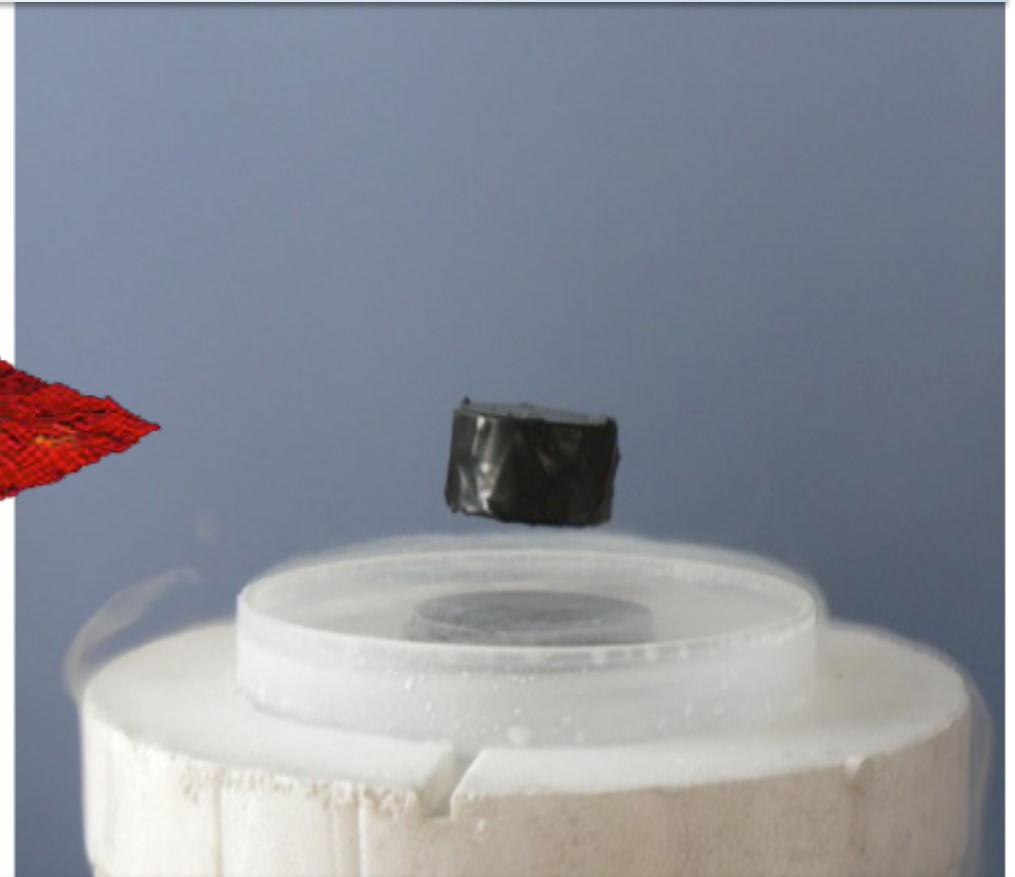
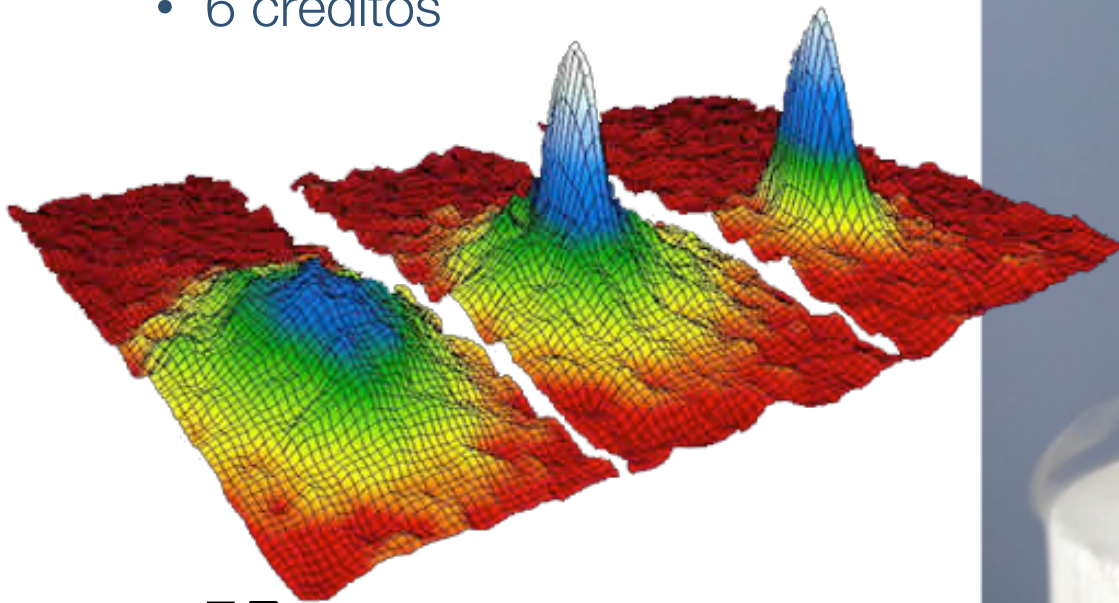


