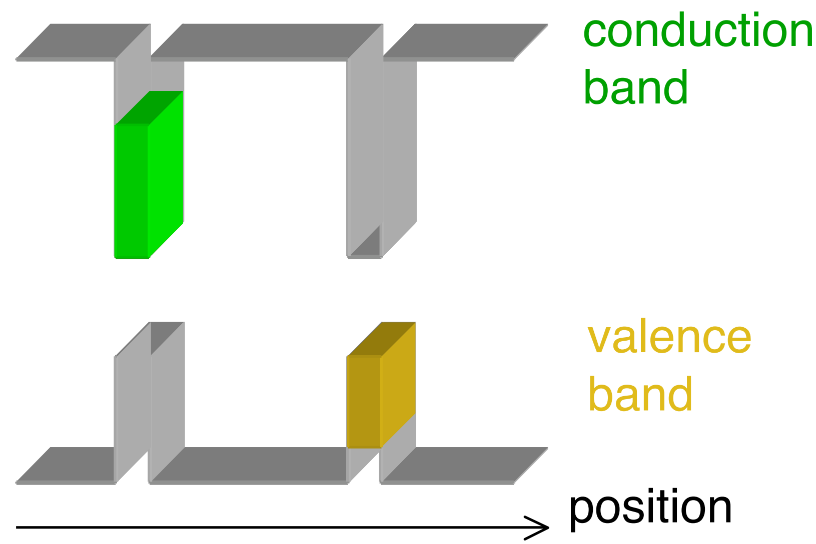
Why interesting?

- Magnetised superconductors (Zeeman)



- Quantum Chromodynamics (and neutron stars)
- Electron-hole bilayers

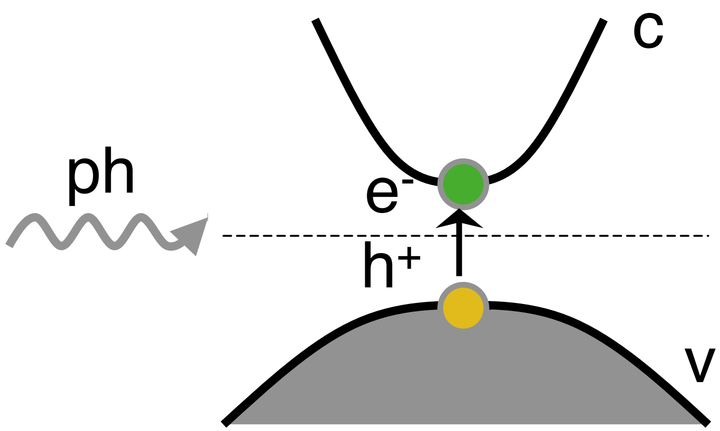




BEC-BCS crossover in electron-hole systems!

[L.V. Keldysh & Yu V. Kopaev, *Sov. Phys. Solid State* **6**, 2219 (1965)]

- Microcavity polaritons
  (1/2-exciton 1/2-photon quasi-particles)



[see M.Szymanska's talk]

[J. Kasprzak *et al.*, *Nature* **443**, 409 (2006)]

Outline

- BEC-BCS crossover

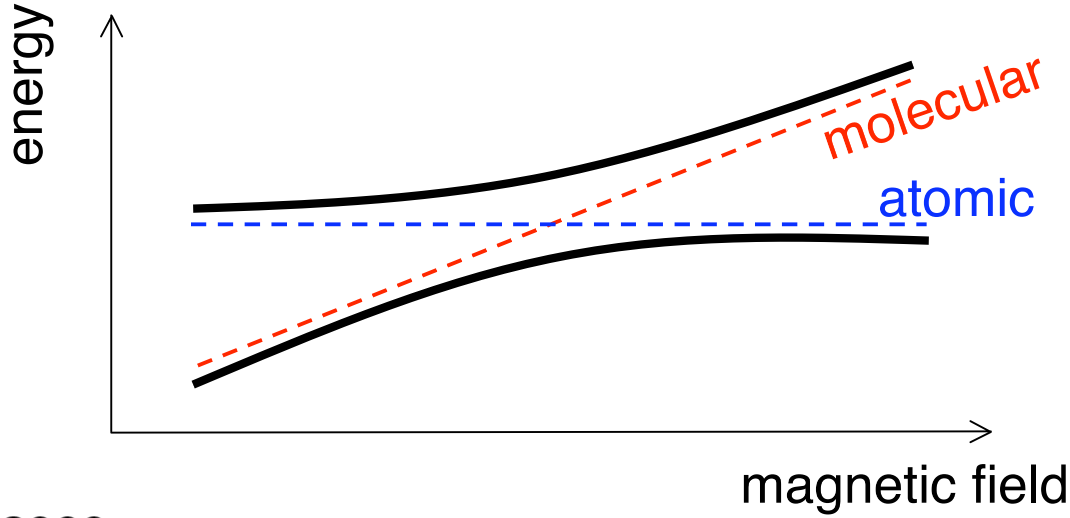
1. Unbalanced populations
   - Homogeneous phase diagram: T=0 & finite T
   - Trap
   - Experiments
2. Unequal masses
3. Dynamics of phase separation

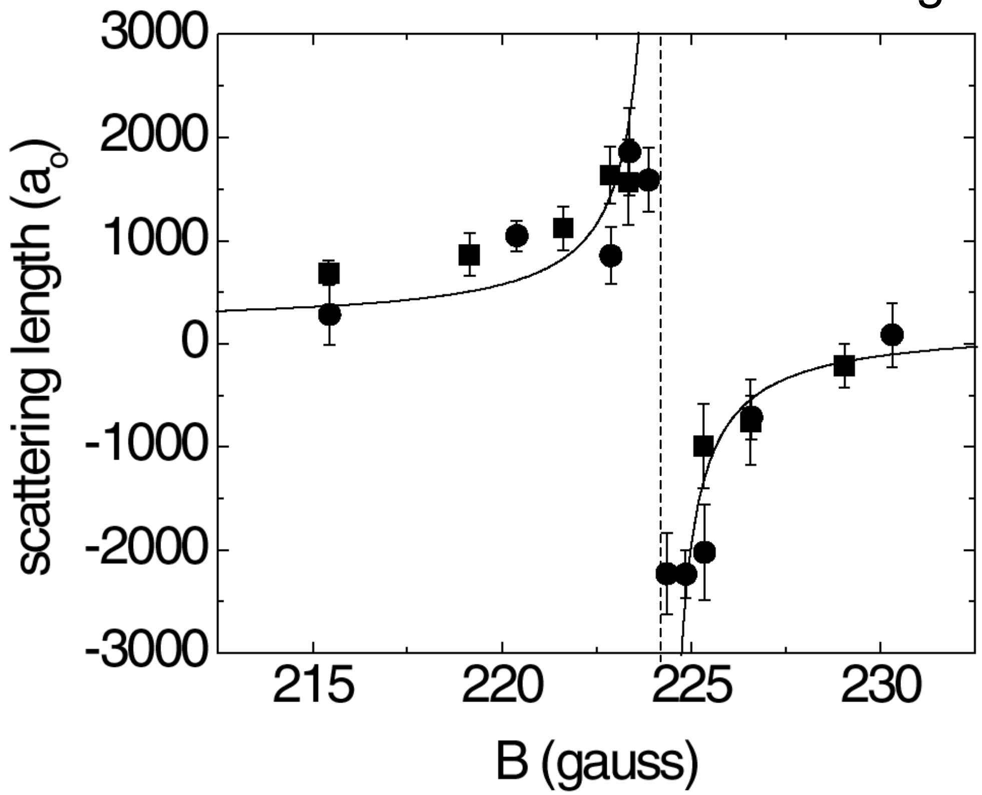
- Conclusions & prospectives

M.M. Parish A. Lamacraft B.D. Simons

1. [M.M. Parish, F.M. Marchetti, A. Lamacraft, & B.D. Simons, *Nature Physics* **3**, 124 (2007)]
2. [M.M. Parish, F.M. Marchetti, A. Lamacraft, & B.D. Simons, *Phys. Rev. Lett.* **98**, 160402 (2007)]
3. [A. Lamacraft & F.M. Marchetti, preprint cond-mat/0701692]
4. [F.M. Marchetti, C. Mathy, & M.M. Parish, (related work on BF mixtures!)]

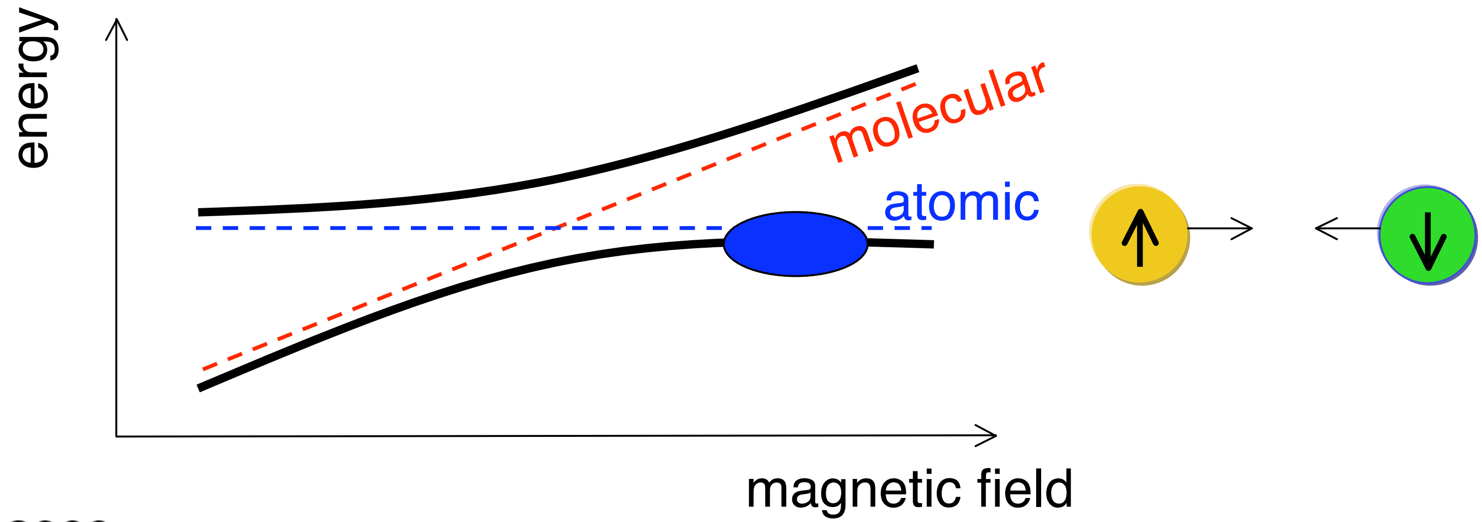
Feshbach resonances

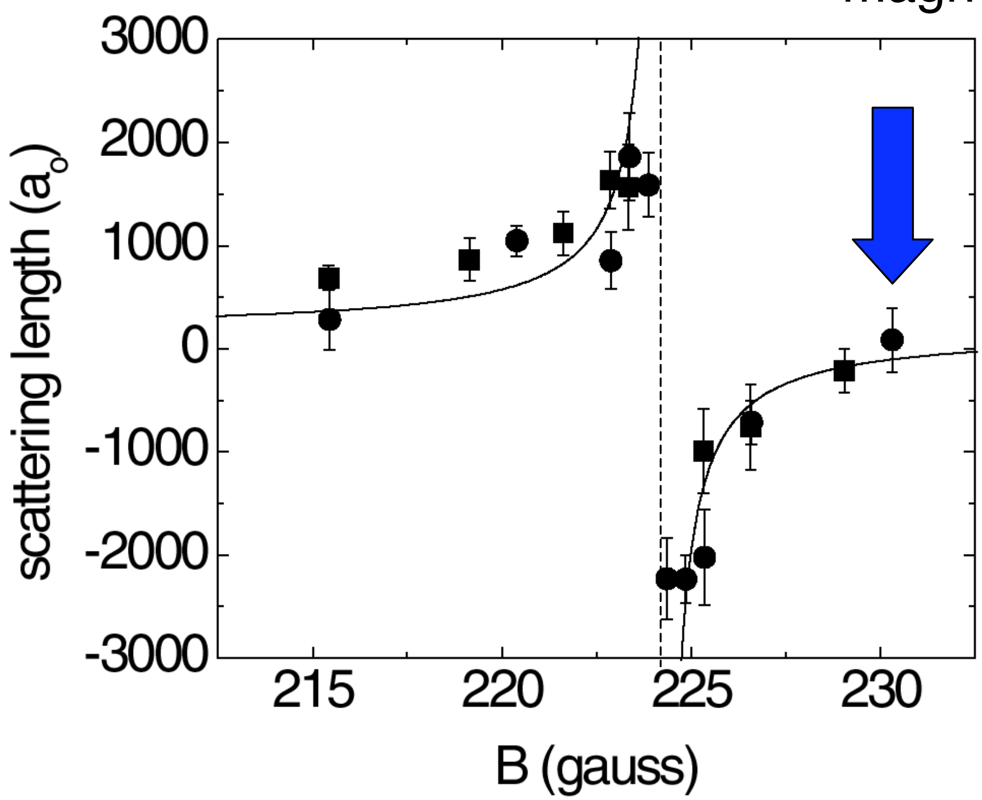




[C. A. Regal *et al.*, *Nature* **424**, 47 (2003)]

Feshbach resonances





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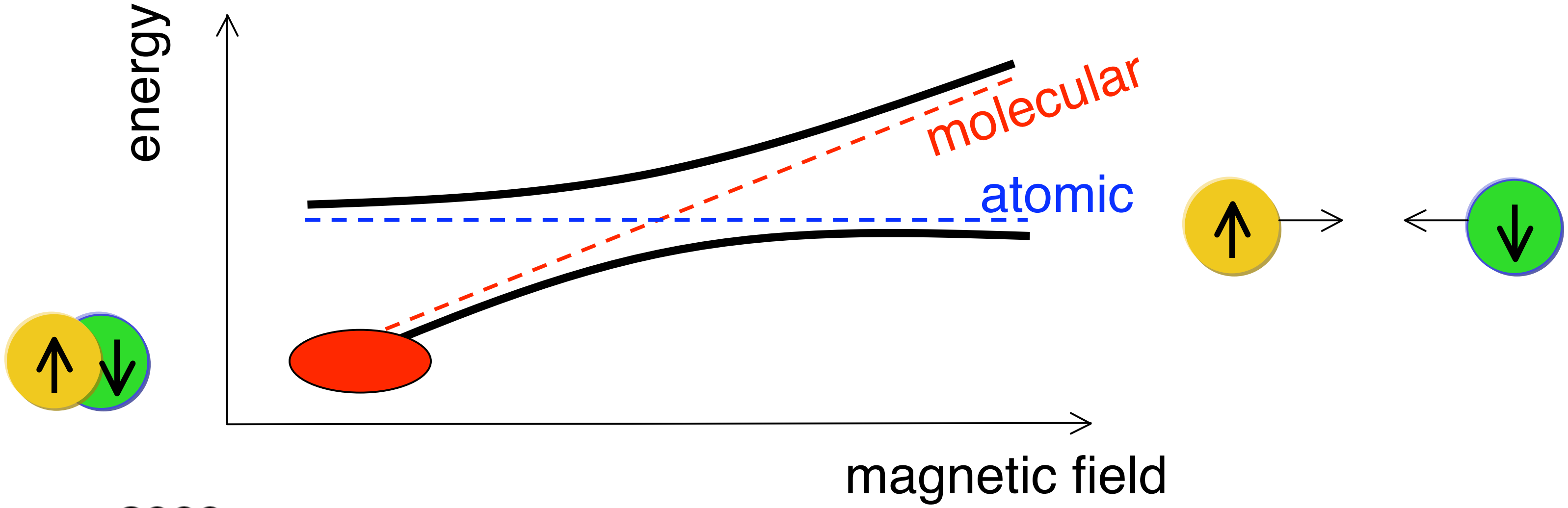
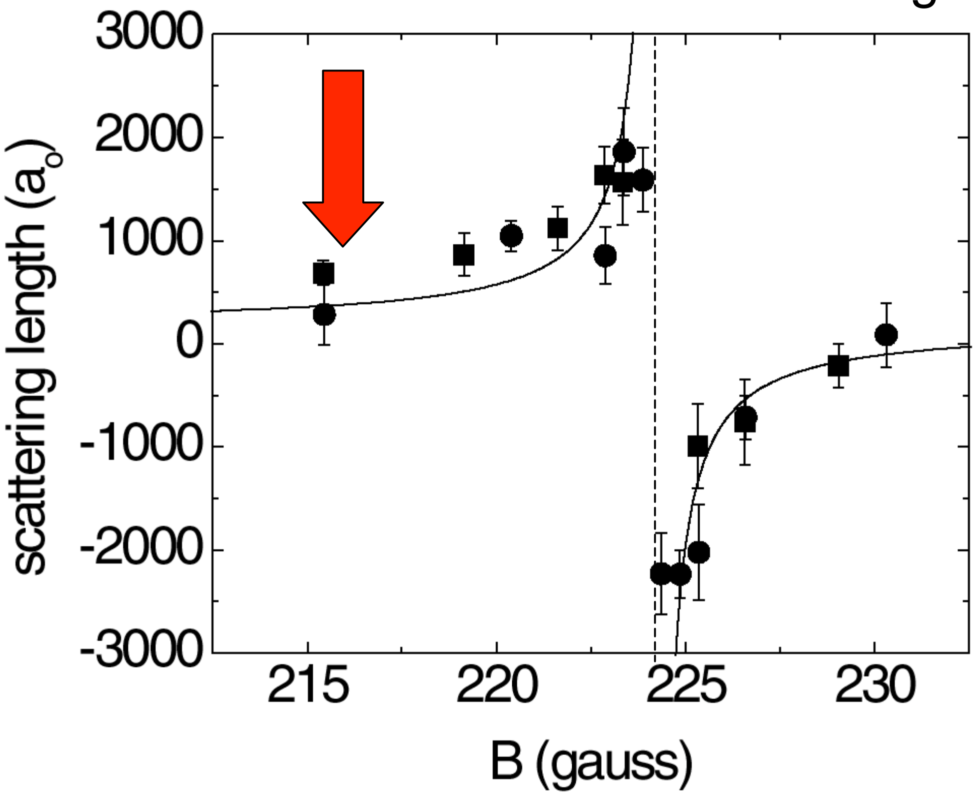
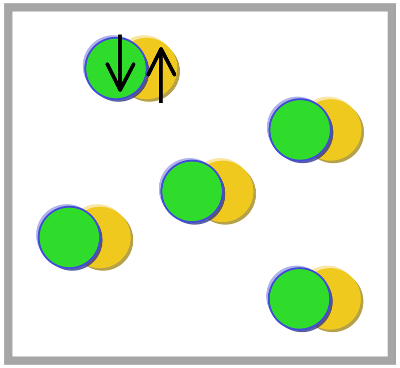




[C. A. Regal *et al.*, *Nature* **424**, 47 (2003)]

