Control 2 — Computación I (curso 2015-2016)

February 7, 2016

Please send your your scripts (name the main script file as `your_name_control2-CI.m` and don't forget to also send the external functions files.m if you use them) at both the following e-mail addresses `francesca.marchetti@uam.es` and `fabrice.laussy@gmail.com`. Before leaving the classroom, wait for a confirmation from Fabrice Laussy that he has received your scripts.

A mass of $m = 50$ g can move in one direction only, specified by the coordinate x, and is coupled to an anharmonic spring which follows the force law $F(x) = -k_1 x - k_3 x^3$, with values of the constants $k_1 = 3$ N/m and $k_3 = 0.9$ N/m^3.

1. Solve the equation of motion (Newton's law) in an interval of time $t \in [0, 4]$ s for the initial condition $x_0 = x(t = 0) = 50$ cm and $v_0 = v(t = 0) = -8$ ms^{-1} by writing an external function routine which uses a second-order Runge-Kutta method. Use the number of points N necessary to reach convergence (i.e., for larger values of N the result does not change). Plot in figure(1) the position $x(t)$, the velocity $v(t)$ and the acceleration $a(t)$ in three separate subplots as a function of time $t \in [0, 4]$ s (**3 points**).

2. Evaluate and plot in figure(2) the kinetic energy $T(v) = \frac{1}{2}mv^2$, the energy potential $U(x)$, and the total energy $E = T(v) + U(x)$ of the mass as a function of time. Use three different colors for the three curves. Check that the value of the total energy is constant in time and coincides with $E = \frac{1}{2}mv_0^2 + U(x_0)$ (**1.5 points**).

3. Plot in figure(3) the energy potential $U(x)$ and the total energy E as a function of the position x and write an external function routine which evaluates the points where the motion inverts, i.e., $E - U(x_{1,2}) = 0$, via a root-finding Newton-Raphson method. Mark those points in figure(3) and compare the values you obtain for x_1 and x_2 with the ones obtained previously by solving the equation of motion (**2 points**).

4. Plot in figure(4) the numerical solution of the equation of motion in phase space, i.e., plot the velocity v versus the position x, and compare it with the exact solution $v_{\text{ex}}(x) = \pm\sqrt{\frac{2}{m}[E - U(x)]}$ (**1.5 points**).

5. Find the period of oscillations by evaluating numerically the following integral
$$T = \sqrt{2m} \int_{x_1}^{x_2} \frac{dx}{\sqrt{E - U(x)}} \; .$$
Compare the result you find this way with the one you find by solving the equation of motion numerically (**2 points**).

For each plot and subplot, do not forget to label both axis, assigning the correct units.