Introducción a la Superconductividad y Superfluidez

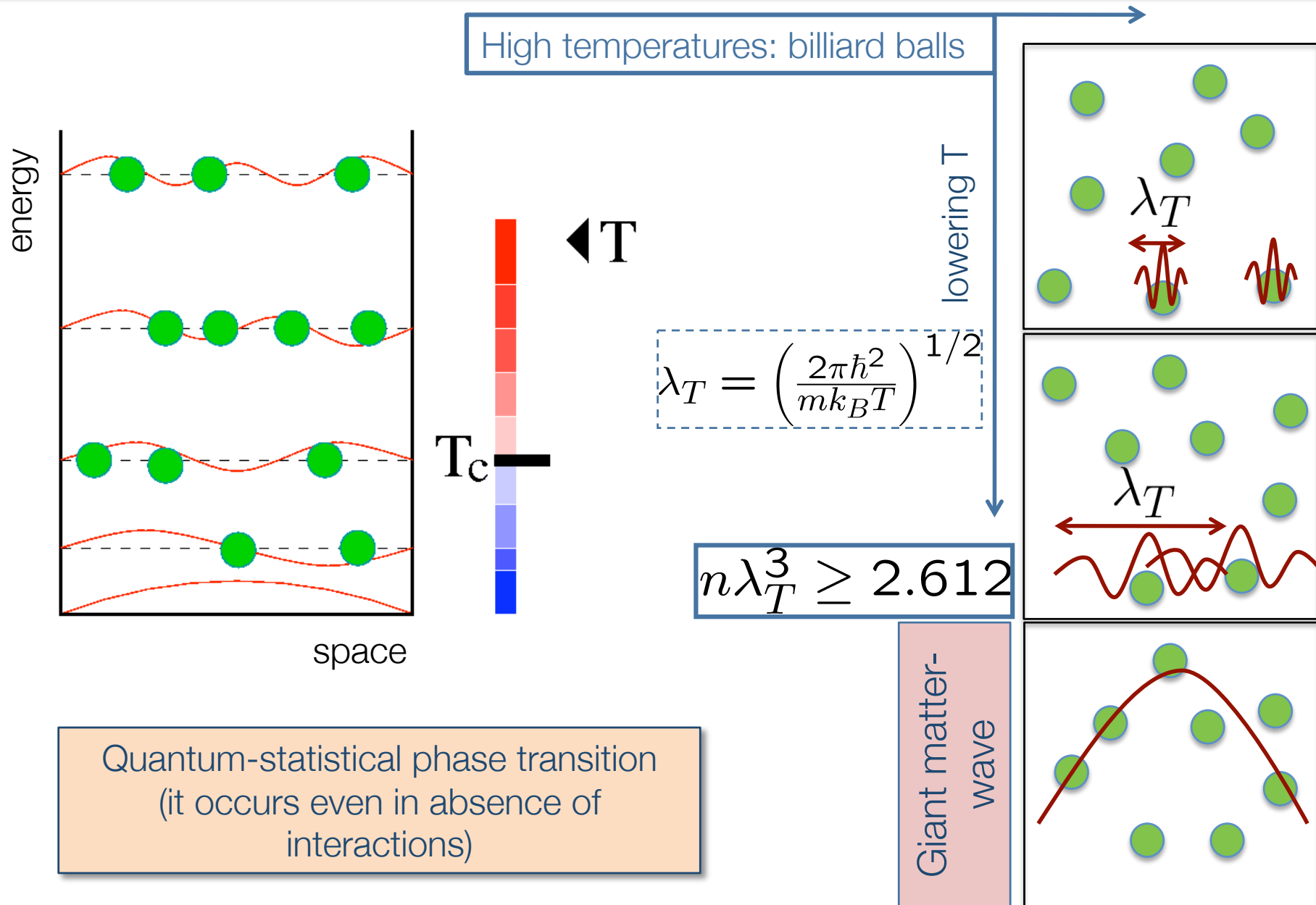
Curso de Mecánica Estadística Avanzada

- asignatura optativa (segundo semestre)
- 6 créditos



Introduction to BEC & superfluidity

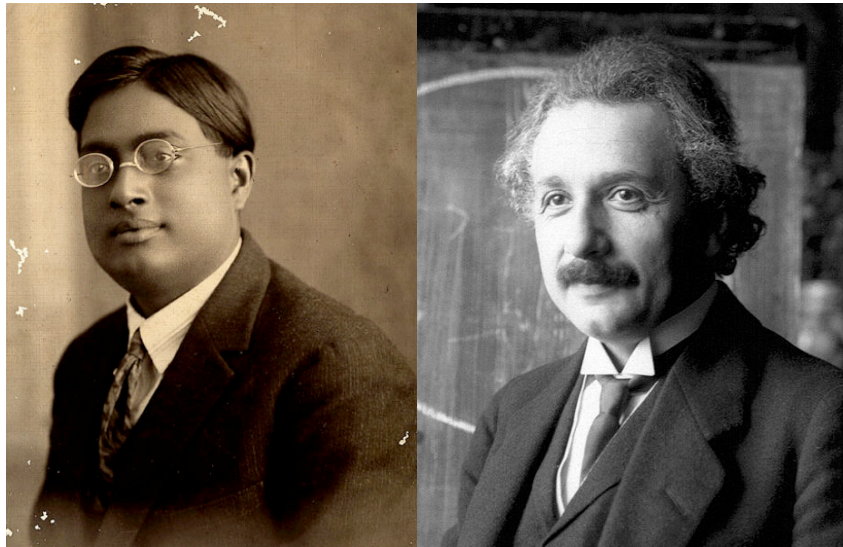
Macroscopic occupation of the ground state



BEC: from 1925 to 1995

- 1924/1925

Following the work of Bose on the statistical description of light quanta, Einstein predicted that a gas of non-interacting massive bosons, below a critical temperature, undergoes a phase transition associated with the condensation of the atoms in the lowest energy state: Bose-Einstein condensation (BEC)



Satyendranath Bose Albert Einstein

“...A separation is affected; some part condenses, the rest remains a ‘saturated ideal gas’...”

“...condensation without attractive forces...”

BEC

NON INTERACTING IDEAL BOSE GAS

⇒ paradigm of quantum statistical mechanics

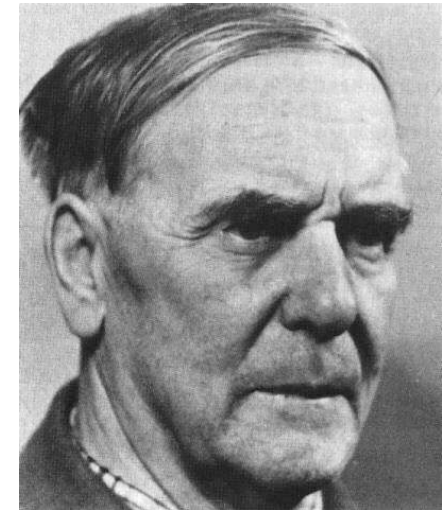
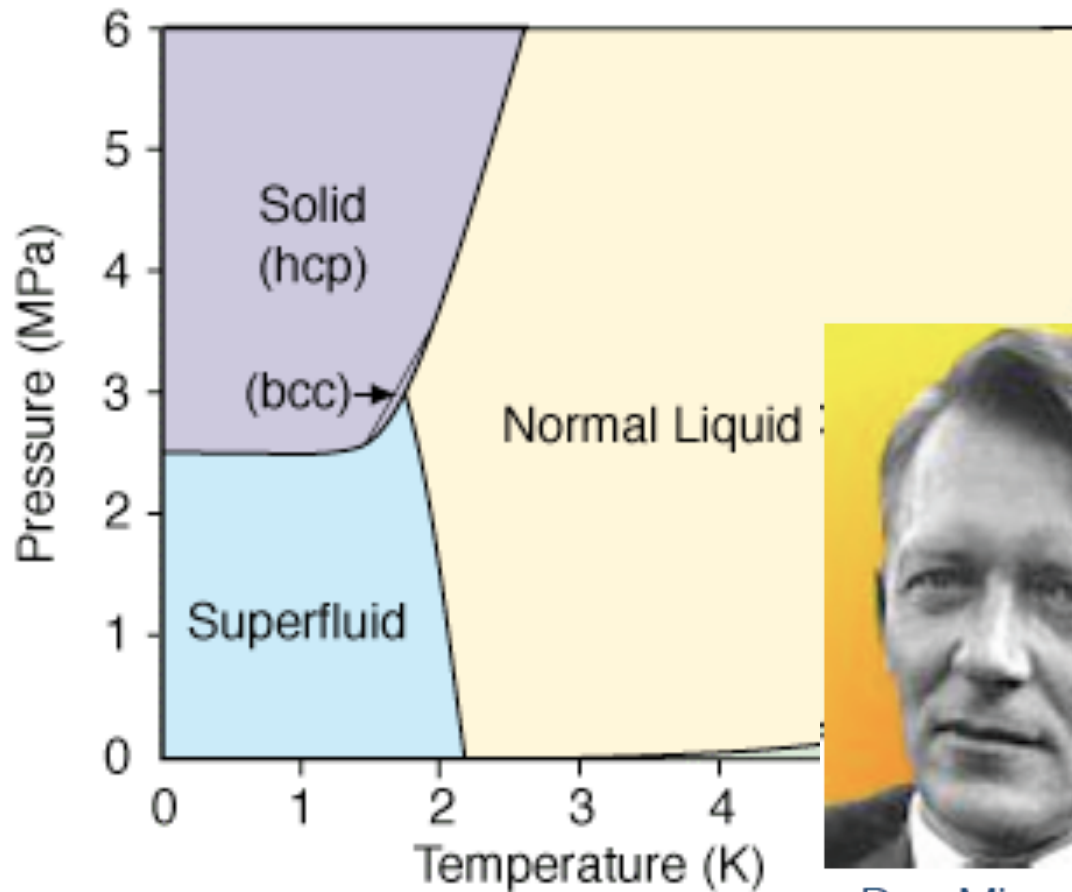
- indistinguishable particles
- wave nature of particles
- thermal equilibrium

⇒ macroscopic quantum phenomena

- macroscopic wavefunction
(many-body ground state wavefunction is the product of N identical single-particle ground-state wavefunctions)

BEC: from 1925 to 1995

- 1938 Discovery of superfluidity in liquid helium ^4He (Allen & Misener; Kapitza).



Peter L. Kapitza



Don Misener



John F. Allen

BEC: from 1925 to 1995

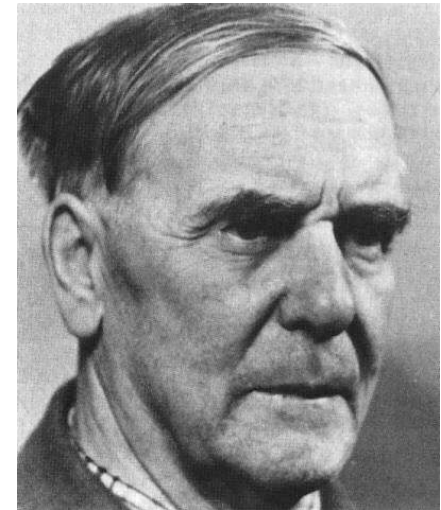
- 1938 Discovery of superfluidity in liquid helium ^4He (Allen & Misener; Kapitza).

What's the relation between BEC & superfluidity?

- Immediately after, London suggested the connection between the superfluidity of ^4He and BEC

⇒ first to bring out the idea of BEC displaying quantum behaviour on a macroscopic scale

- It was a source of debate for decades
- e.g., the Landau's criterion for superfluidity does not explicitly mention the notion of BEC
- it is now recognised that superfluidity in ^4He is related to BEC (though, because of strong interactions in ^4He , there is a strong reduction of the lowest energy state occupancy)



Peter L. Kapitza



Fritz London

BEC: from 1925 to 1995

- 1947 microscopic theory of interacting Bose gases (Bogoliubov)

...provides the microscopic picture behind Landau's theory

- 1951 off-diagonal long range order (Landau&Lifshitz; Penrose)
- and much more theoretical work... (which we are going to see in class)



Nikolai N. Bogoliubov



Lev D. Landau

BEC

vs.