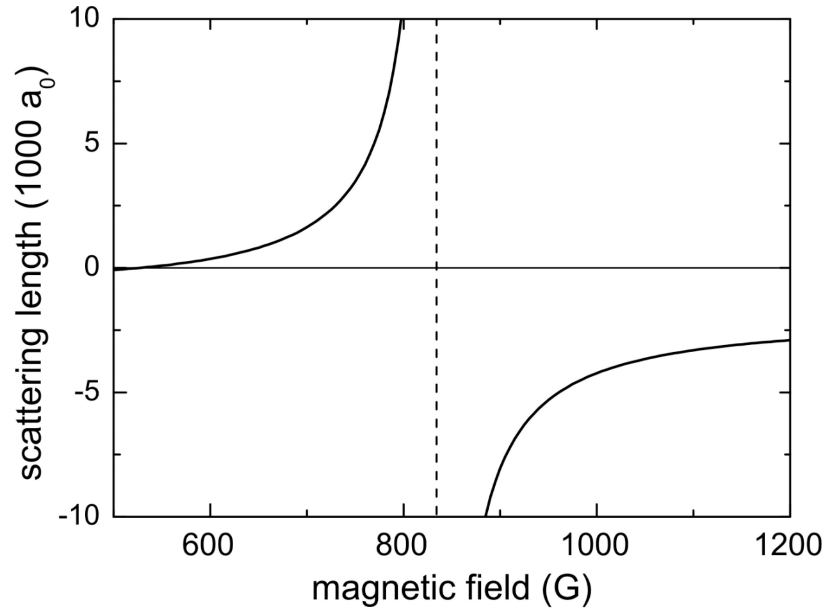
BEC-BCS crossover

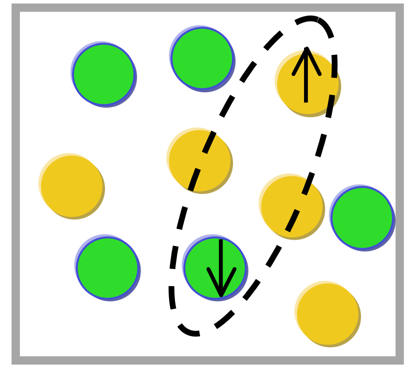
$\frac{1}{a}$ 0



BEC

a>0





BCS

a<0

- weakly repulsive diatomic molecules

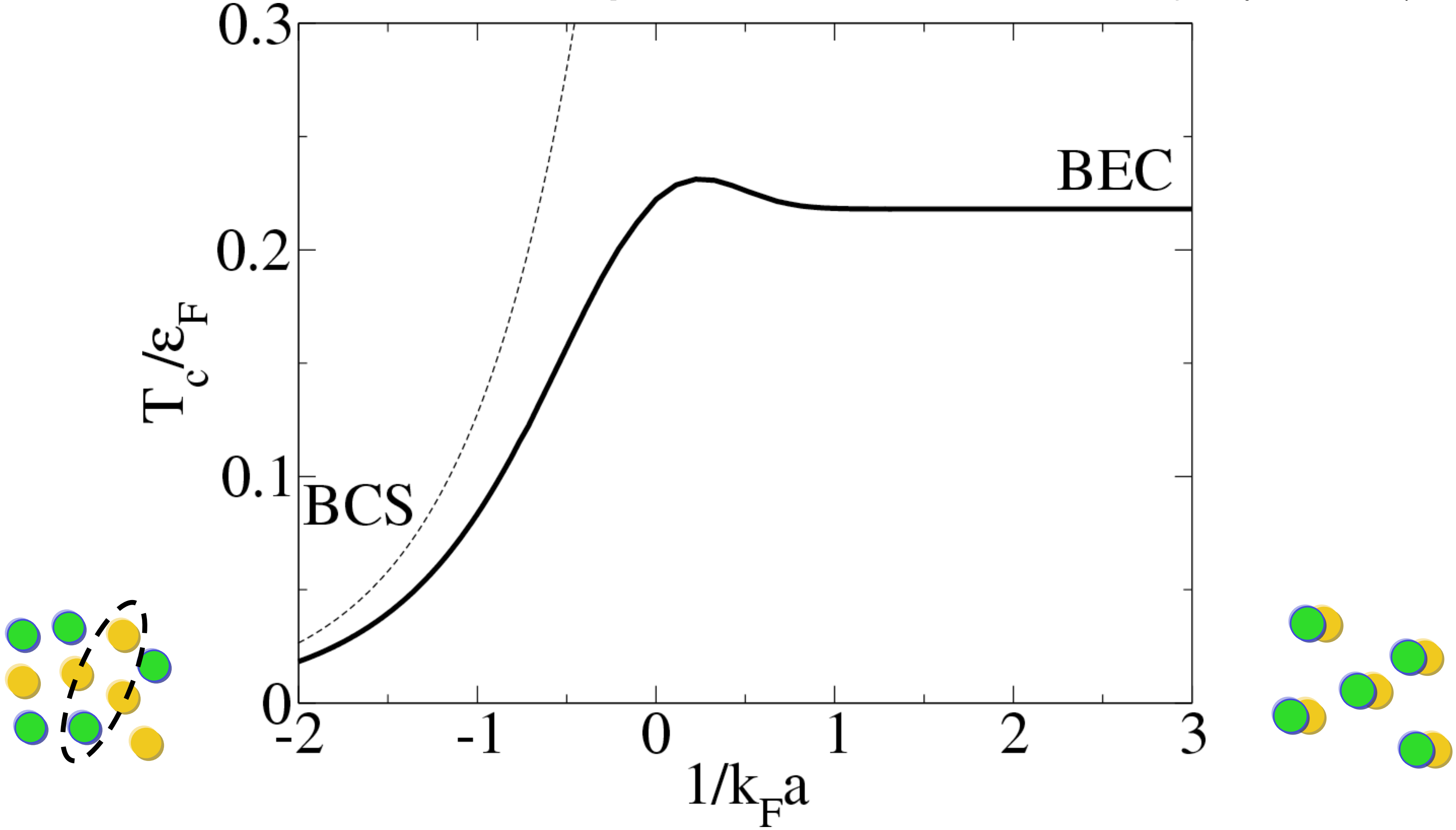
- weakly attractive fermionic atoms

At T=0, described by the same ground state

$$e^{\lambda \sum_{\mathbf{k}} \varphi_{\mathbf{k}} c^{\dagger}_{\mathbf{k},\uparrow} c^{\dagger}_{-\mathbf{k},\downarrow}} |0\rangle \qquad = \qquad \prod_{\mathbf{k}} \left(u_{\mathbf{k}} + v_{\mathbf{k}} c^{\dagger}_{\mathbf{k},\uparrow} c^{\dagger}_{-\mathbf{k},\downarrow} \right) |0\rangle$$

Finite T BEC-BCS crossover

[P. Nozieres & S. Schmitt-Rink, J. Low temp. Phys. **59**, 195 (1985)]



BCS: pairing instability

$$\Delta_{\mathrm{BCS}} \sim k_B T_{\mathrm{BCS}} \ll \varepsilon_F$$

BEC: condensate forms out of preformed molecules

$$T_{\mathrm{BEC}} \sim T_F \ll T_{\mathrm{diss}}$$

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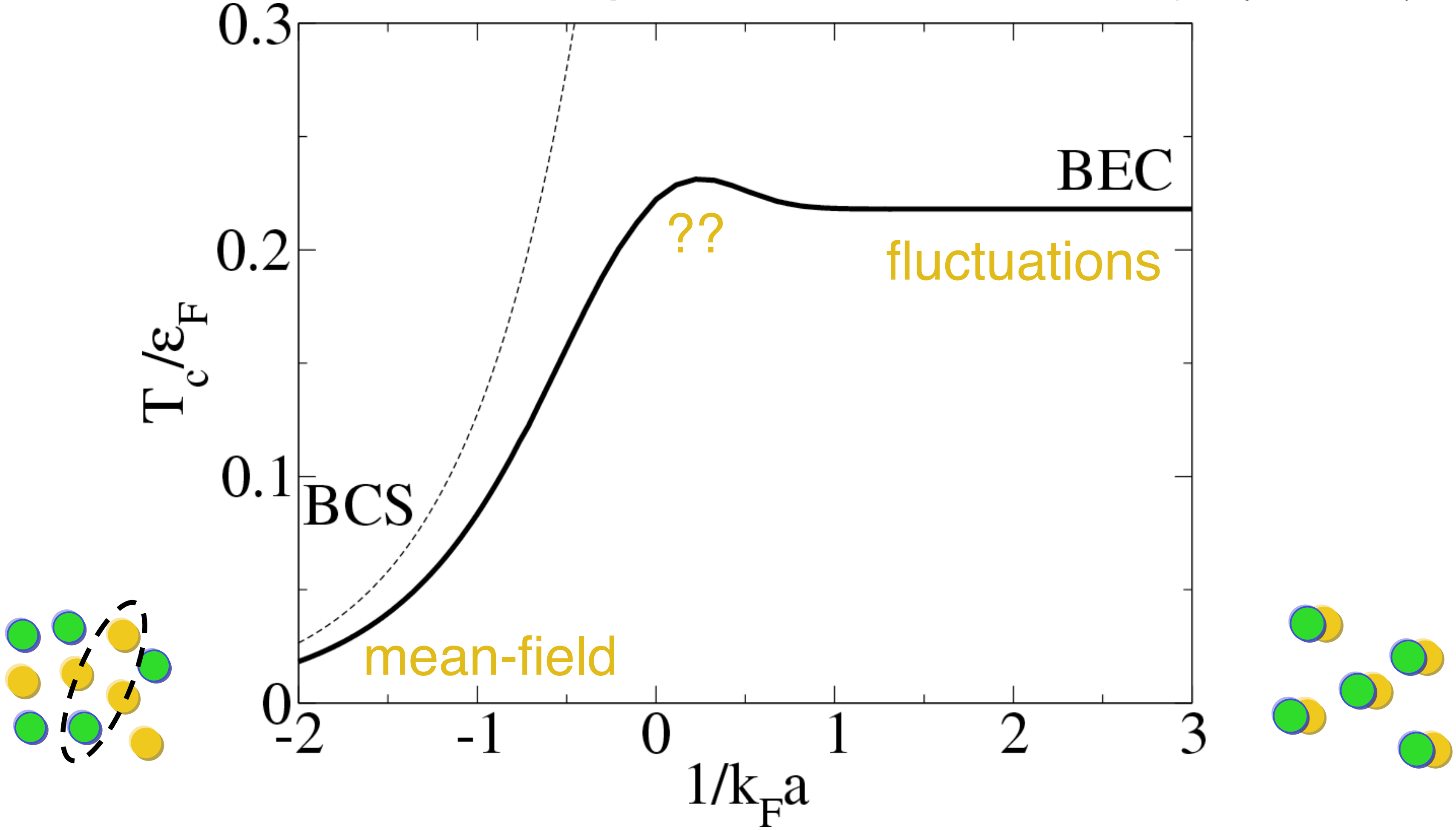


BCS: pairing instability

$$\Delta_{\text{BCS}} \sim k_B T_{\text{BCS}} \ll \varepsilon_F$$

BEC: condensate forms out of preformed molecules

$$T_{\text{BEC}} \sim T_F \ll T_{\text{diss}}$$

What if not every fermion can pair up?

Single-channel model

$$\hat{\mathcal{H}} - \sum_{\sigma=\uparrow,\downarrow} \mu_\sigma \hat{N}_\sigma = \sum_{\mathbf{k},\sigma} (\epsilon_{\mathbf{k}} - \mu_\sigma) c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{U}{V} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} c^\dagger_\uparrow c^\dagger_\downarrow c_\downarrow c_\uparrow$$

- Contact interaction $\frac{1}{U} = \frac{m}{4\pi a} - \frac{1}{V} \sum_{\mathbf{k}} \frac{1}{2\epsilon_{\mathbf{k}}}$
- Allow for different populations

$$\hat{n}_\uparrow = \frac{1}{V} \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} c_{\mathbf{k}\uparrow}$$

$$\hat{n}_\downarrow = \frac{1}{V} \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\downarrow} c_{\mathbf{k}\downarrow}$$

- Averaged chemical potential & 'Zeeman' term
  [total density & population imbalance (or 'magnetisation')]

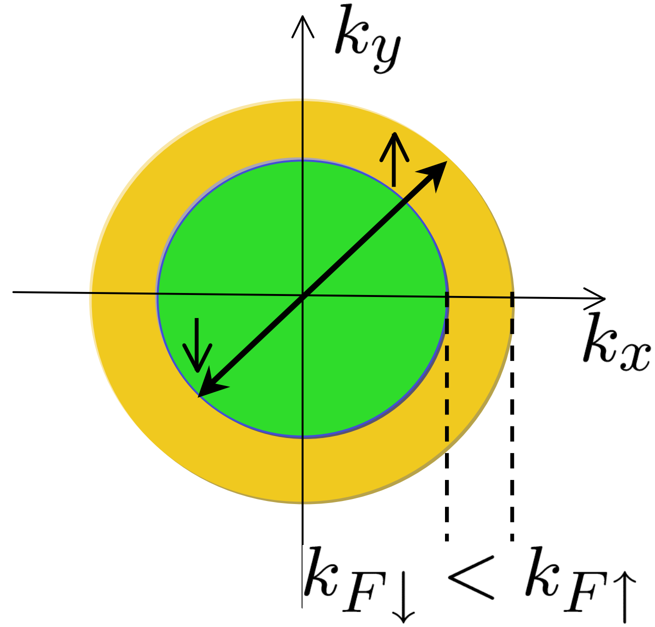
$$\mu = (\mu_\uparrow + \mu_\downarrow)/2 \qquad \hat{n} = \hat{n}_\uparrow + \hat{n}_\downarrow$$

$$h = (\mu_\uparrow - \mu_\downarrow)/2 \qquad \hat{m} = \hat{n}_\uparrow - \hat{n}_\downarrow$$

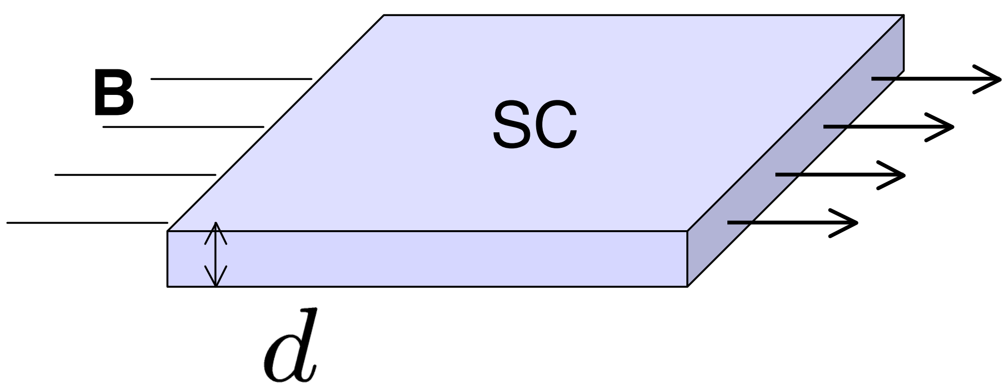
Analogy with magnetised superconductors

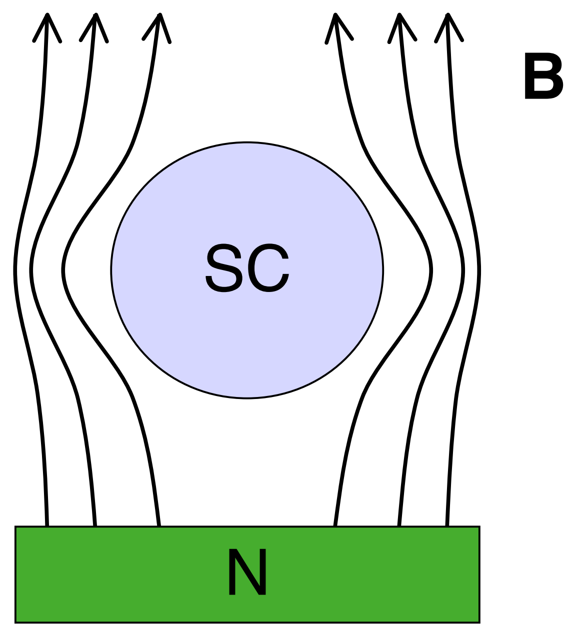
- A population imbalance like a Zeeman term in a superconductor

$$H_{\mathrm{BdG}} = \begin{pmatrix} \epsilon_{\mathbf{k}} - \mu - h & -\Delta \\ -\Delta & -(\epsilon_{\mathbf{k}} - \mu) - h \end{pmatrix}$$



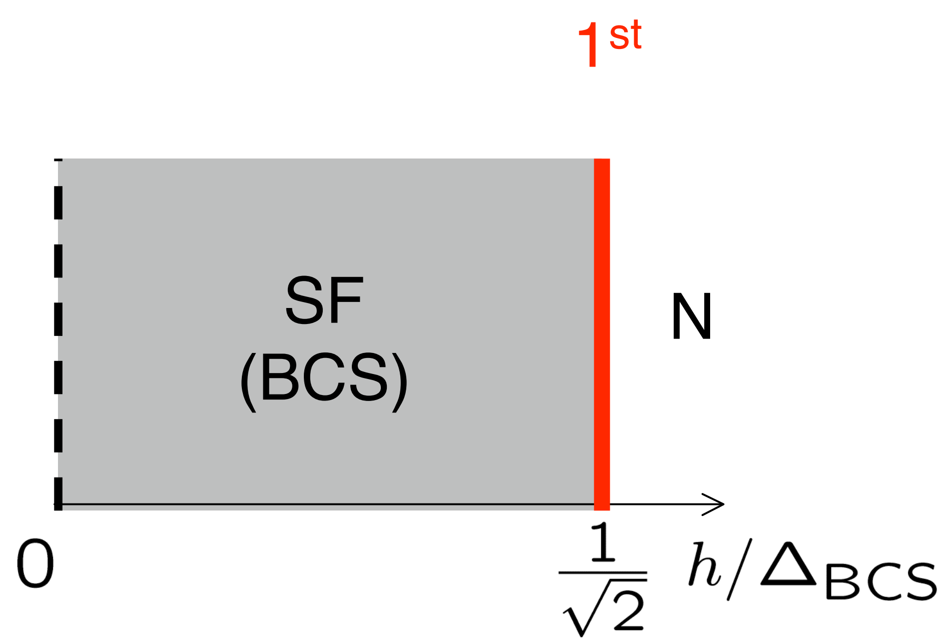
- Neglect the orbital effect?