Superfluidity

NON INTERACTING
IDEAL BOSE GAS

⇒ paradigm of quantum statistical mechanics

- indistinguishable particles
- wave nature of particles
- thermal equilibrium

⇒ macroscopic quantum phenomena

- macroscopic wavefunction (many-body ground state wavefunction is the product of N identical single-particle ground-state wavefunctions)

IDEAL BECs ARE NOT SUPERFLUID

INTERACTING
BOSE GAS

⇒ mainly related to transport phenomena (flow without friction)

- is essential the form of the dispersion of the elementary excitations (Landau criterion)
- interactions are essential (change the dispersion from quadratic to phonon-like)
- superfluidity is possible even with few% of atoms in the ground state (see ^4He)

LINK BETWEEN BEC &
SUPERFLUIDITY: ORDER
PARAMETER and ODLRO

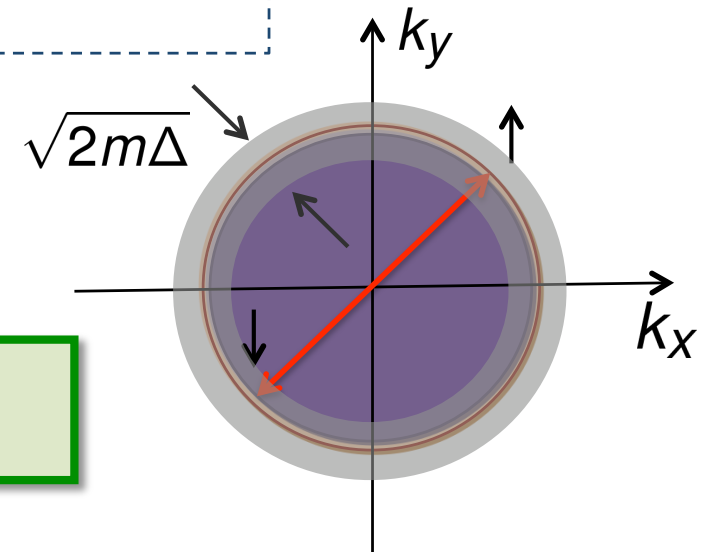
BEC in other systems

- BEC is involved in several macroscopic quantum phenomena (even if some systems are not ideal Bose gases):

macroscopic
occupation of a single
quantum state

- ⇒ ^4He (but is a strongly interacting system)
- ⇒ superconductors (BEC of Cooper pairs)
(we will see the BEC-BCS crossover)
- ⇒ ^3He (also fermions)
- ⇒ lasers (but out of equilibrium: requires inversion of the population)
- ⇒ ...

QUEST TO REALISE A BEC:
Search of weakly interacting Bose gases

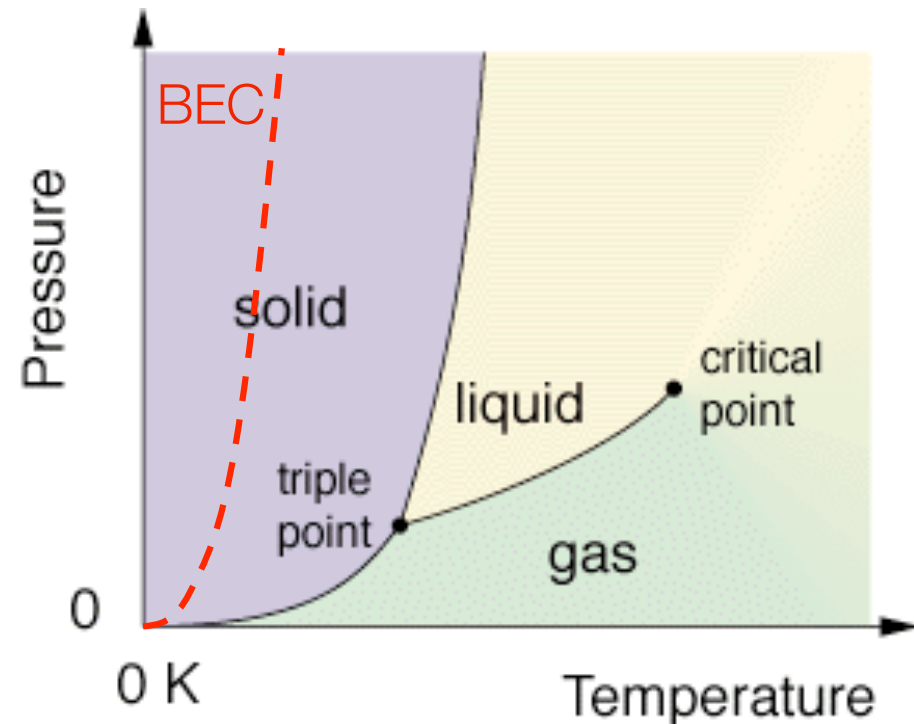


Searching for weakly interacting Bose gases

Why so hard?

⇒ At very low T most substances are in the solid (or liquid) phase & interaction becomes strong

- BEC in its ideal form can be realised only in conditions of metastability
- Thermal equilibrium but the gas has a finite lifetime

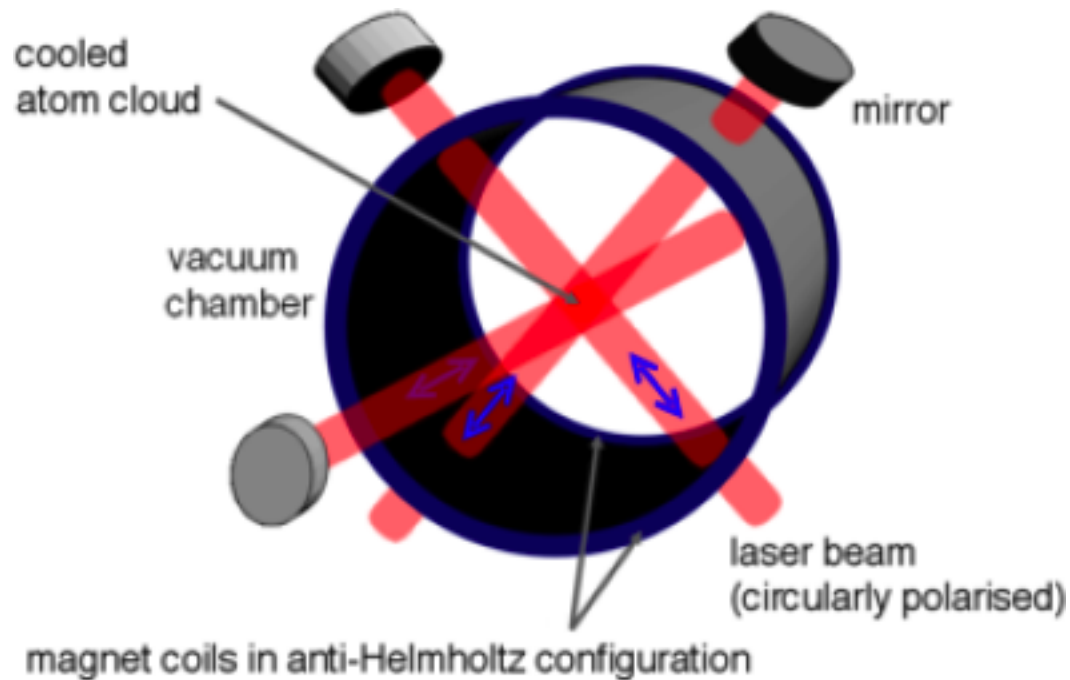
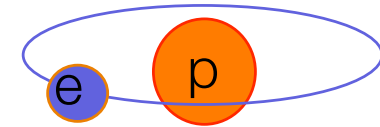