T=0 magnetised superconductors

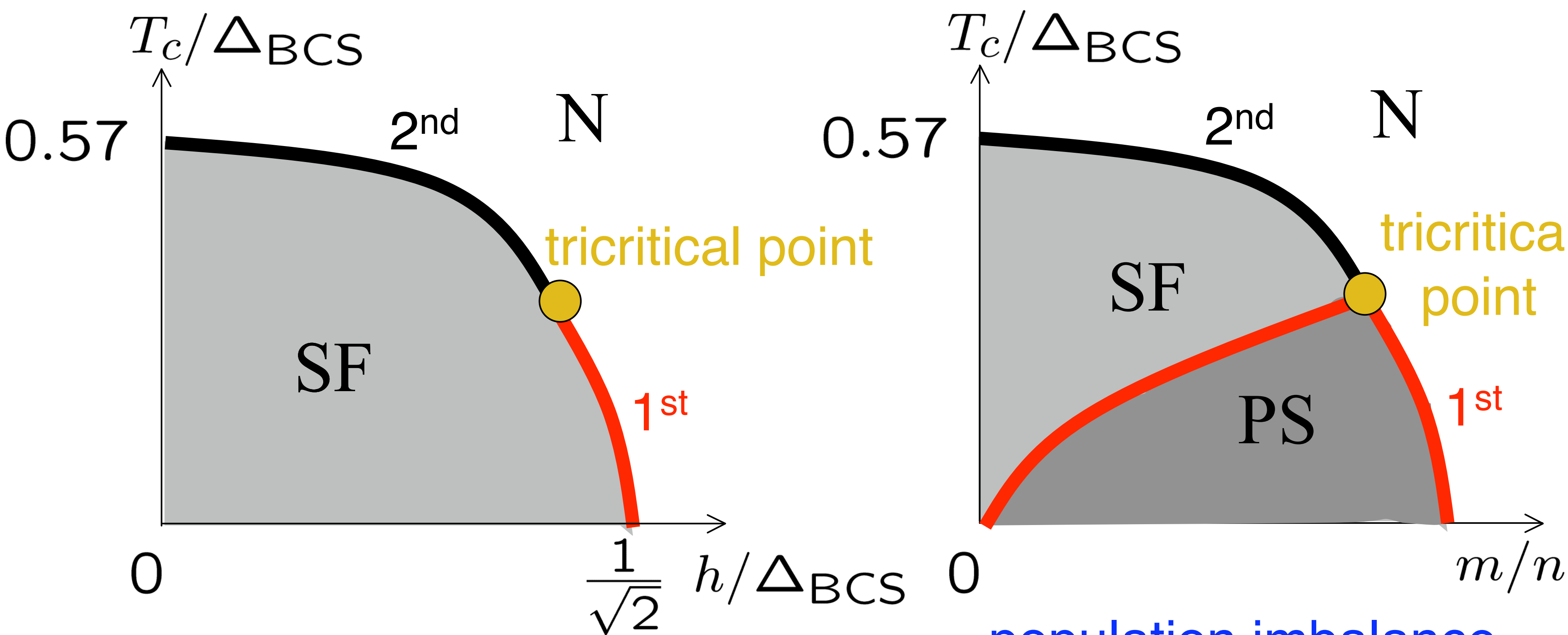
BCS side of the resonance



[G. Sarma, J. Phys. Chem. Solids **24** 1029 (1963)]

T≠0 magnetised superconductor

BCS side of the resonance

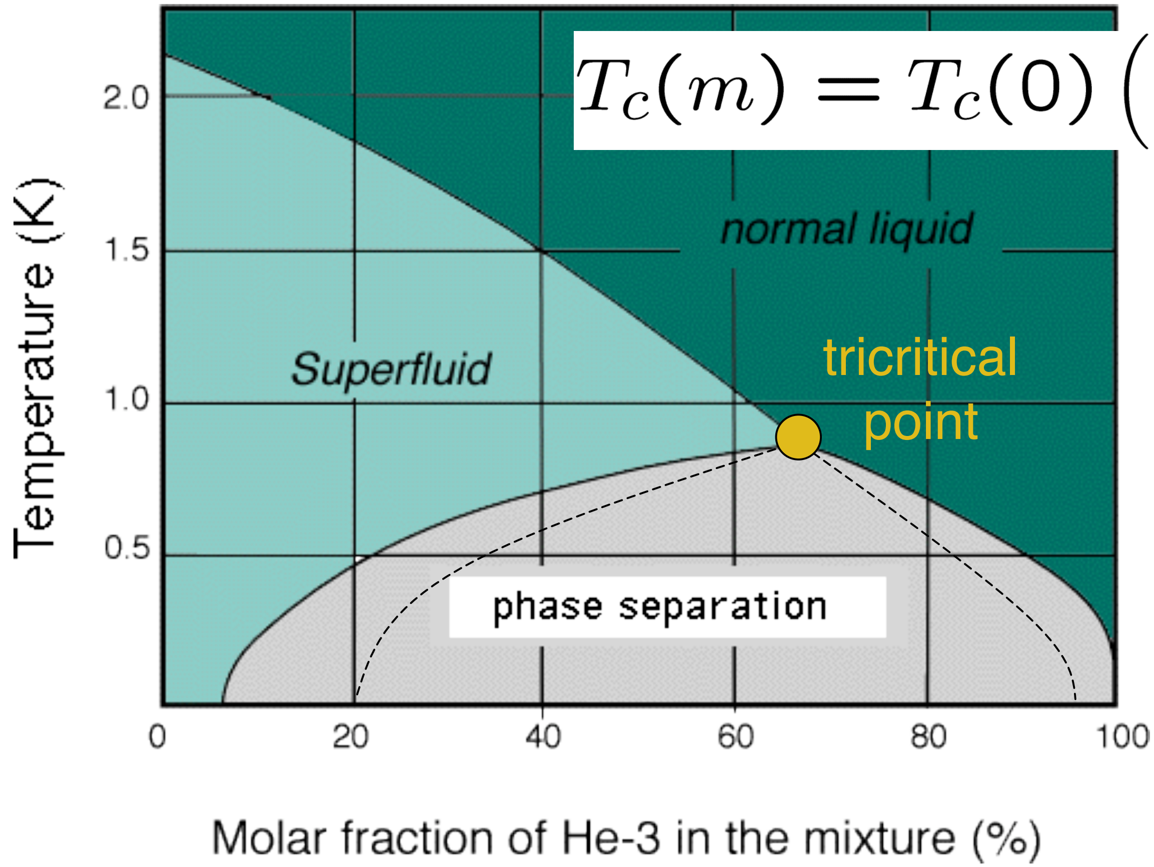


population imbalance

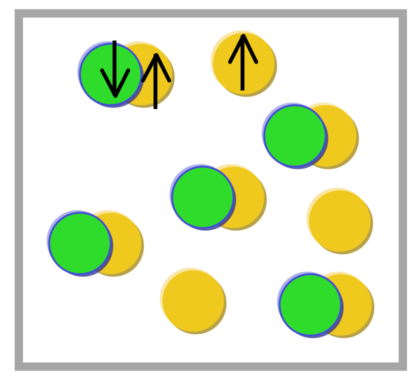
$$m = n_\uparrow - n_\downarrow$$

[G. Sarma, J. Phys. Chem. Solids **24** 1029 (1963)]

Analogy with ^{3}He-^{4}He mixtures



$$T_c(m) = T_c(0)\left(1 - \frac{m}{n}\right)^{\alpha}$$



- ^{3}He-^{4}He = Bose-Fermi mixture…
  … and the polarised Fermi gas is a Bose-Fermi mixture on the BEC side of the resonance

Expect the same structure on the BEC side?

Mean-field: excitation spectrum

- Paired states start to be depleted when:

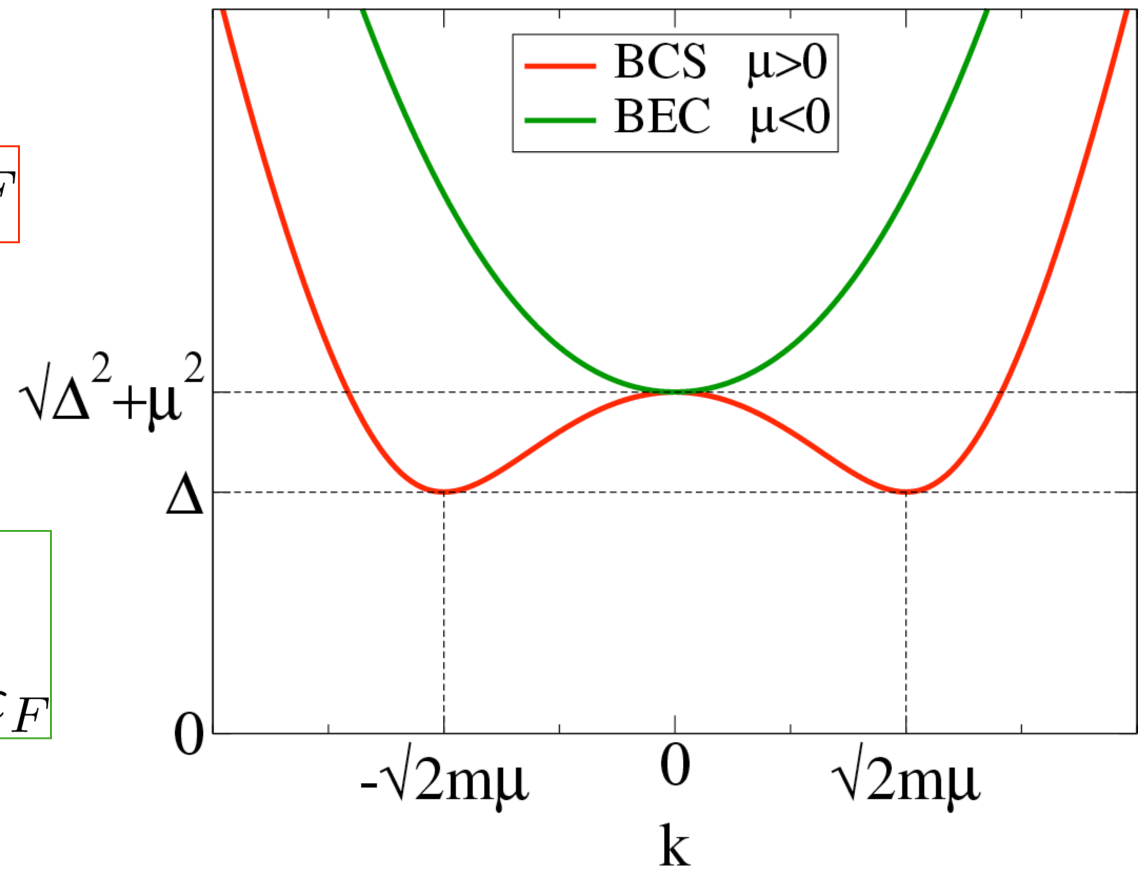
$$E_{\mathbf{k},\sigma=\uparrow,\downarrow} = \underbrace{\sqrt{(\epsilon_{\mathbf{k}}-\mu)^2+\Delta^2}}_{E_{\mathbf{k}}} \pm h$$

1. BCS side

$$h = \min_{\mathbf{k}} E_{\mathbf{k}} = \Delta \ll \varepsilon_F$$

2. BEC side

$$h = \min_{\mathbf{k}} E_{\mathbf{k}} = \sqrt{\Delta^2+\mu^2} \sim \mu \gg \varepsilon_F$$



Mean-field grand-canonical free energy

$$\Omega^{(0)}(\mu, h) = \min_{\Delta} f^{(0)}(\Delta; \mu, h)$$

$\Rightarrow$ $f^{(0)}(\frac{\Delta}{|\mu|}; \frac{h}{|\mu|})$

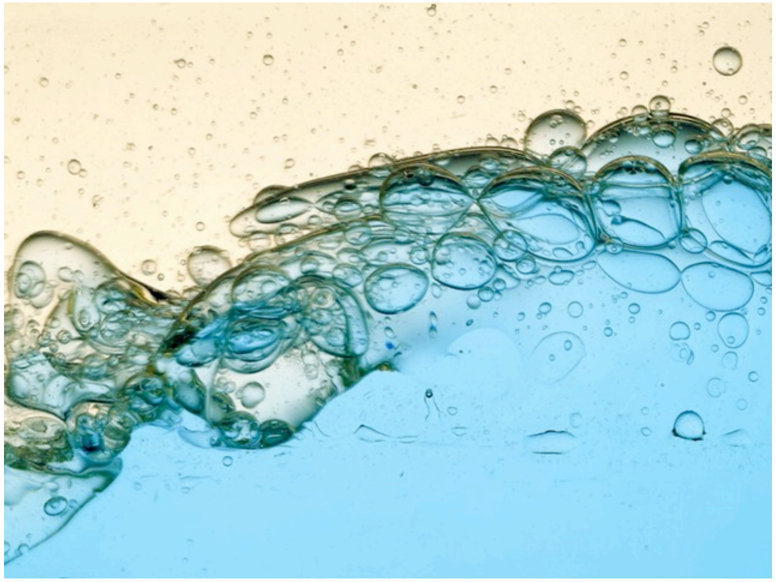
$$\begin{cases} n = -\dfrac{\partial \Omega^{(0)}}{\partial \mu} \\ m = -\dfrac{\partial \Omega^{(0)}}{\partial h} \end{cases}$$

$\Rightarrow$ $\dfrac{m}{n}$

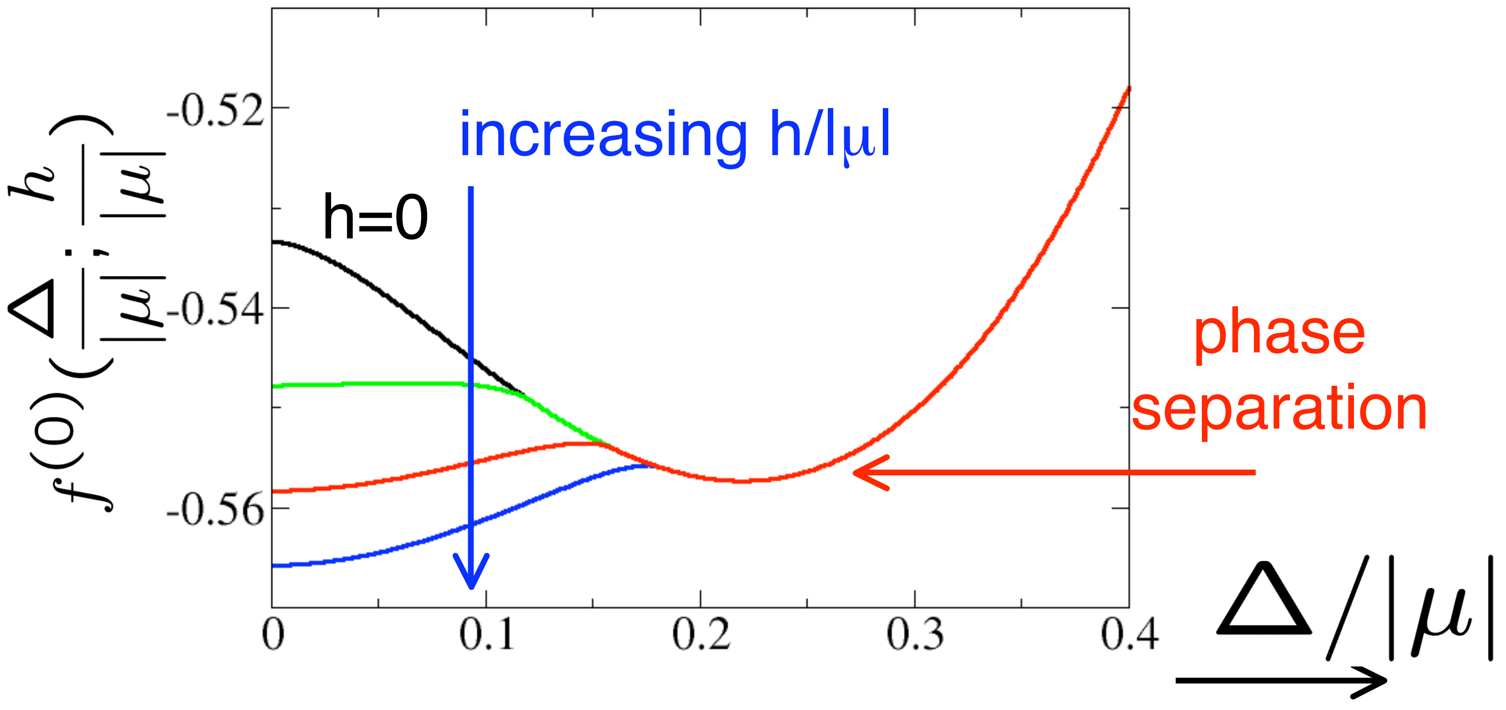
polarisation

1^{st} order phase transition

- $\frac{1}{k_F a} < \left(\frac{1}{k_F a}\right)_{\text{tricrit}}$ and $\frac{T}{\varepsilon_F} < \left(\frac{T}{\varepsilon_F}\right)_{\text{tricrit}}$



(oil&water)



1st order phase transition

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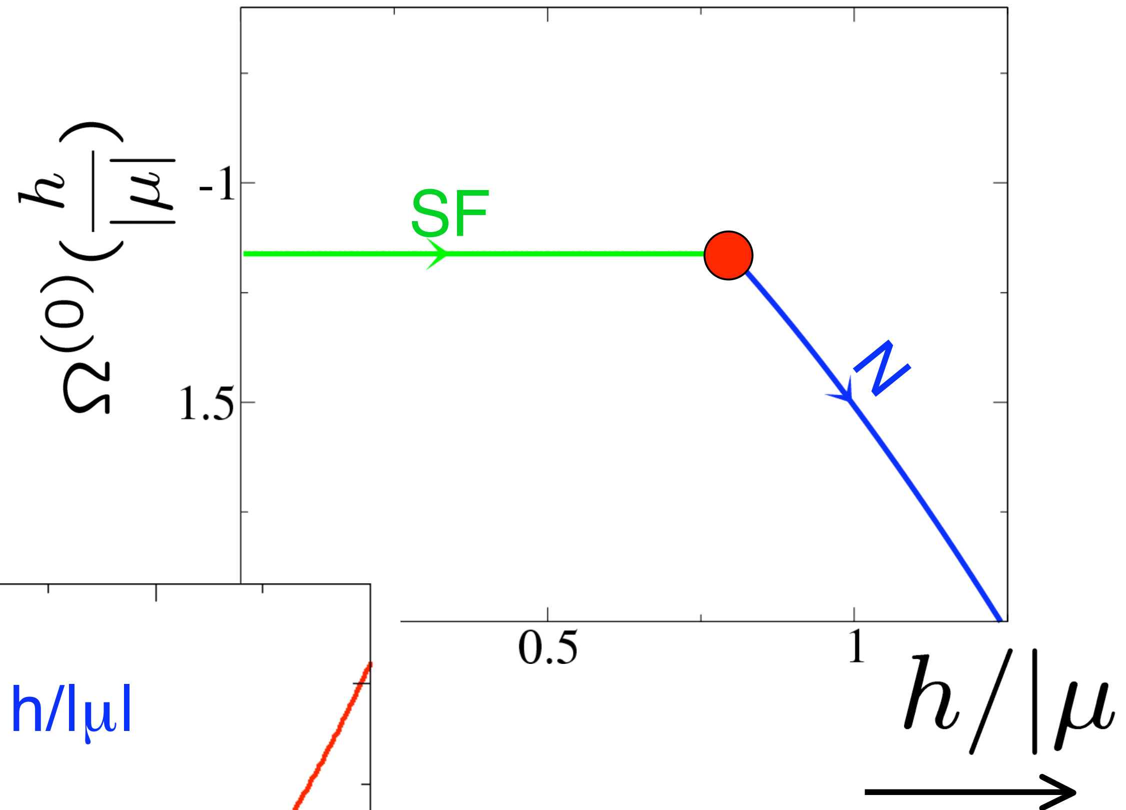
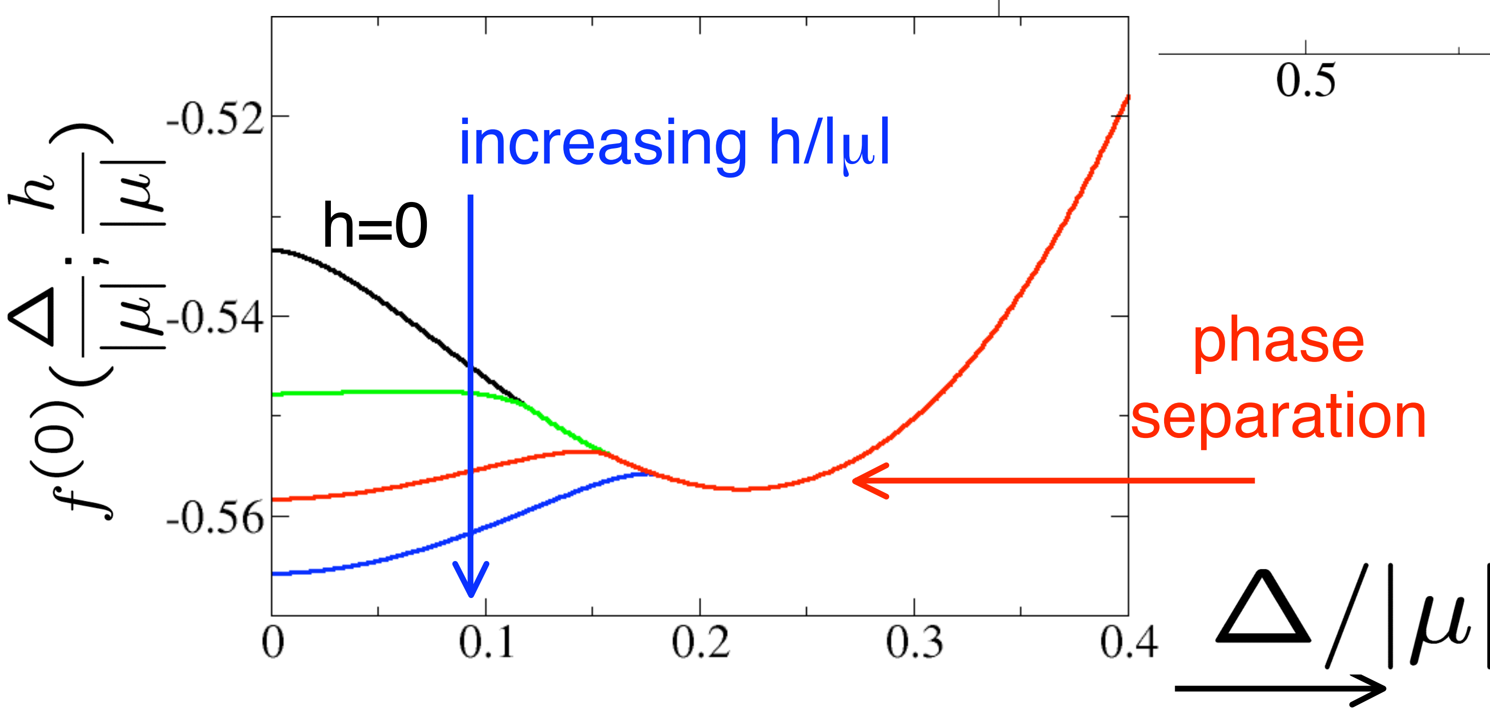
  and $$\frac{T}{\varepsilon_F} < \left(\frac{T}{\varepsilon_F}\right)_{\text{tricrit}}$$