70 years to realise a BEC in dilute atomic gases

very sophisticated cooling and trapping techniques

Dilute ultracold atomic gases: Experiments

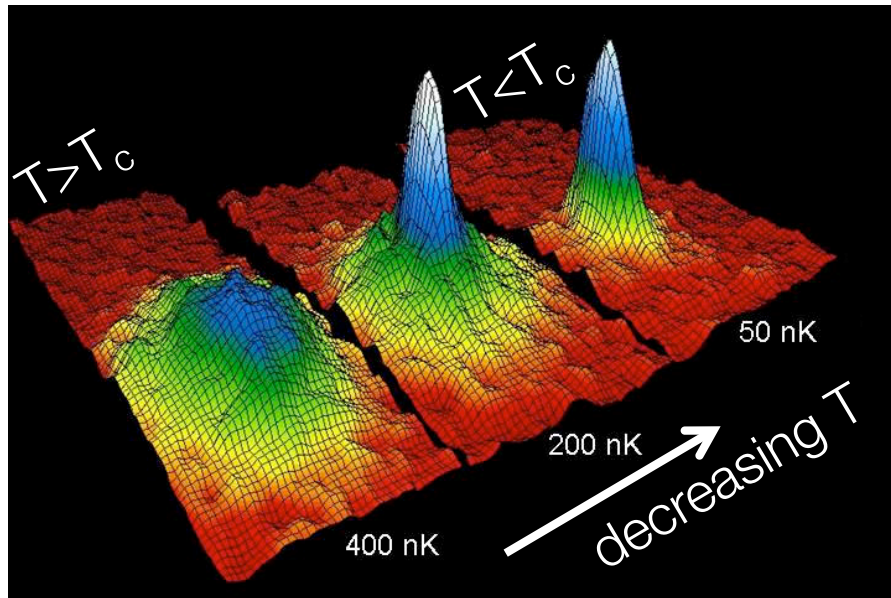
- 1959 spin-polarized (by a magnetic field) hydrogen proposed as a good candidate for a weakly interacting Bose gas
- '80 Developments in magnetic trapping, laser and evaporative cooling of alkali atoms



Group →	1	2	3
↓ Period			
1	1 H		
2	3 Li		
3	11 Na		
4	19 K		
5	37 Rb		
6	55 Cs		
7	87 Fr		

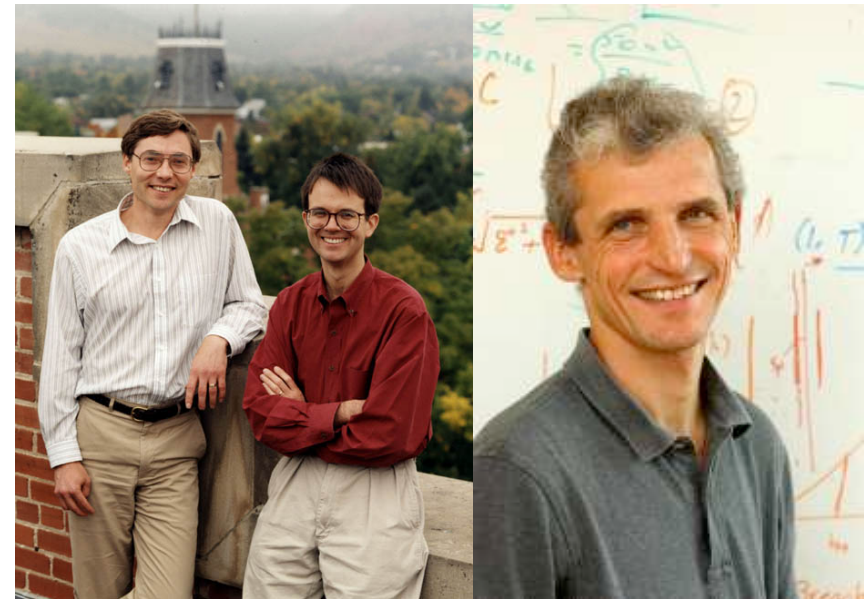
First realisation of a BEC in ultracold gases

- 1995 BEC in alkali atoms (^{87}Rb , ^{23}Na , ^7Li , ...)



Coollest system in the universe!

Nobel prize (2001)



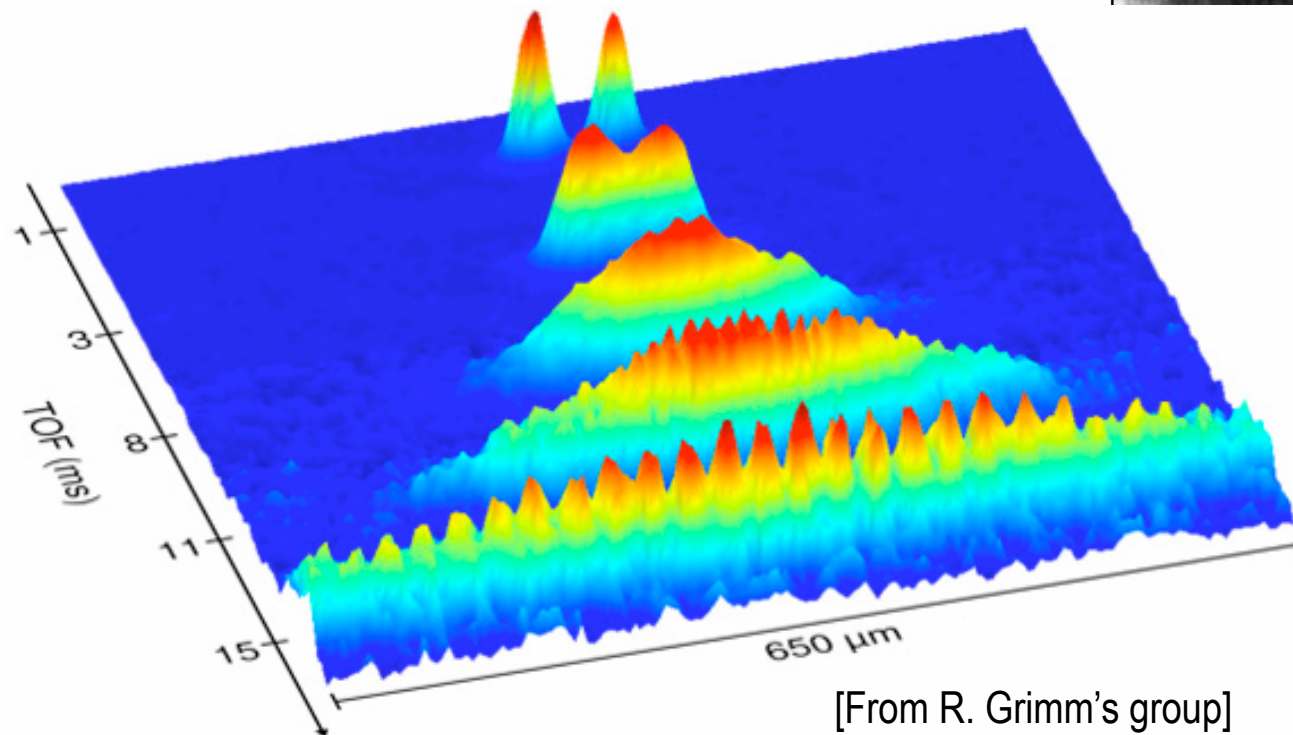
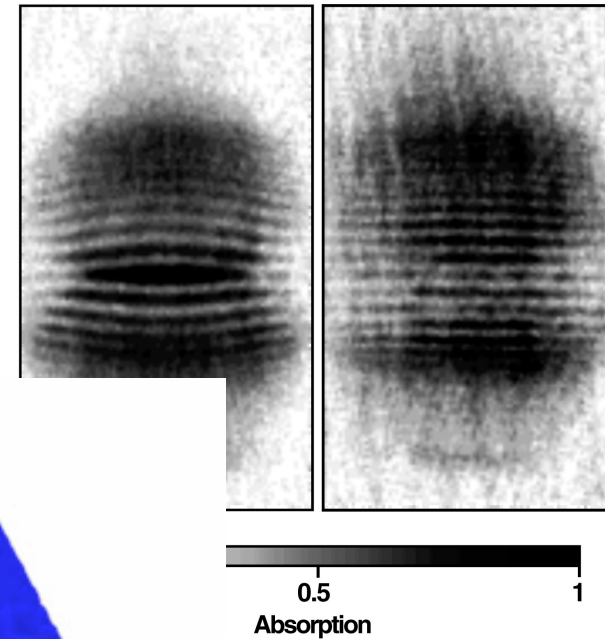
Carl Wieman & Eric Cornell Wolfgang Ketterle

$$T \sim 500\text{nK} - \mu\text{K}$$
$$n \sim 10^{11} - 10^{13}\text{cm}^{-3}$$

BEC & superfluidity

- Landau criterion
- Macroscopic phase coherence

[Andrews *et al.* Science (1997)]

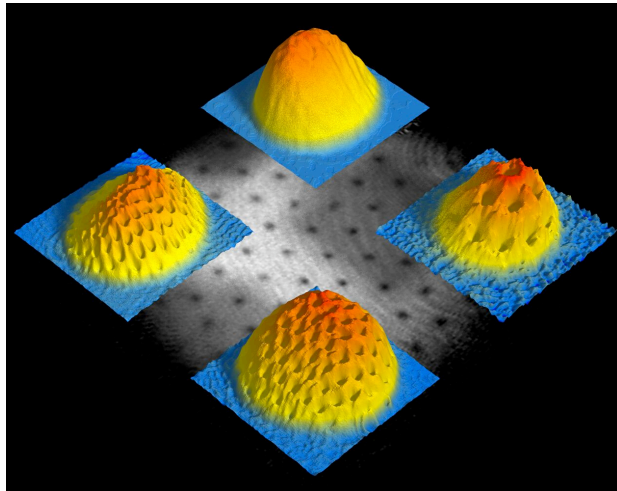


[From R. Grimm's group]

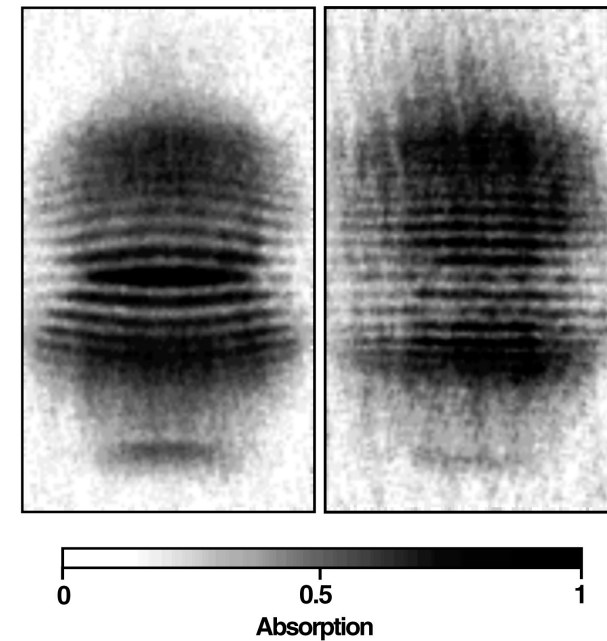
BEC & superfluidity

- Landau criterion
- Macroscopic phase coherence
- Quantised vortices
(rotating condensates)

[Abo-Shaeer *et al.* *Science* (2001)]



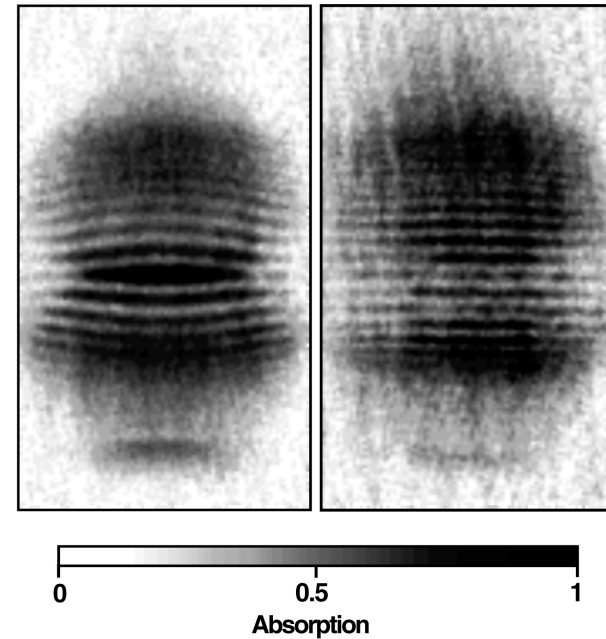
[Andrews *et al.* *Science* (1997)]



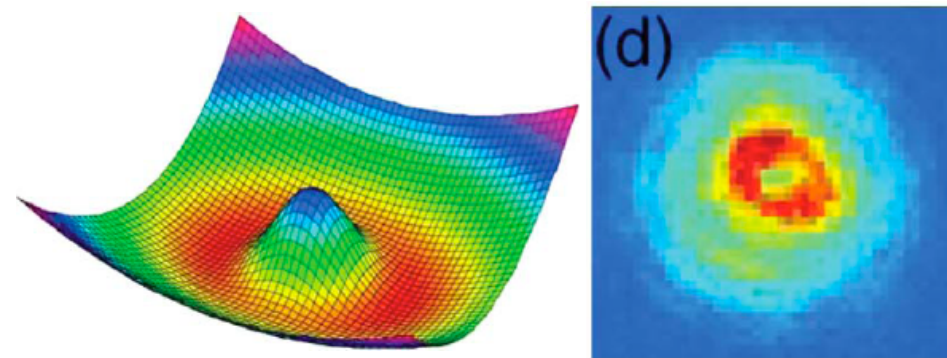
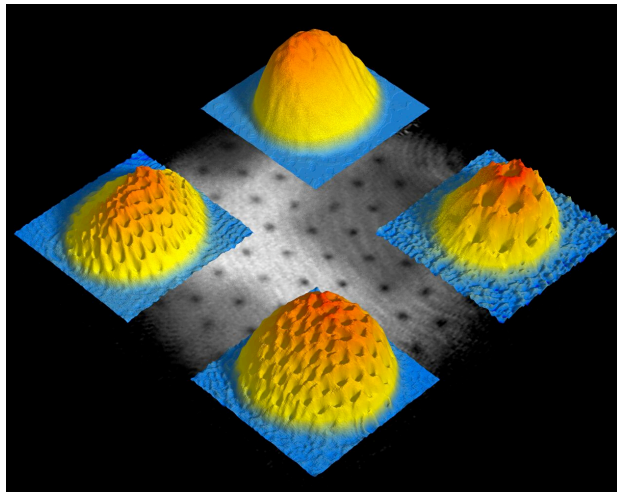
BEC & superfluidity

- Landau criterion
- Macroscopic phase coherence
- Quantised vortices
- Metastable persistent flow

[Andrews *et al.* Science (1997)]



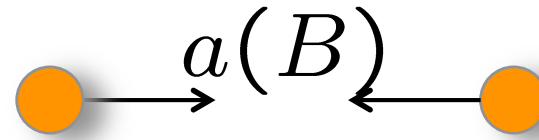
[Abo-Shaeer *et al.* Science (2001)]



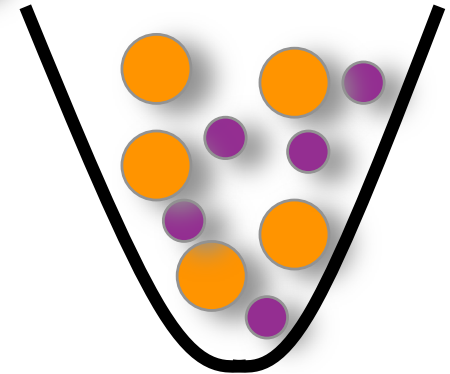
[Ryu *et al.* PRL (2007)]

Ultracold atoms today

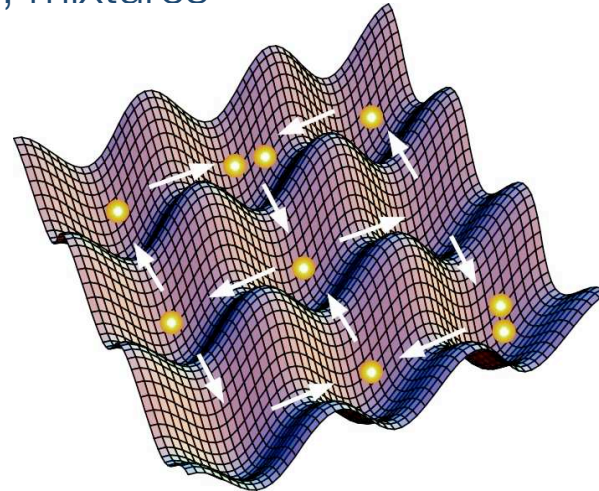
- Tune the interaction strength (Feshbach resonances)



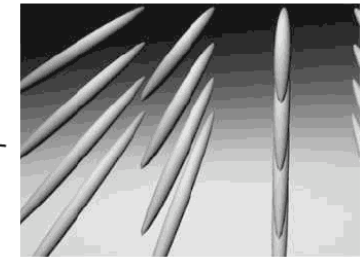
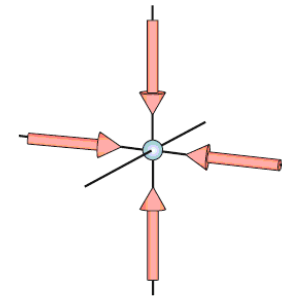
- Bosons, Fermions, mixtures