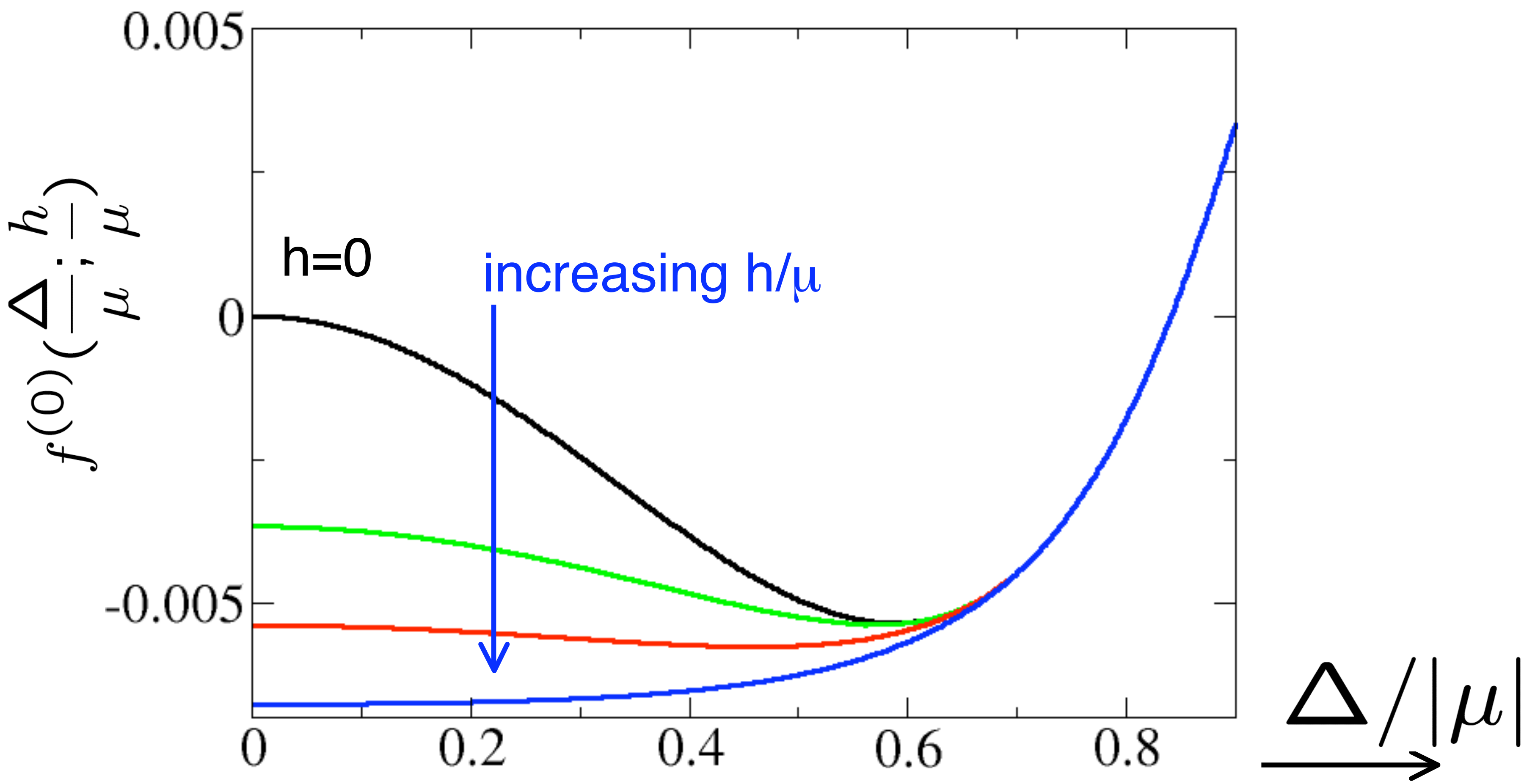




2^{nd} order phase transition

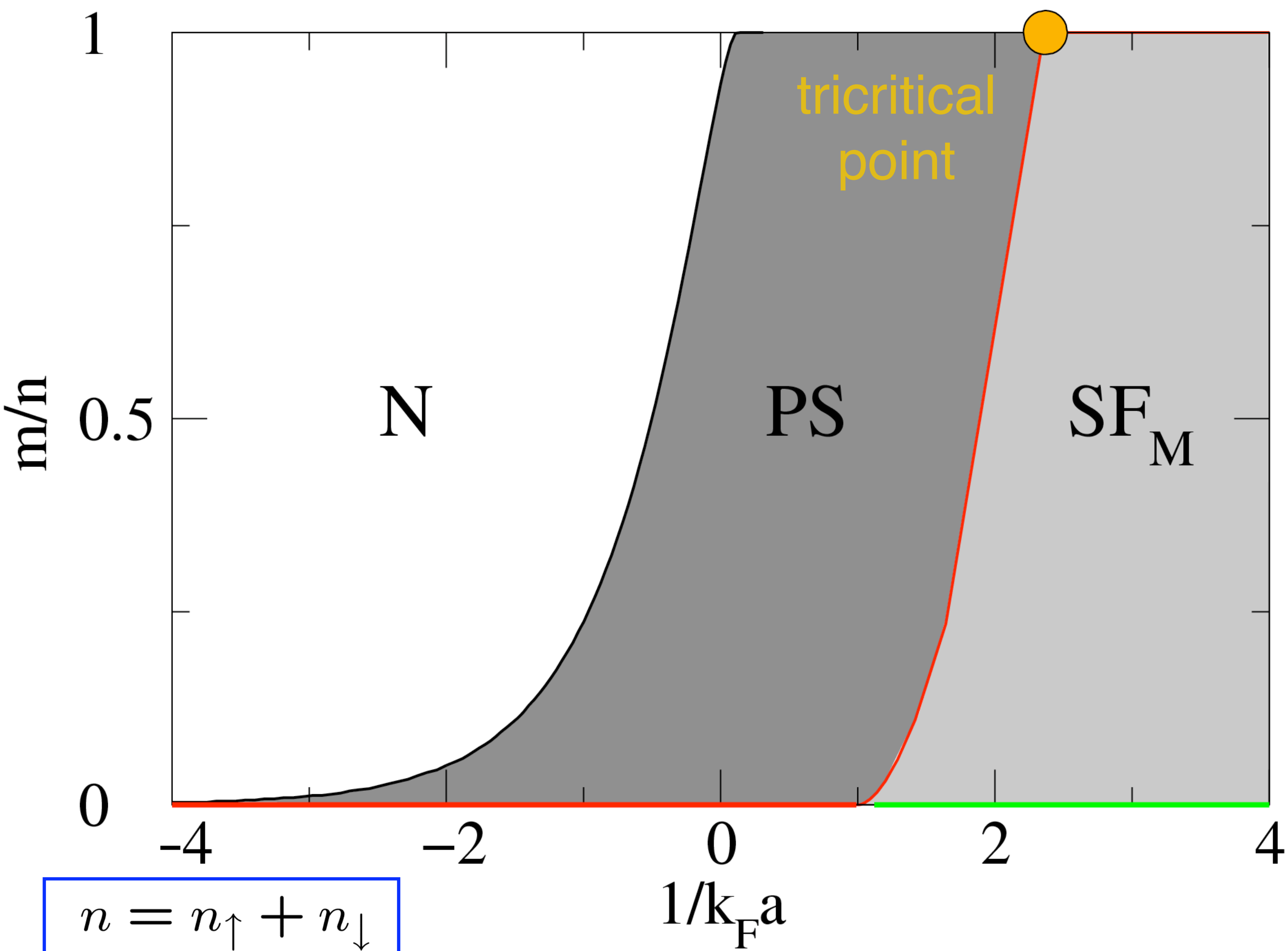
- $\frac{1}{k_F a} > \left(\frac{1}{k_F a}\right)_{\text{tricrit}}$ (towards BEC

or $\frac{T}{\varepsilon_F} > \left(\frac{T}{\varepsilon_F}\right)_{\text{tricrit}}$ or high temperature)



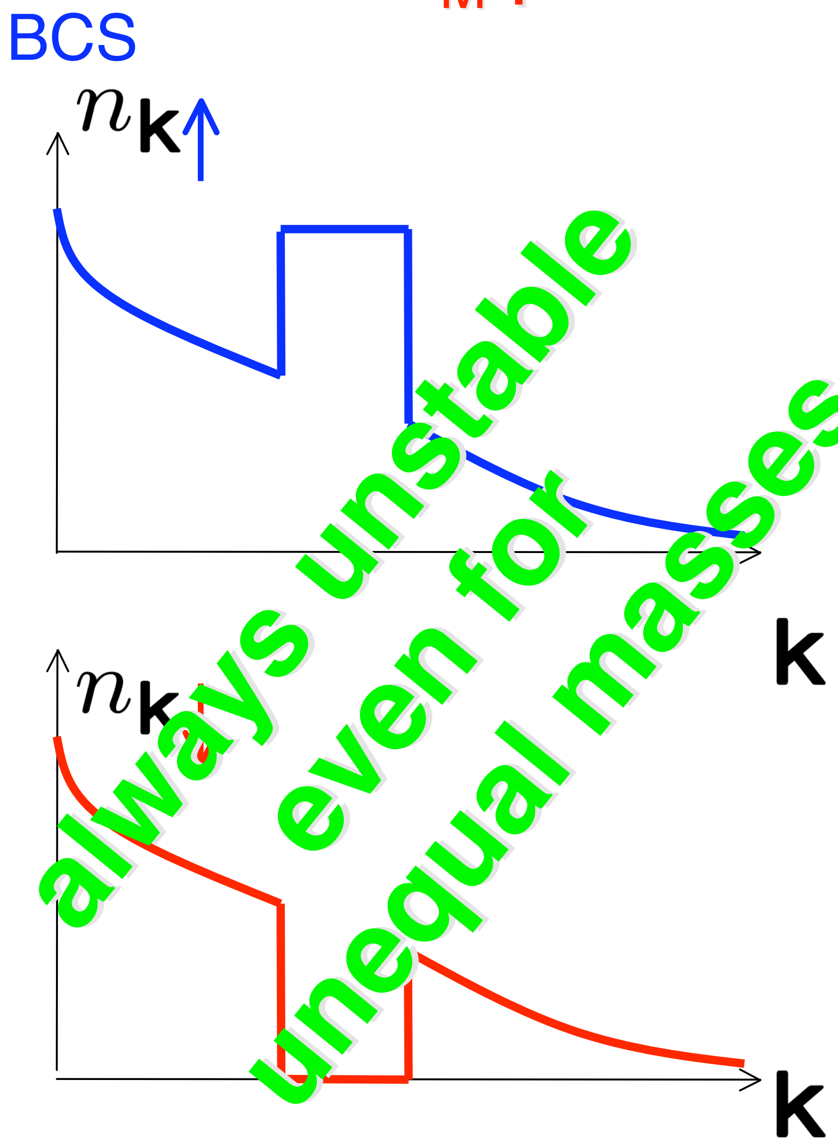
T=0 phase diagram

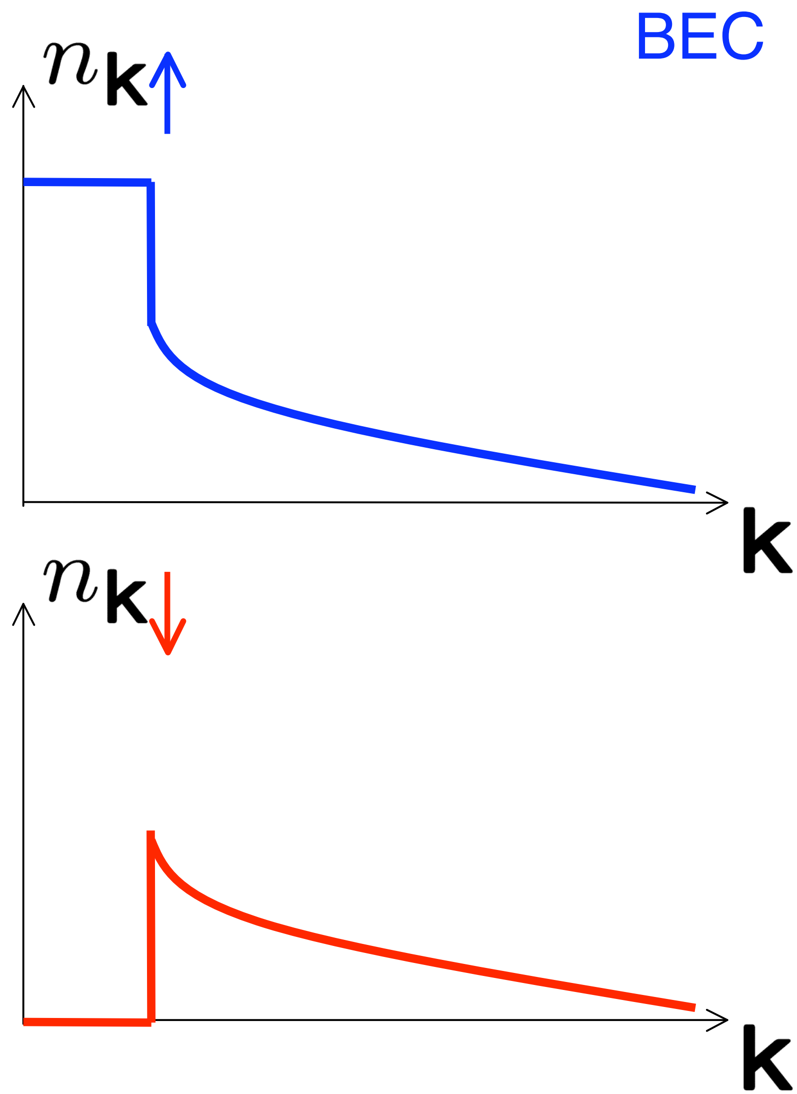
[M. Parish, F.M. Marchetti *et al.*, *Nature Physics* **3**, 124 (2007)]



$$n = n_\uparrow + n_\downarrow$$
$$m = n_\uparrow - n_\downarrow$$

SF_M phase: Breached Pairing



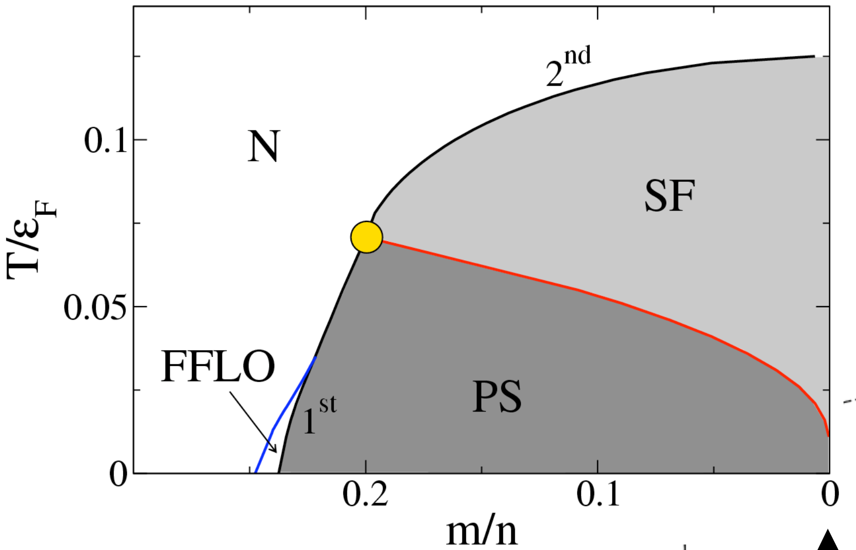


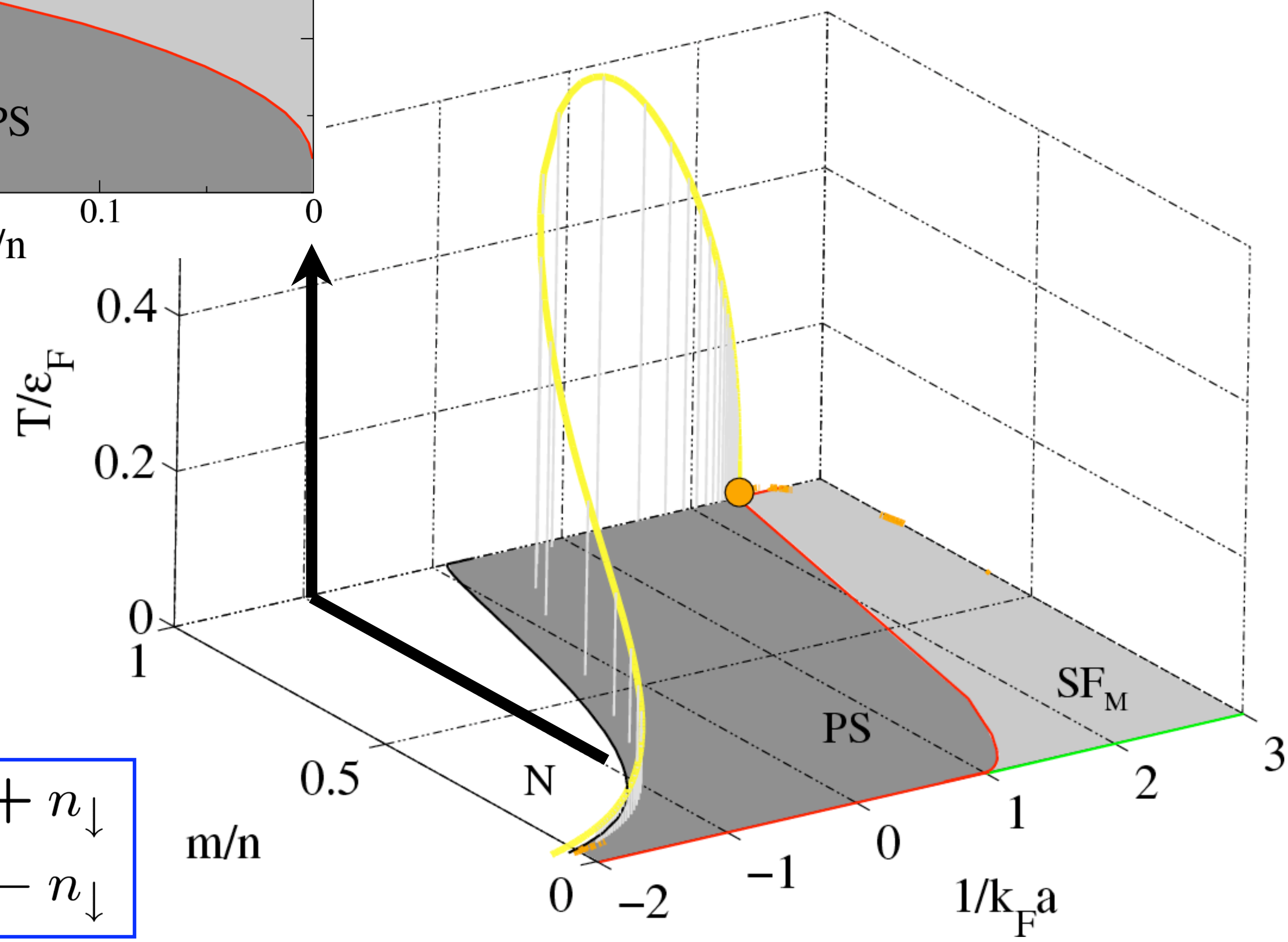
[W. V. Liu & F. Wilczek, PRL **90** 047002 (2003)]