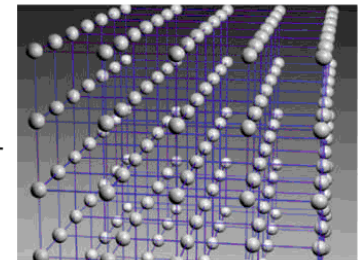
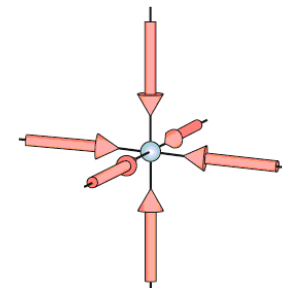
- Optical lattices



- Reduced dimensions (2D, 1D, 0D)



- Disorder



Simulate solid state systems
(with advantage of external control)

[Bloch *Nature Physics* (2005)]

Synopsis for the first half of the course

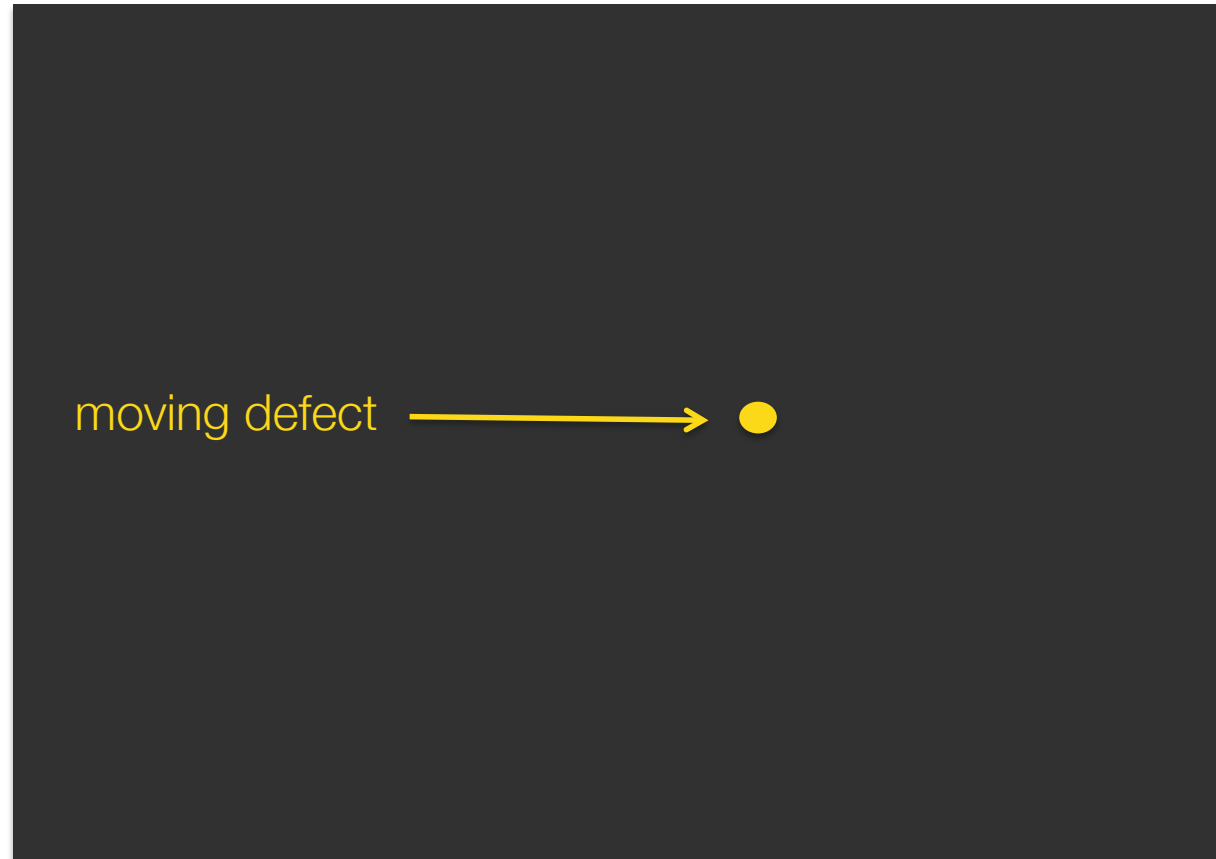
1. BEC: ideal Bose gas & weak interactions
2. BEC & superfluidity (Landau criterion and response to a moving defect)
3. Gross-Pitaevskii equation (non uniform condensates)
4. experiments in ultracold atoms (elements)
5. Applications: interference between two BECs
⇒ Josephson coupling & oscillations: analogy with superconductors
6. Applications: rotating BEC & vortices
⇒ vortex lattices
7. ...

List of possible topics for the presentation

1. How to measure the speed of sound in a superfluid (ultracold atomic BEC)
2. Defect moving through a superfluid and Cherenkov radiation
3. Interference between two expanding condensates and the Josephson effect
4. Rotating superfluids and vortices
5. ...
6. BEC-BCS crossover
7. Imbalanced Fermi mixtures and analogy with a BCS superconductor in a Zeeman magnetic field
8. Your proposals!

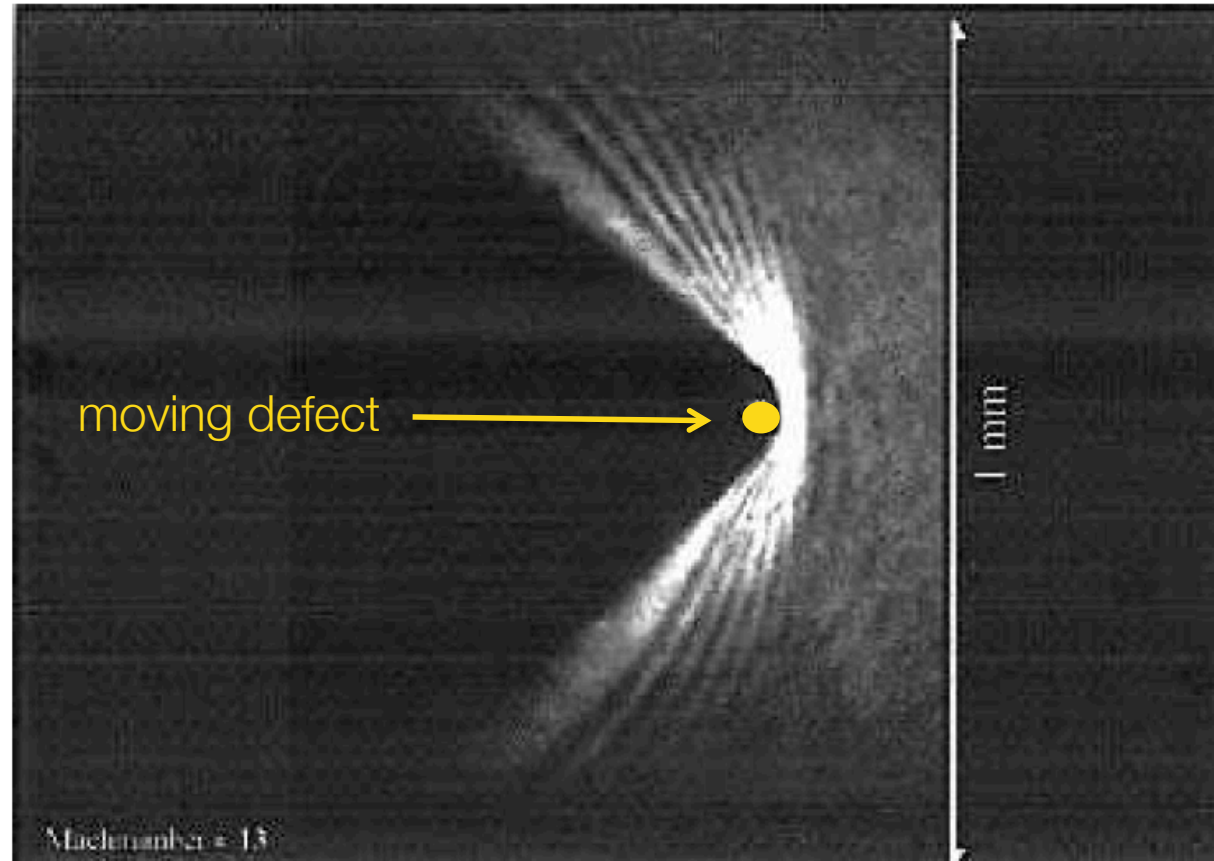
Moving defect in a BEC & Cherenkov radiation

$$v < v_c$$



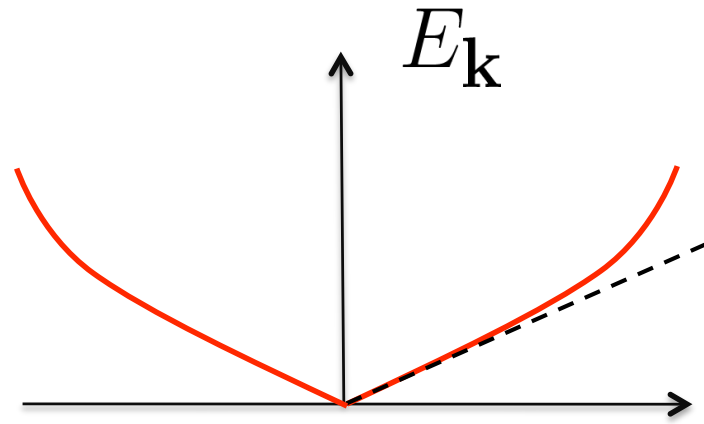
Moving defect in a BEC & Cherenkov radiation

$$v > v_c$$



[from E. Cornell's group]

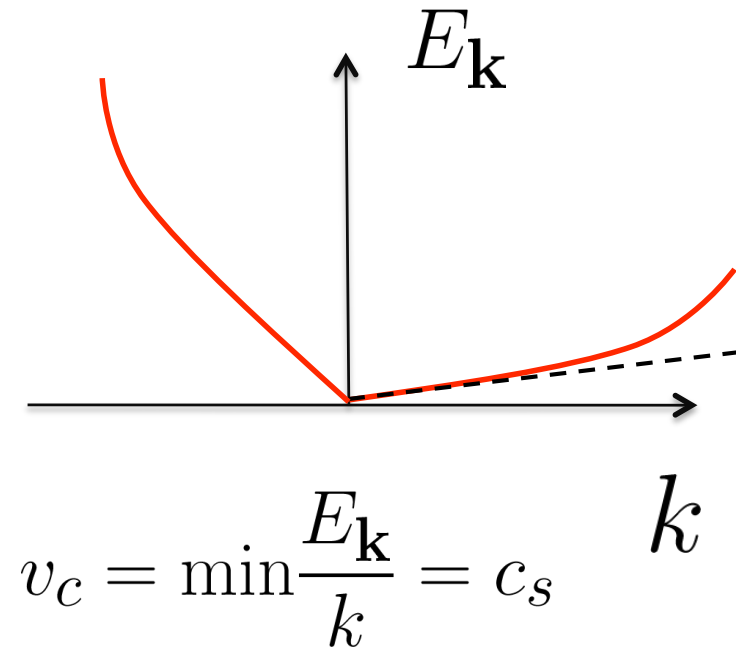
Moving defect in a BEC & Cherenkov radiation



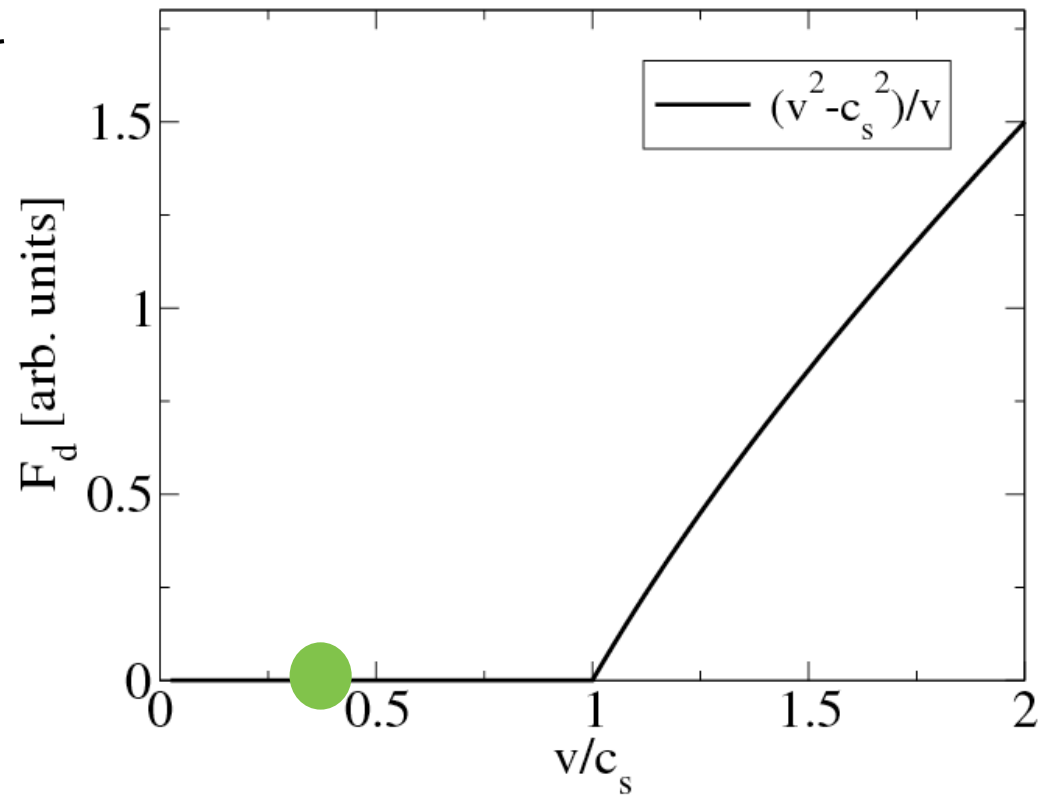
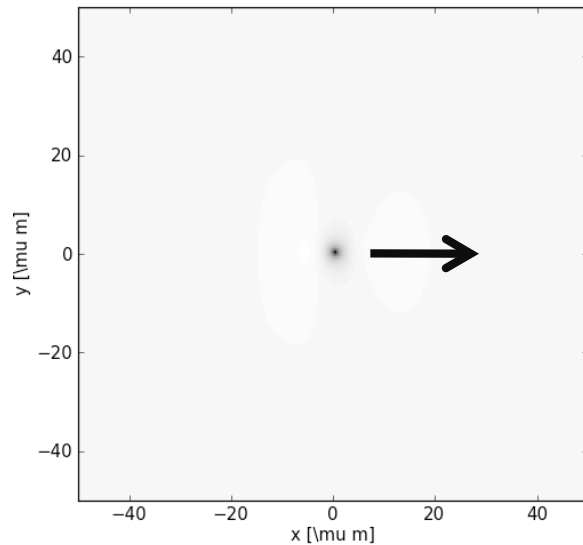
sound
velocity $c_s = \sqrt{\frac{gn}{m}}$