[M. Parish, F.M. Marchetti *et al.*, *Nature Physics* **3**, 124 (2007)]
[M. Parish, F.M. Marchetti *et al.*, *PRL* **98**, 160402 (2007)]

Finite T phase diagram

[M. Parish, F.M. Marchetti *et al.*, *Nature Physics* **3**, 124 (2007)]





$$n = n_\uparrow + n_\downarrow$$
$$m = n_\uparrow - n_\downarrow$$

Adding pair fluctuations (finite T)

[P. Nozieres & S. Schmitt-Rink, J. Low temp. Phys. **59**, 195 (1985)]

- One loop correction to mean-field T_c ($\Delta = 0$)

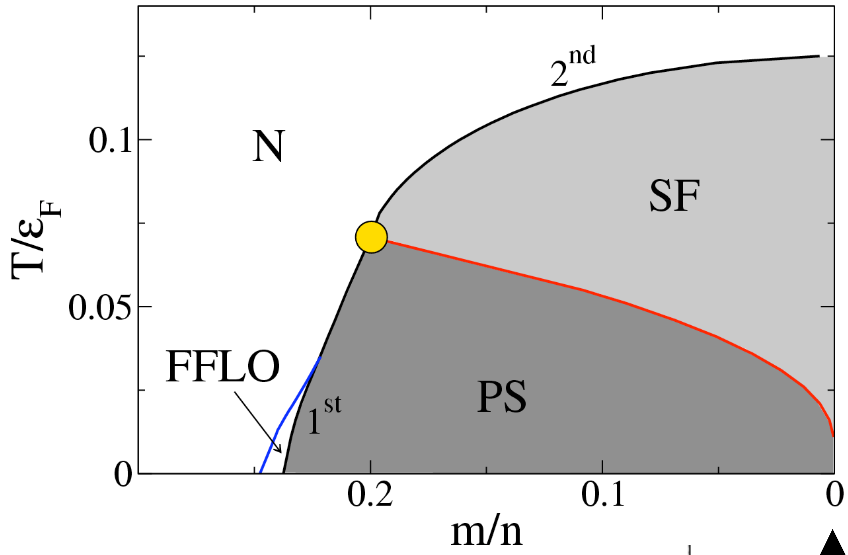
$$\Omega(\mu, h) = \Omega^{(0)}(\mu, h) + \Omega^{(1)}(\mu, h)$$

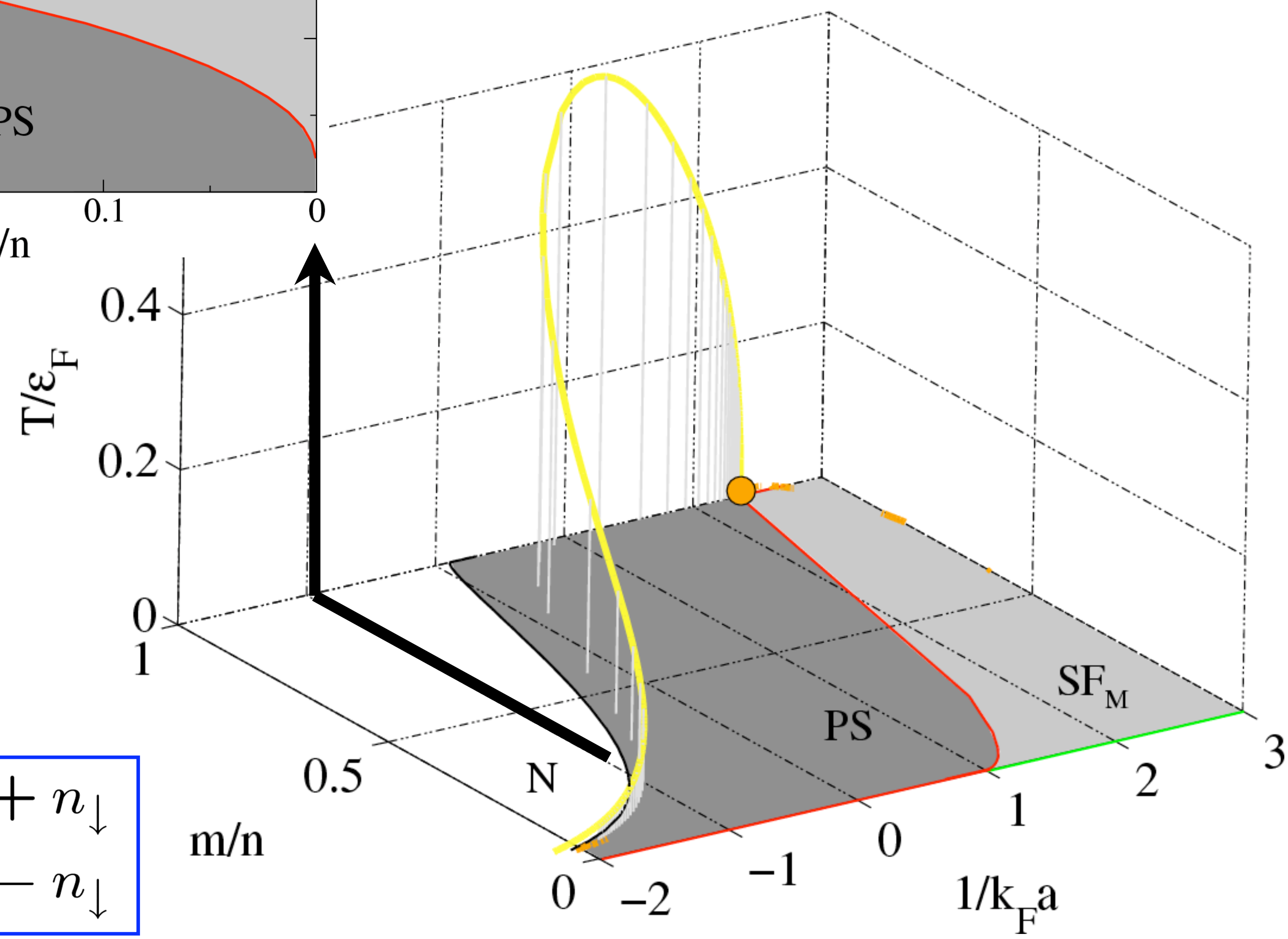
$$n = -\frac{\partial \Omega}{\partial \mu} = n^{(0)} + n^{(1)}$$
$$m = -\frac{\partial \Omega}{\partial h} = m^{(0)} + m^{(1)}$$

condensed pairs + qp's — thermal pairs

Finite T phase diagram

[M. Parish, F.M. Marchetti *et al.*, *Nature Physics* **3**, 124 (2007)]





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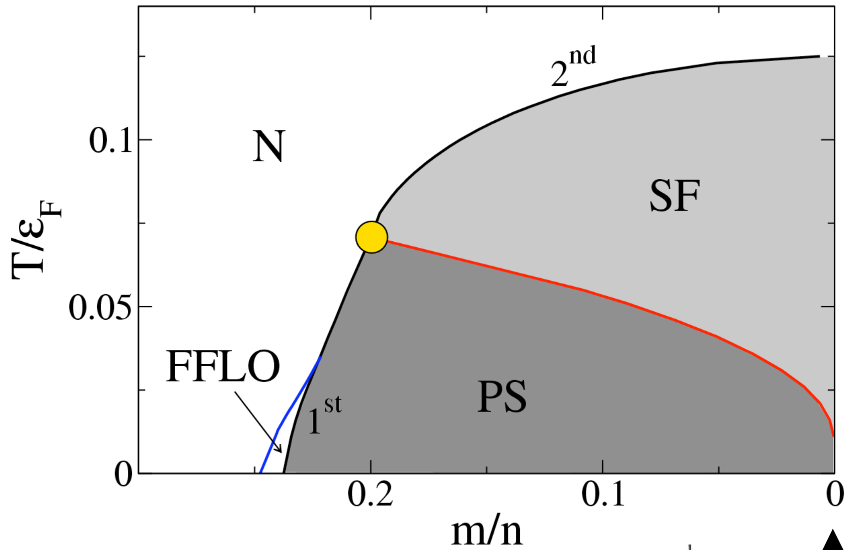
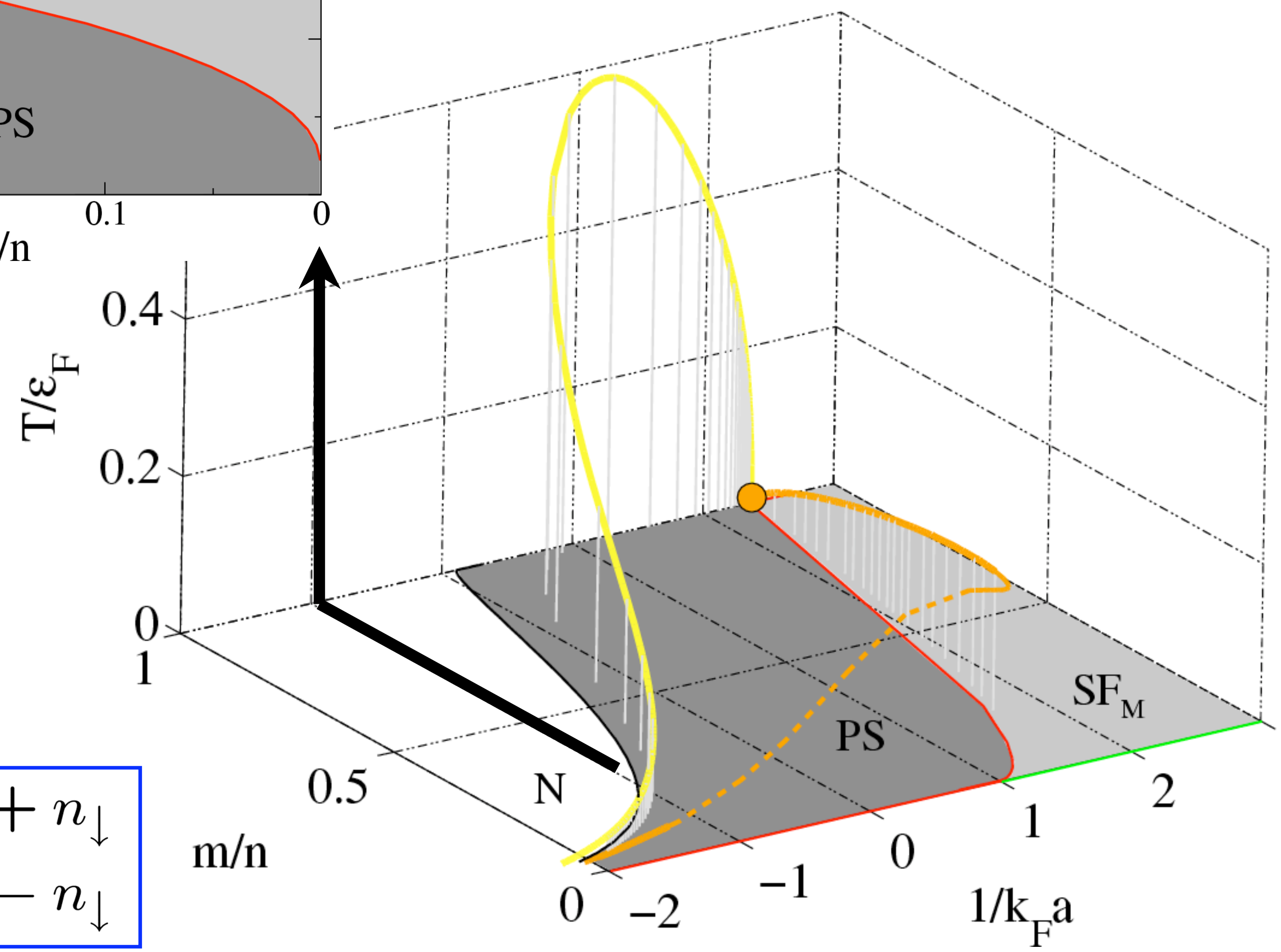




$$n = n_\uparrow + n_\downarrow$$
$$m = n_\uparrow - n_\downarrow$$

Single- vs. two-channel model

[A. Andreev *et al.*, *PRL* **93**, 130402 (2004)]

$$\hat{\mathcal{H}}_{1C} = \sum_{\mathbf{k},\sigma=\uparrow,\downarrow} \epsilon_{\mathbf{k}} c^\dagger_\sigma c_\sigma + \frac{U}{V} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} c^\dagger_\uparrow c^\dagger_\downarrow c_\downarrow c_\uparrow$$

$$\hat{\mathcal{H}}_{2C} = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} c^\dagger_\sigma c_\sigma + \sum_{\mathbf{k}} \left(\frac{\epsilon_{\mathbf{k}}}{2} + \delta_0\right) b^\dagger b + \frac{g}{\sqrt{V}} \sum_{\mathbf{k},\mathbf{k}'} \left(b c^\dagger_\uparrow c^\dagger_\downarrow + \text{h.c.}\right)$$

- The single-channel model is recovered in the limit

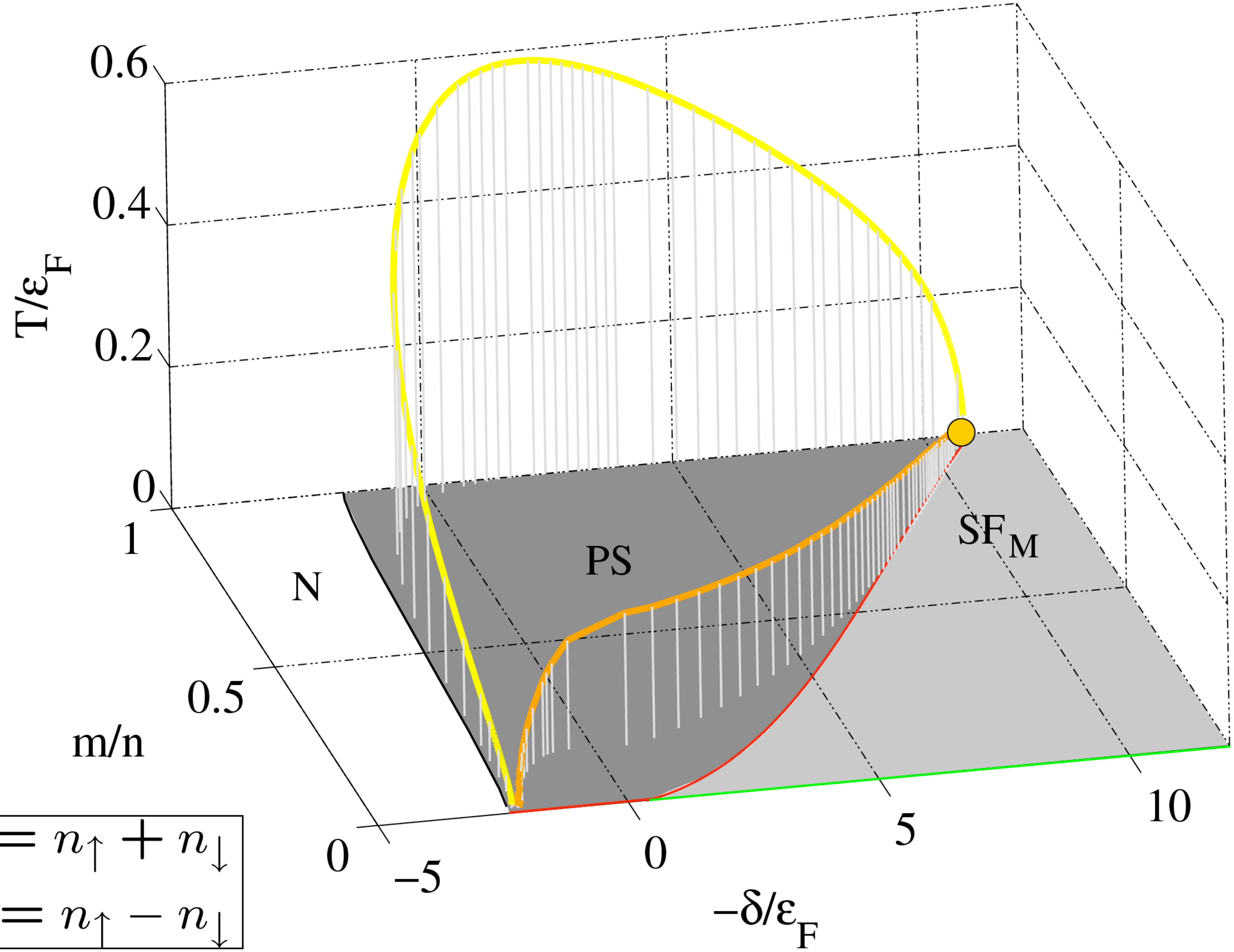
$$\frac{4\pi a}{m} = \frac{-g^2}{\delta_0 - \frac{g^2}{V}\sum_{\mathbf{k}} \frac{1}{2\epsilon_{\mathbf{k}}}} \equiv \frac{-g^2 \to \infty}{\delta \to \infty} = \text{const}$$

- g can be a small parameter (narrow resonances) and controls the fluctuations corrections above mean-field

Single- vs. two-channel model

[A. Andreev *et al.*, *PRL* **93**, 130402 (2004)]

[M. Parish, F.M. Marchetti *et al.*, *Nature Physics* **3**, 124 (2007)]