$$v_c = \min \frac{E_{\mathbf{k}}}{k} = c_s$$

Moving defect in a BEC & Cherenkov radiation



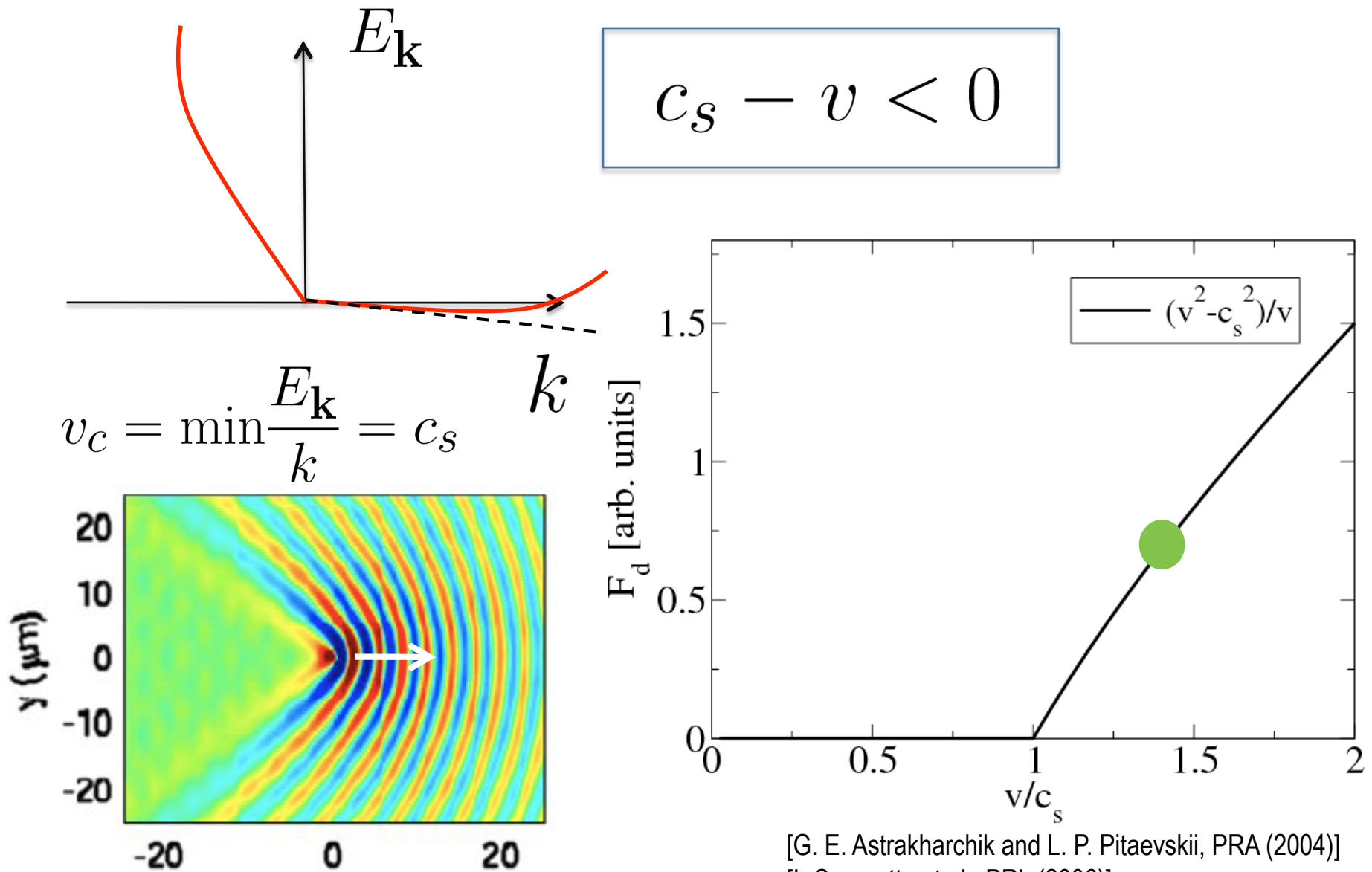
$$c_s - v > 0$$



[G. E. Astrakharchik and L. P. Pitaevskii, PRA (2004)]

[I. Carusotto et al., PRL (2006)]

Moving defect in a BEC & Cherenkov radiation

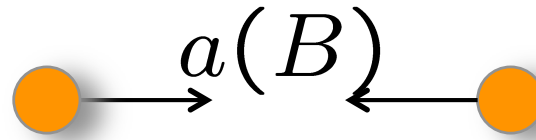


[G. E. Astrakharchik and L. P. Pitaevskii, PRA (2004)]

[I. Carusotto et al., PRL (2006)]

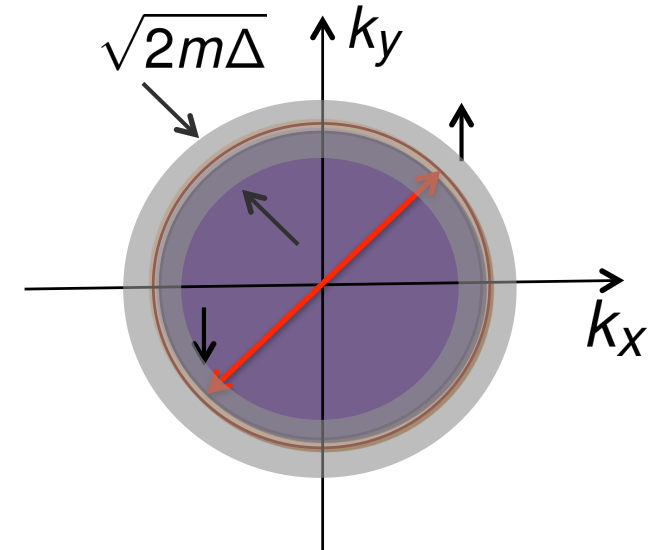
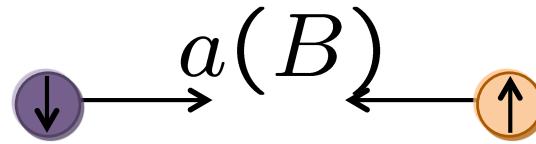
BEC-BCS crossover

- Tune the interaction strength (Feshbach resonances)

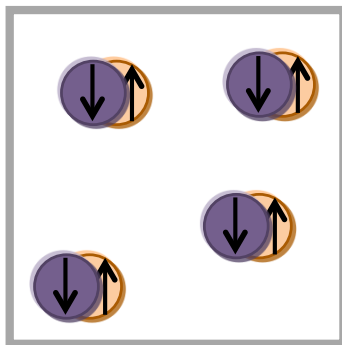


BEC-BCS crossover

- Tune the interaction strength (Feshbach resonances)



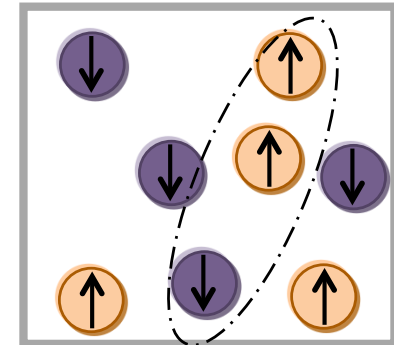
$\frac{1}{k_F a}$



BEC

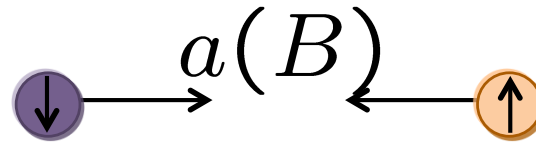
0

BCS



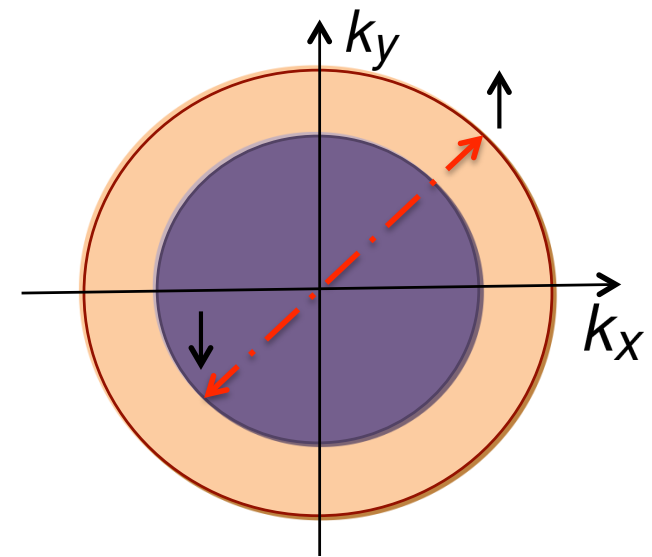
Imbalanced Fermi mixtures

- Tune the interaction strength (Feshbach resonances)



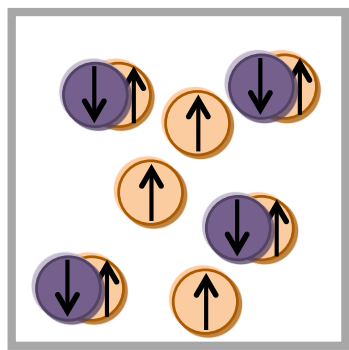
Can superfluidity persist in presence of a population imbalance?

Analogy with a superconductor in a magnetic Zeeman field

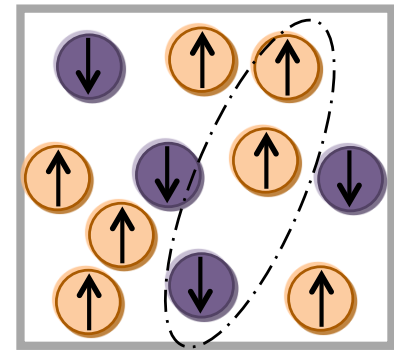


$\frac{1}{k_F a}$

$n_{\uparrow} - n_{\downarrow}$



BEC



BCS