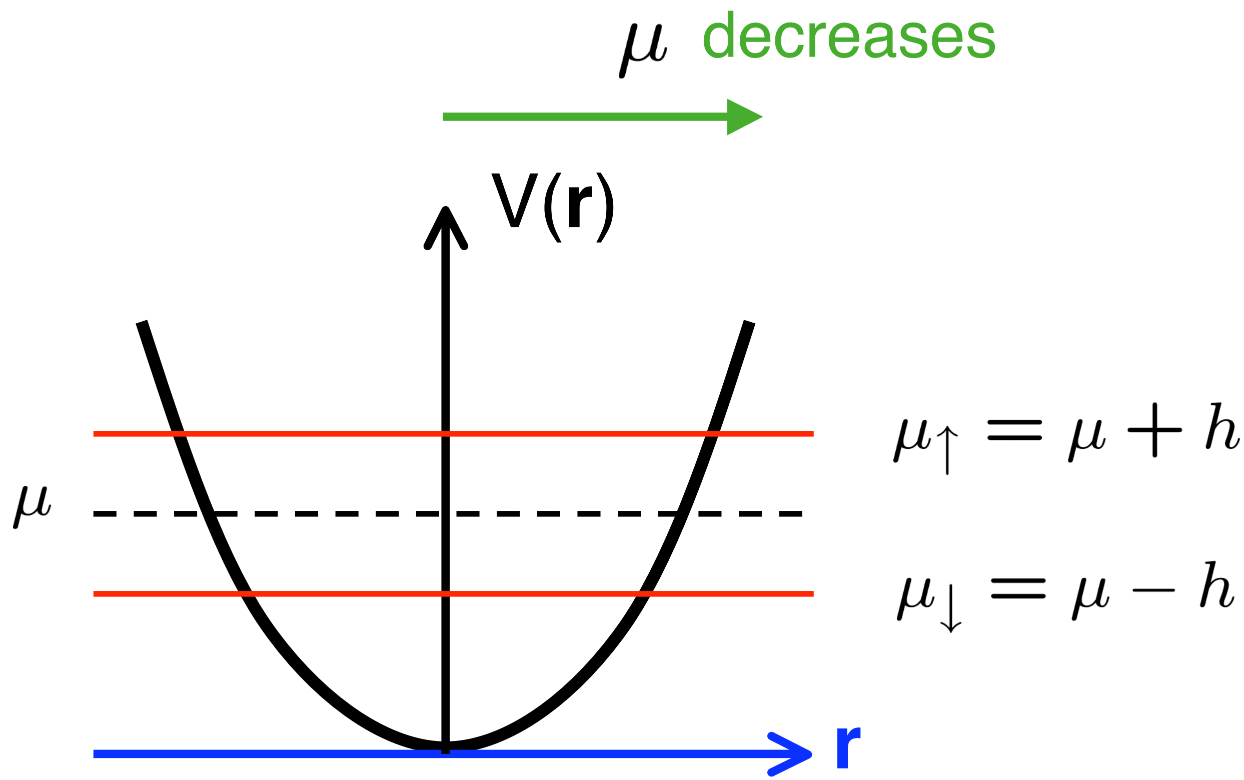


$$n = n_\uparrow + n_\downarrow$$

$$m = n_\uparrow - n_\downarrow$$

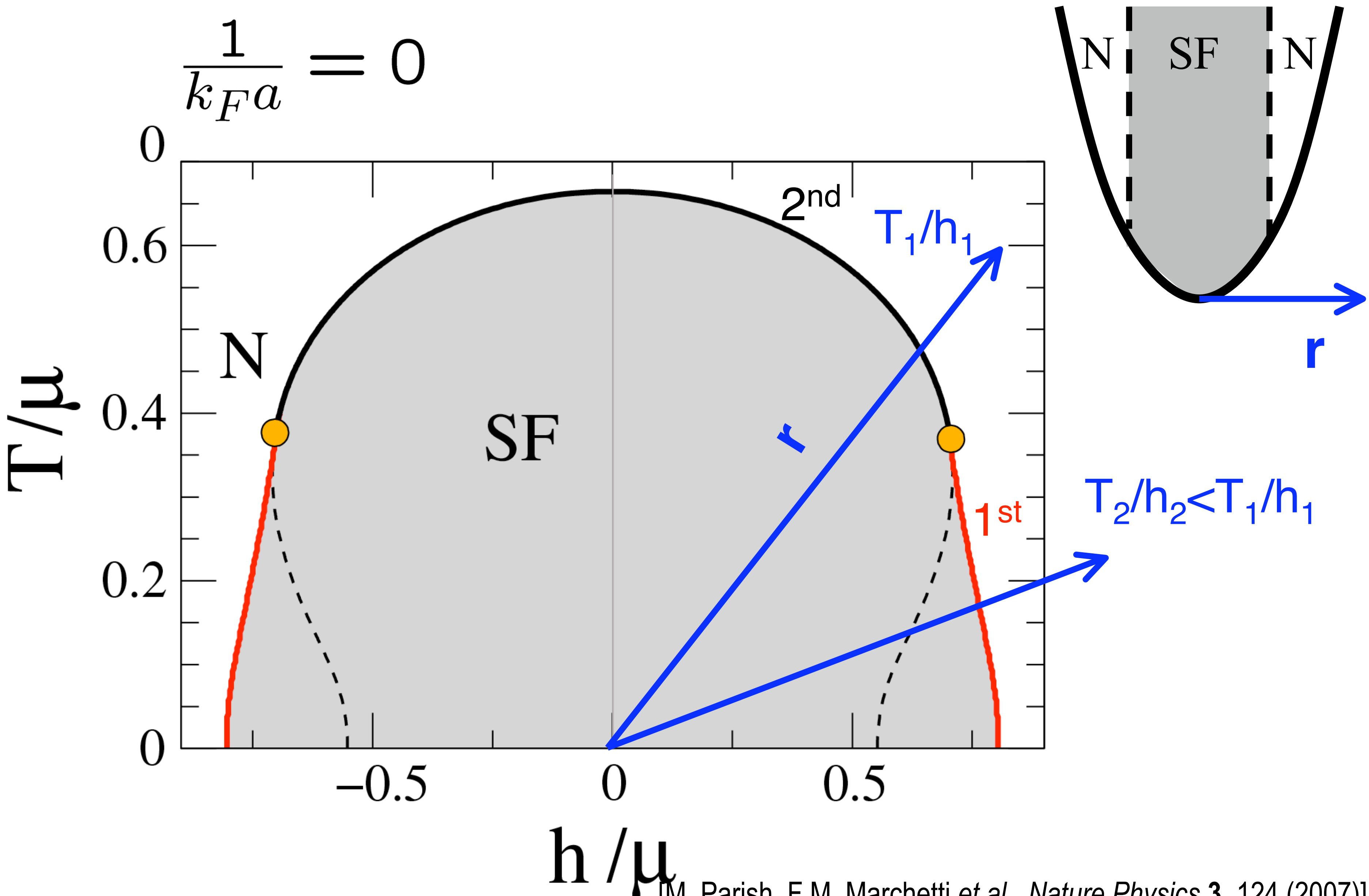
Trapped Fermi Gases

- LDA $\mu_{\uparrow,\downarrow}(\mathbf{r}) = \mu_{\uparrow,\downarrow} - V(\mathbf{r})$



Phase Diagram for Trapped Gases

$$\frac{1}{k_F a} = 0$$

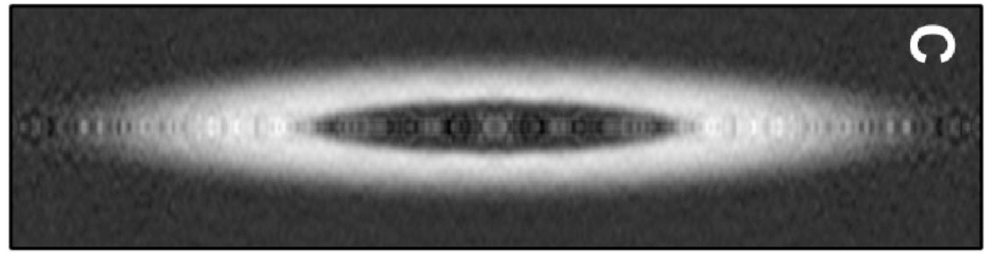


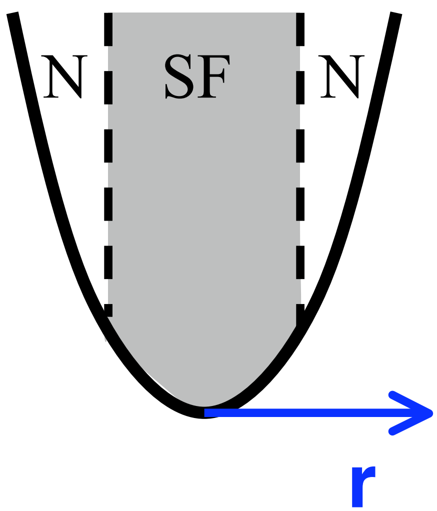
[M. Parish, F.M. Marchetti *et al.*, *Nature Physics* **3**, 124 (2007)]

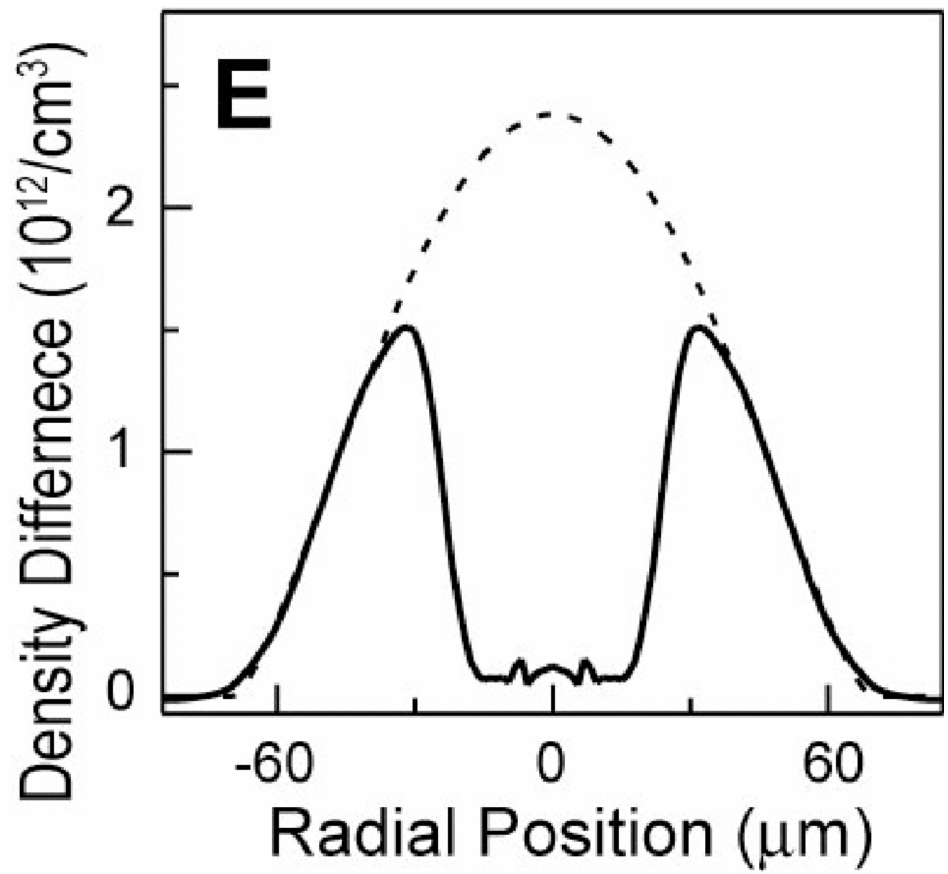
Experiments on Imbalanced Fermi Clouds

- In-situ imaging of phase separation (3D density distribution $n_{\uparrow,\downarrow}(\mathbf{r})$)

$$n_\uparrow(\mathbf{r}) - n_\downarrow(\mathbf{r})$$





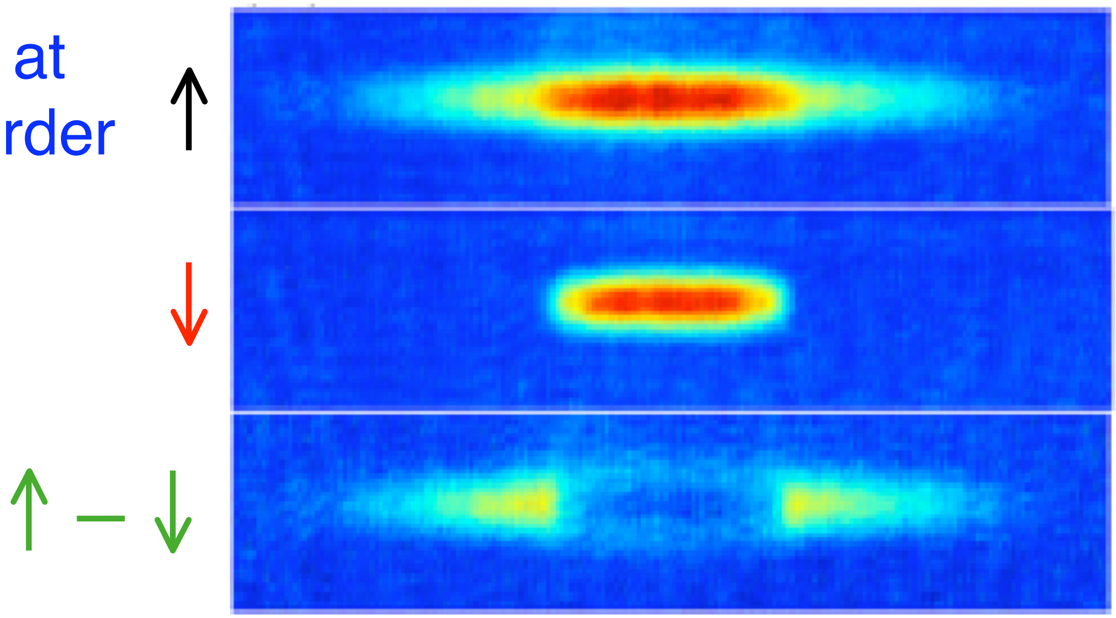


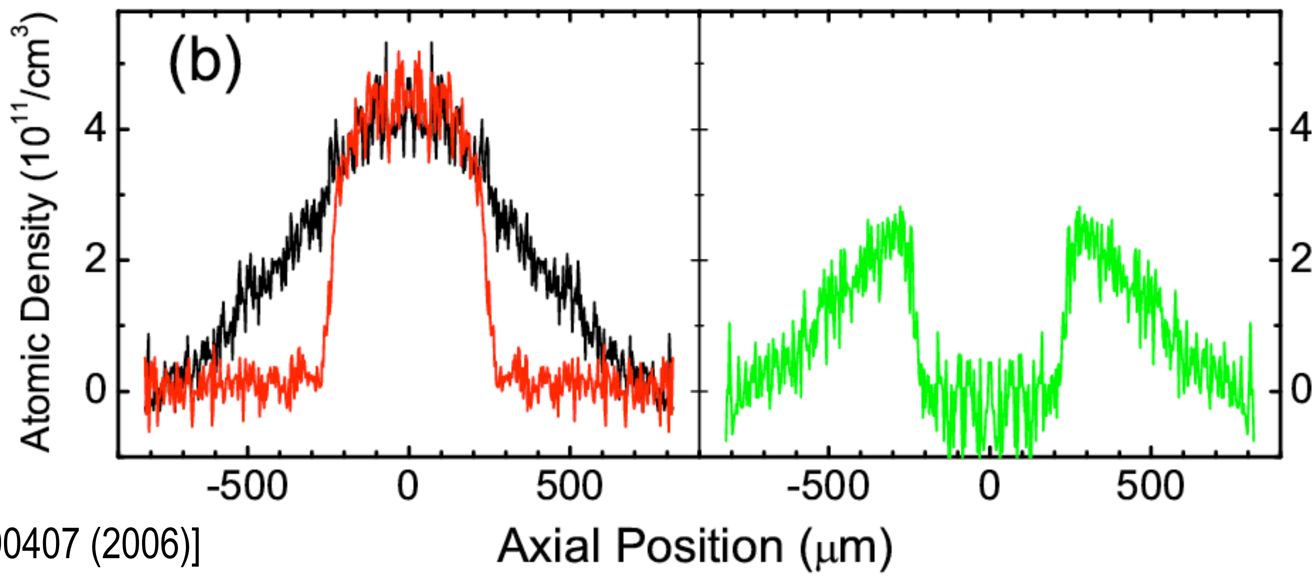
[Y. Shin *et al.*, PRL **97**, 030401 (2006)]

Experiments on Imbalanced Fermi Clouds

- Sharp phase boundary at low temperatures (1st order transition)

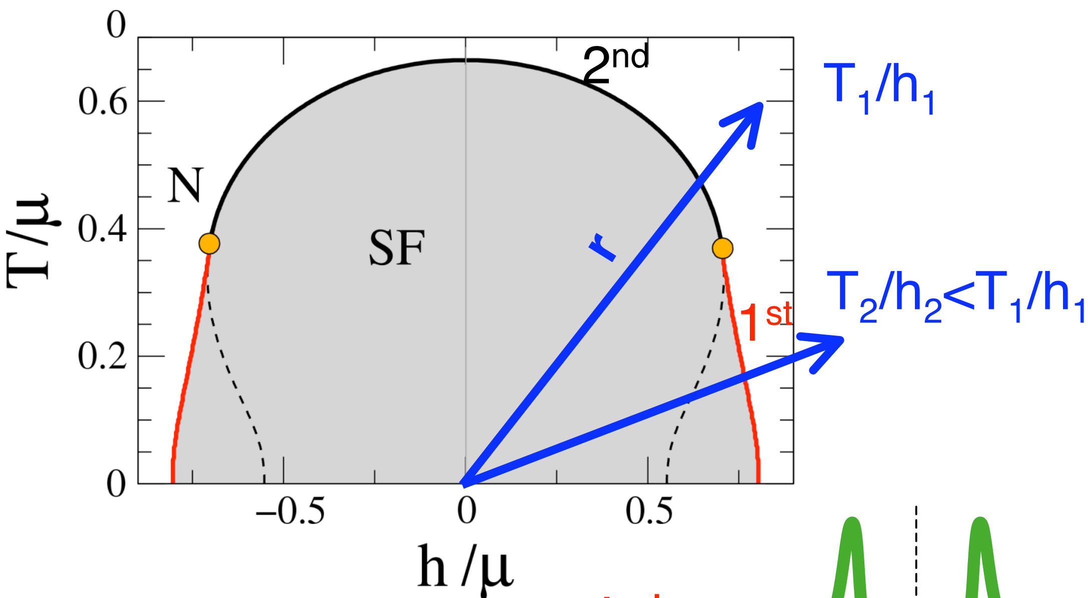
$T < 0.05\, T_F$
$m/n = 0.35$

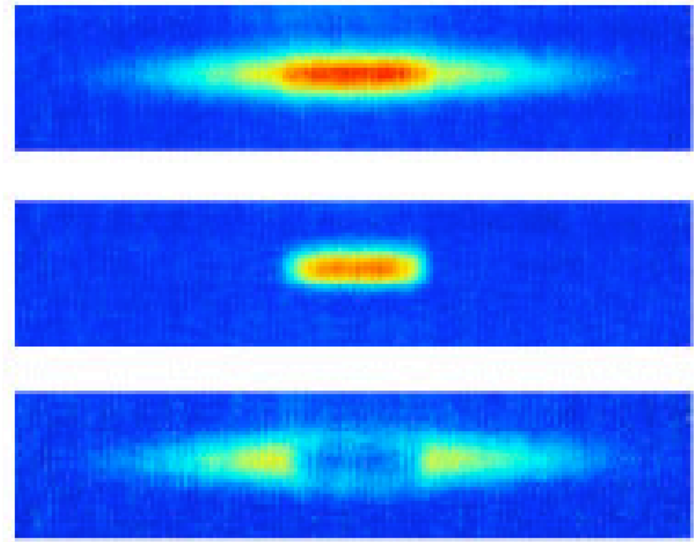


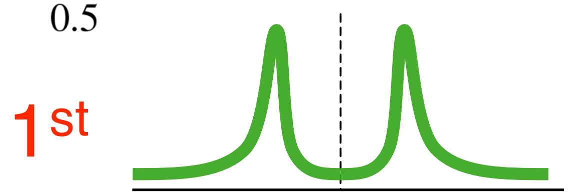


[G. B. Partridge *et al.*, PRL **97**, 190407 (2006)]

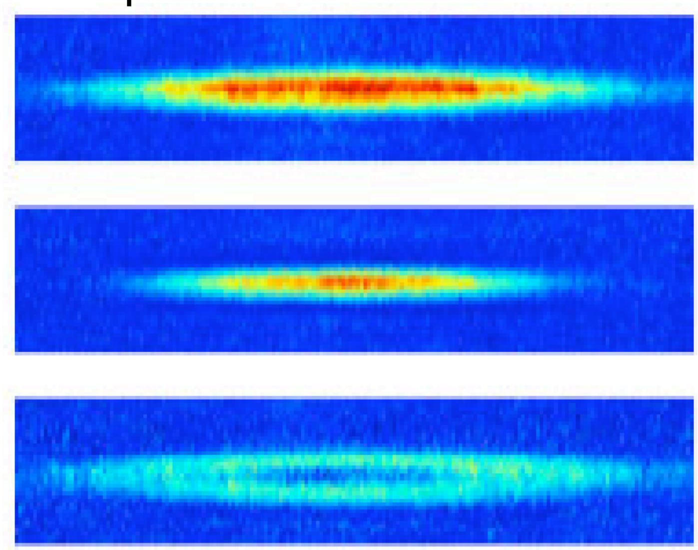
Temperature Dependence of Phase Separation

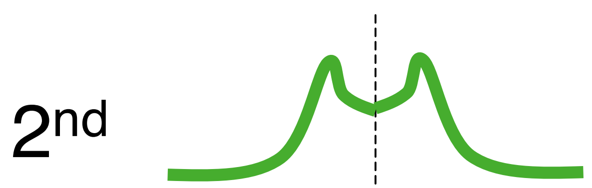




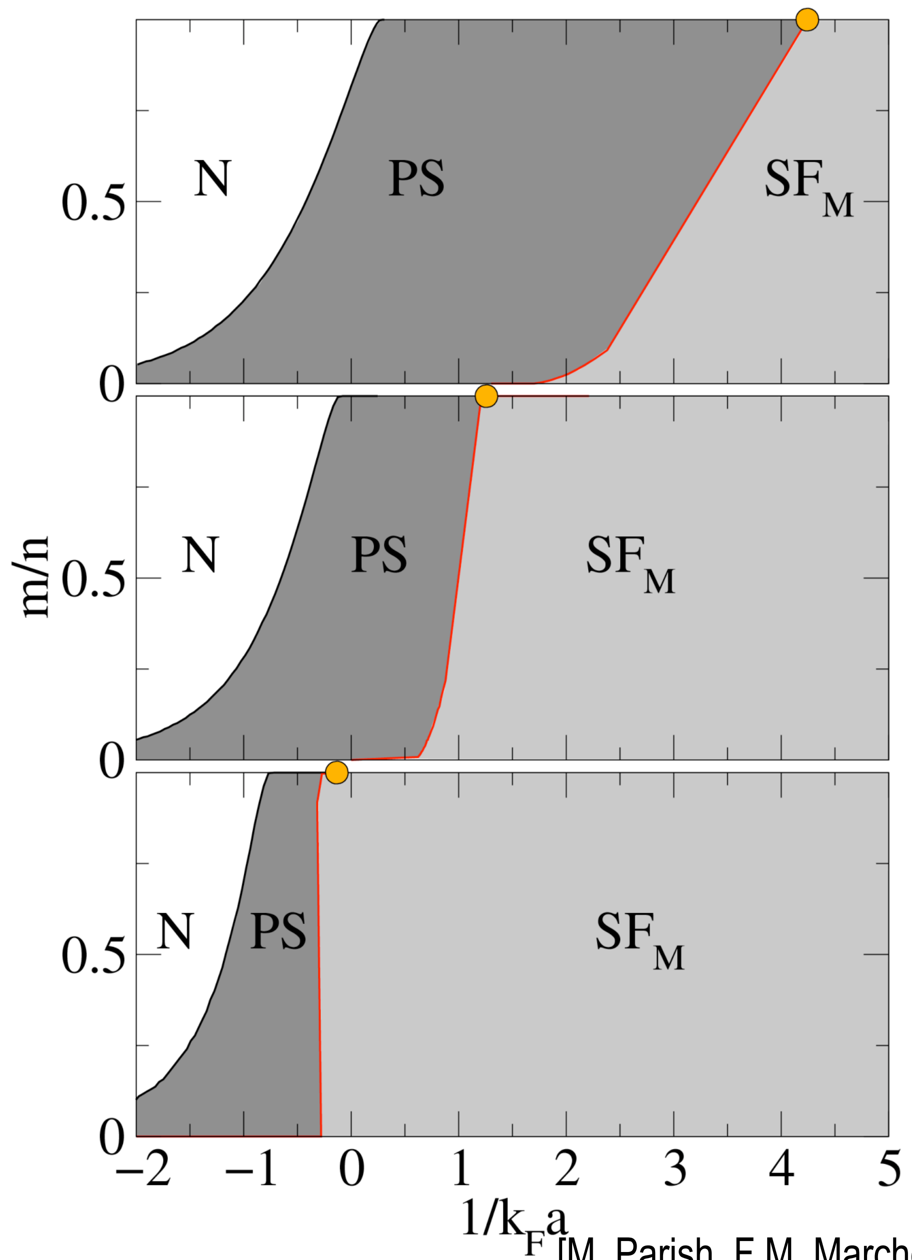


[G. B. Partridge *et al.*, PRL **97**, 190407 (2006)]





Unequal Masses



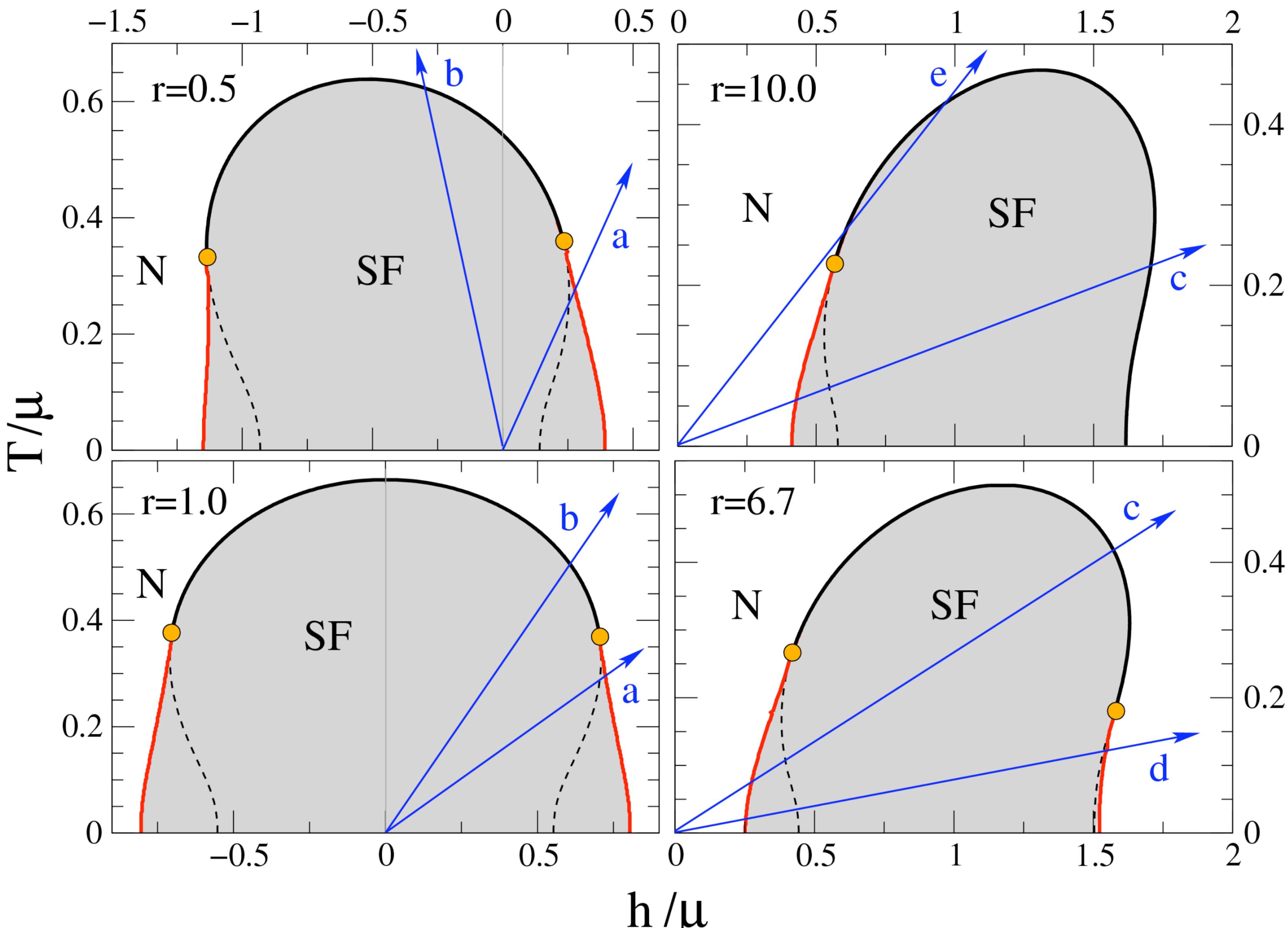
$$\frac{m_\downarrow}{m_\uparrow} = 0.5$$

$$\frac{m_\downarrow}{m_\uparrow} = 2$$

$$\frac{m_\downarrow}{m_\uparrow} = 10$$

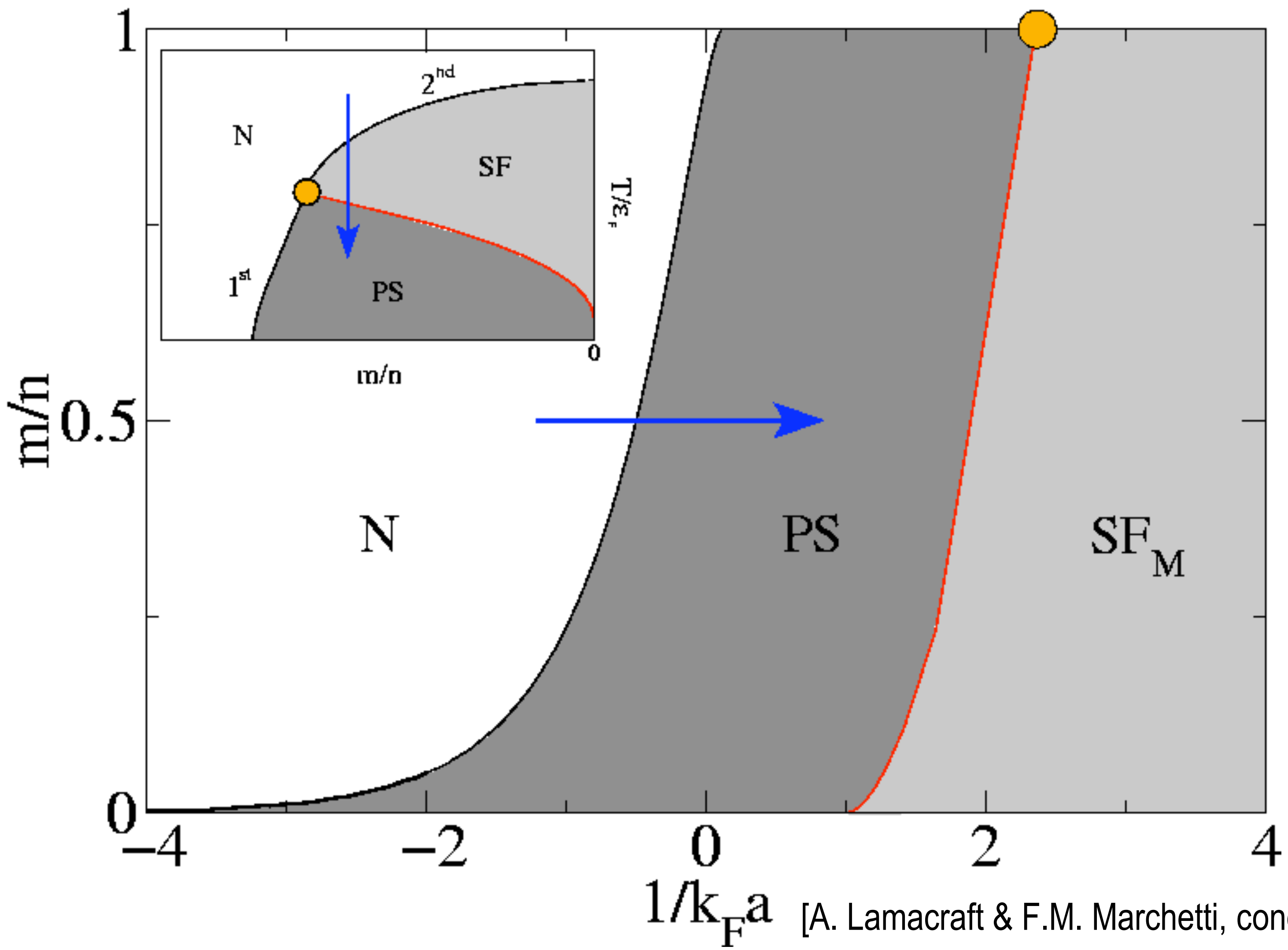
[M. Parish, F.M. Marchetti *et al.*, *PRL* **98**, 160402 (2007)]

Unequal Masses



[M. Parish, F.M. Marchetti *et al.*, *PRL* **98**, 160402 (2007)]

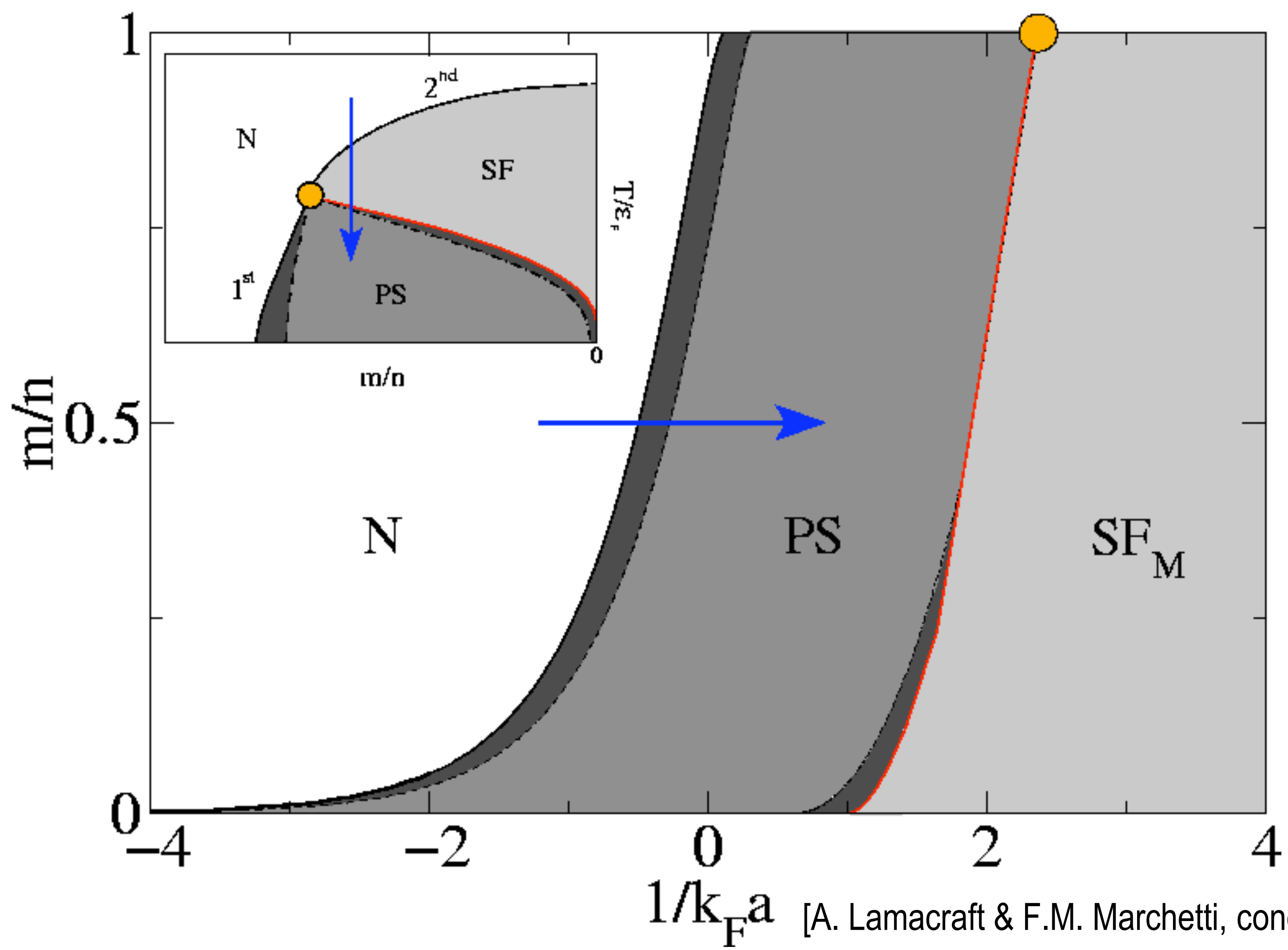
Dynamics of Phase Separation



[A. Lamacraft & F.M. Marchetti, cond-mat/0701692]

Dynamics of Phase Separation

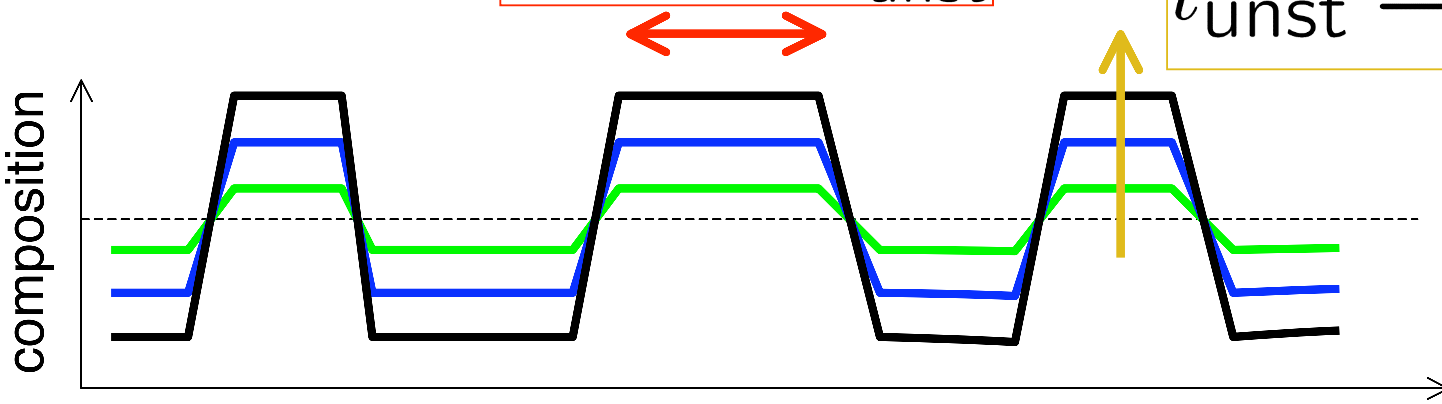
- Spinodal: phase separation starts via a linear instability



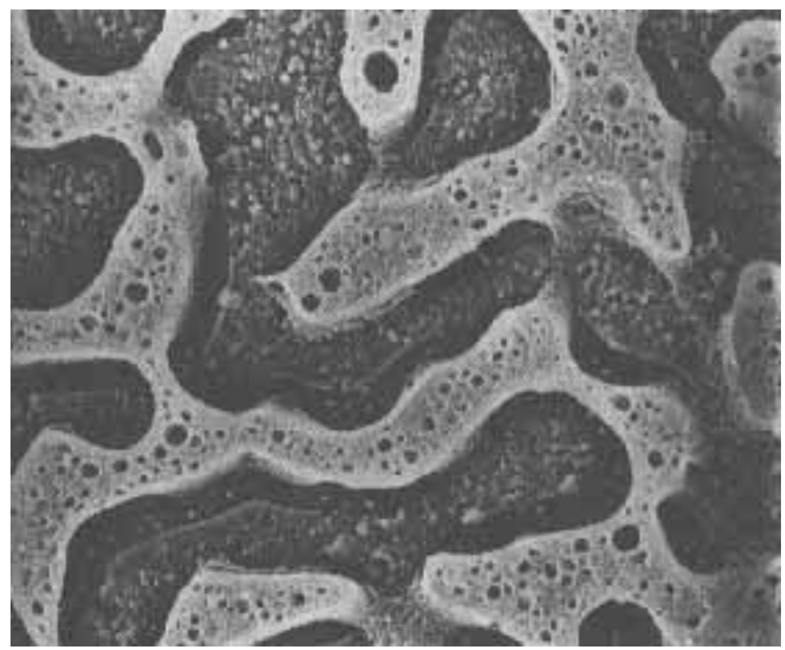
[A. Lamacraft & F.M. Marchetti, cond-mat/0701692]

Spinodal Decomposition

- Early time dynamics $\ell_{\mathrm{unst}} = \frac{1}{\mathbf{q}_{\mathrm{unst}}}$ $t_{\mathrm{unst}} = \frac{1}{\omega_{\mathrm{unst}}}$

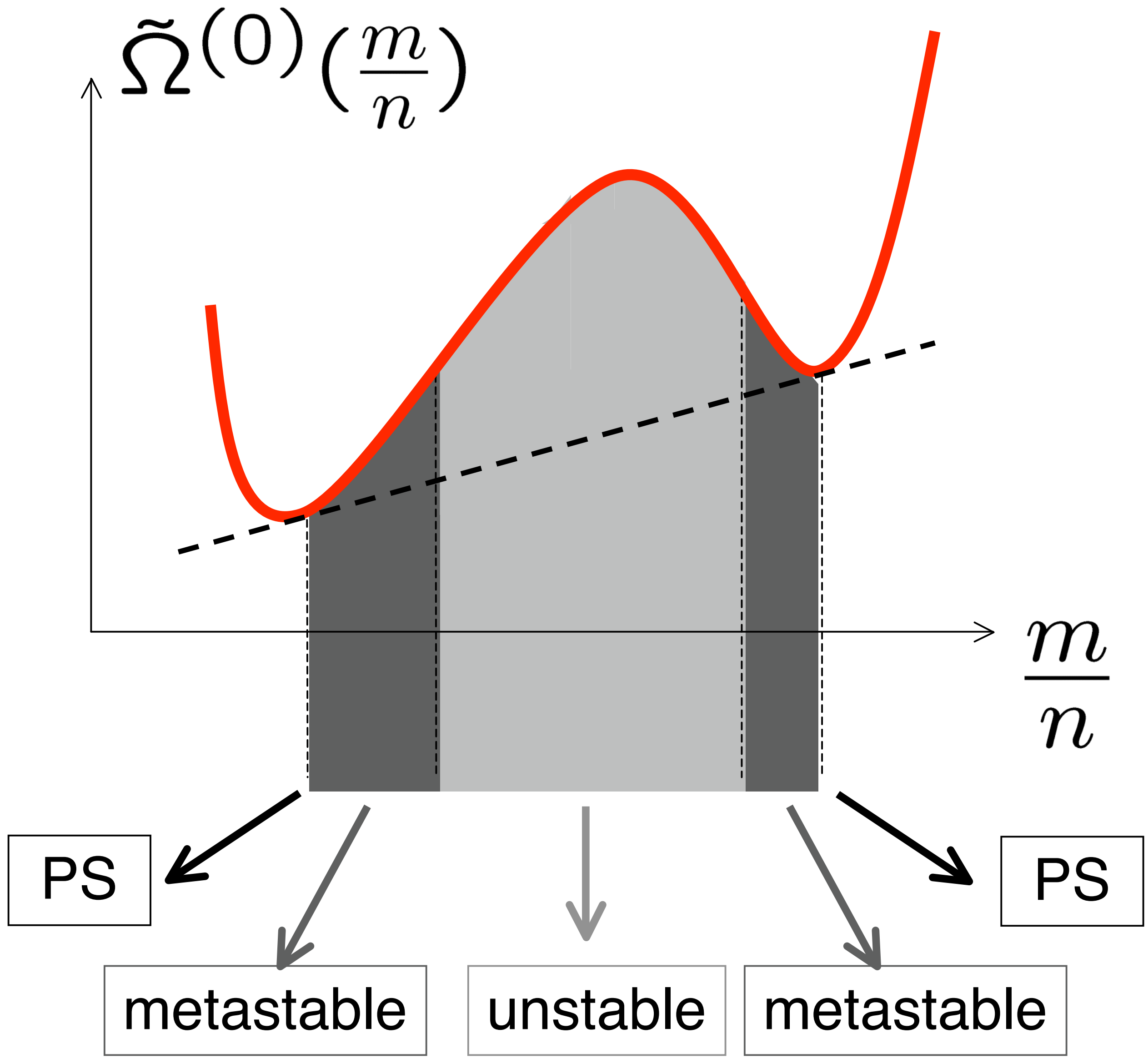


- E.g. Temperature quenches in polymers in solutions,…



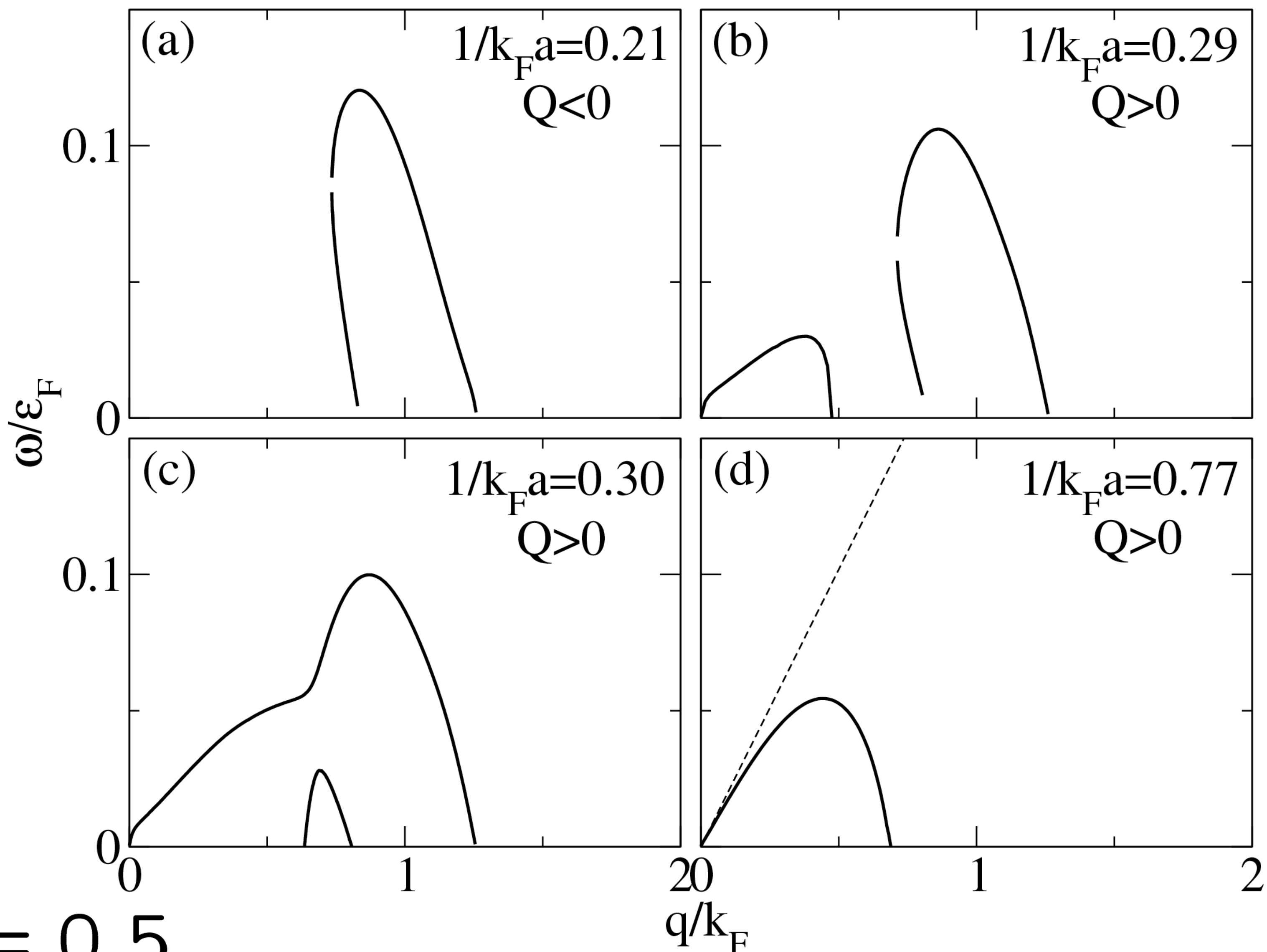
[Courtesy of Nigel Clarke, Polymer IRC]

Spinodal Region



Unstable Modes

- Matrix response function (to changes of the density)



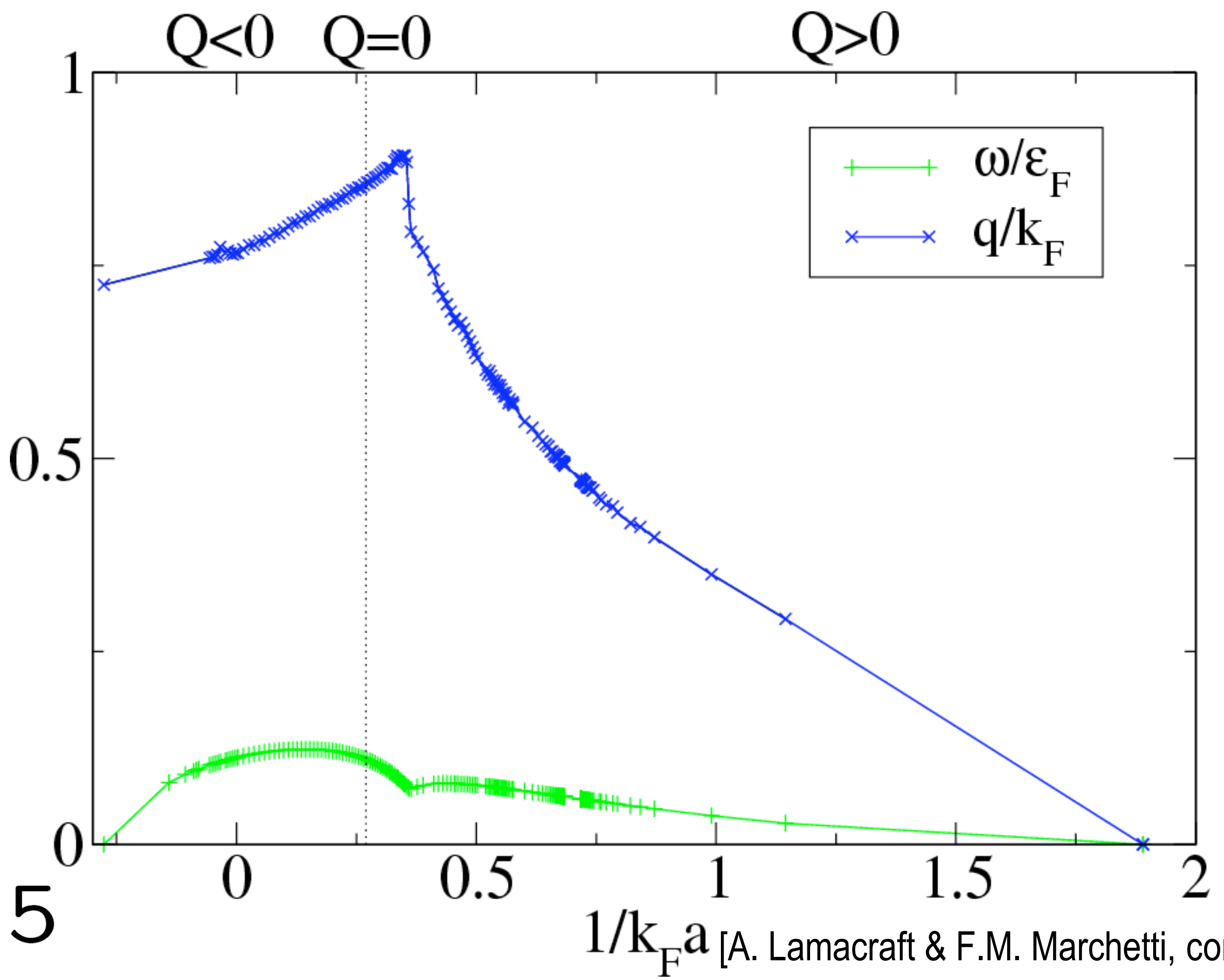
$$\frac{m}{n} = 0.5$$

[A. Lamacraft & F.M. Marchetti, cond-mat/0701692]

Most Unstable Modes

- Characteristic length and time scales (for $T_F = 1\mu\text{K}$)

$\ell_{\text{unst}} \simeq 0.1\mu\text{m}$ $\quad$ $t_{\text{unst}} \simeq 400\mu\text{s}$

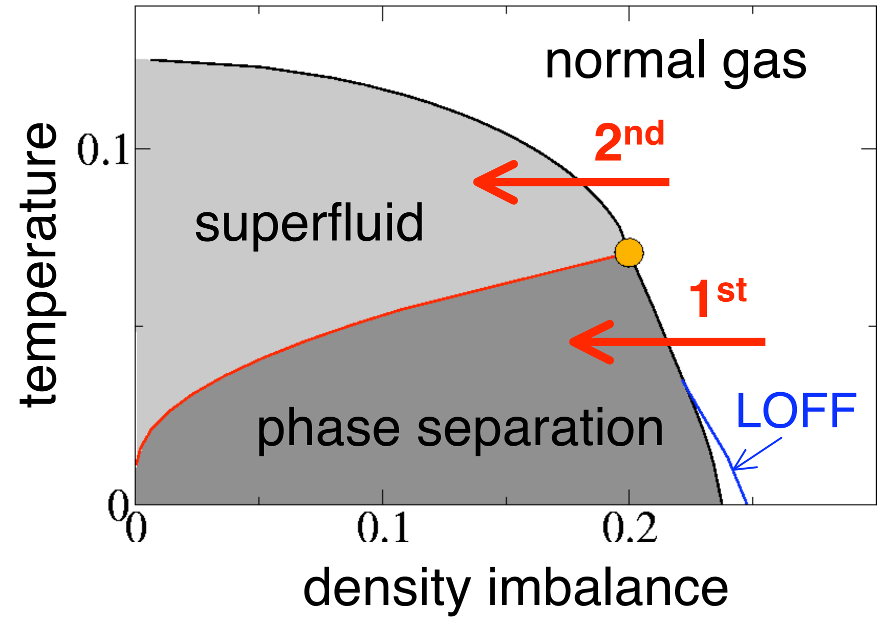


$\frac{m}{n} = 0.5$

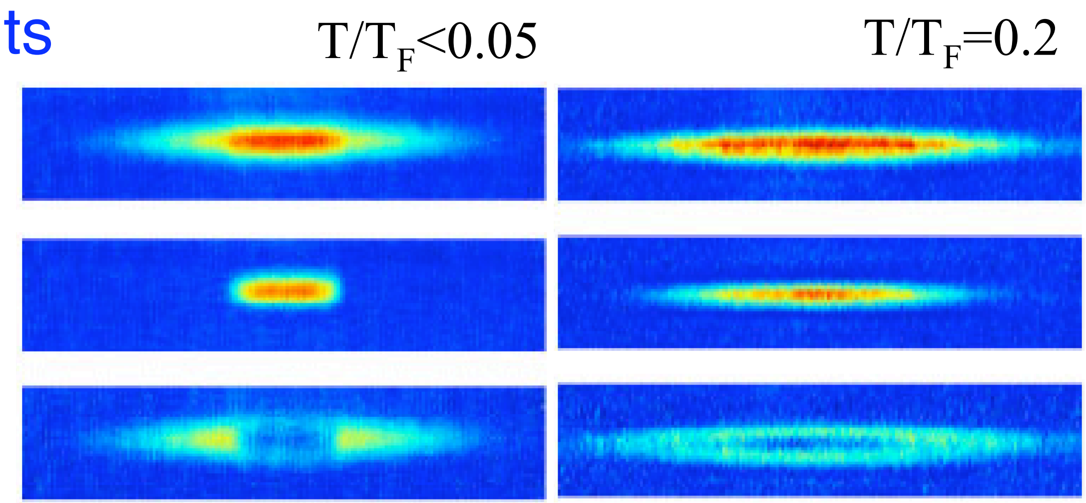
[A. Lamacraft & F.M. Marchetti, cond-mat/0701692]

Conclusions

1. Phase diagram of polarised Fermi superfluids



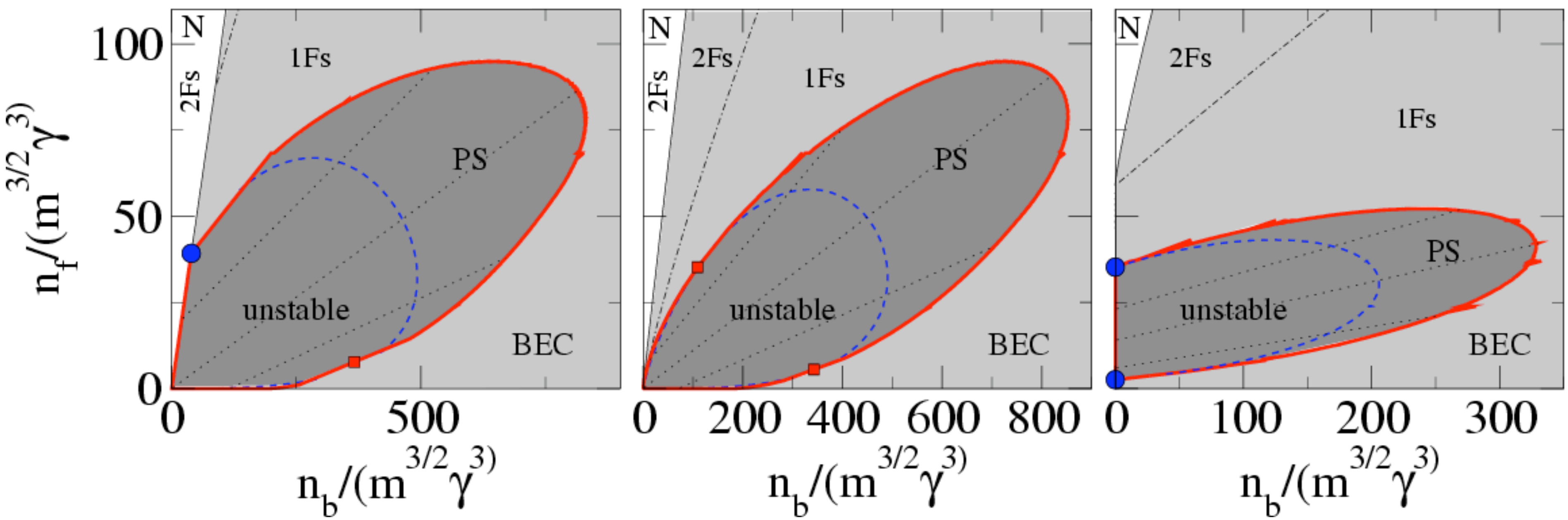
2. Probing the order of the transition in experiments



3. Dynamics of phase separation

Some future work

- Bose-Fermi mixtures (with a Feshbach resonance)



(with C. Mathy and M. Parish)