

Introduction to experiments in ultracold atomic gases

Introduction to BEC & superfluidity

Francesca Maria Marchetti

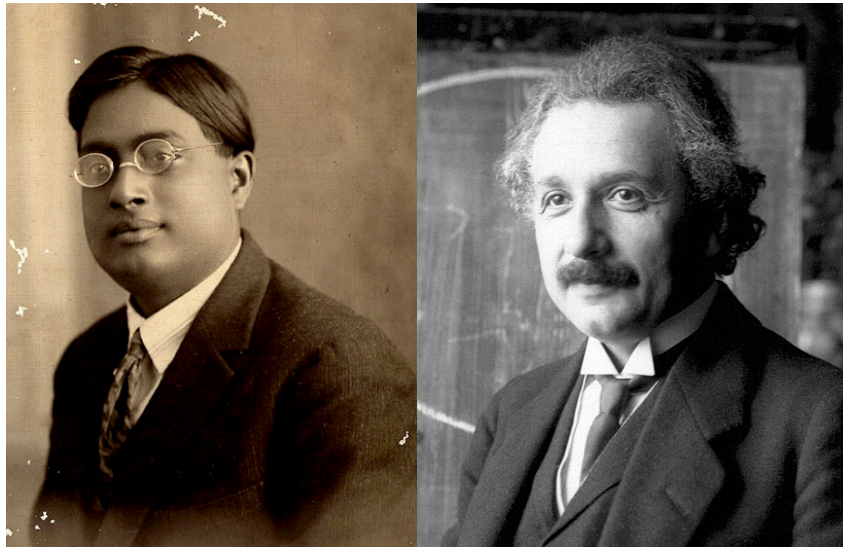


UAM, May 2012

BEC: from 1925 to 1995

- 1924/1925

Following the work of Bose on the statistical description of light quanta, Einstein predicted that a gas of non-interacting massive bosons, below a critical temperature, undergoes a phase transition associated with the condensation of the atoms in the lowest energy state: Bose-Einstein condensation (BEC)



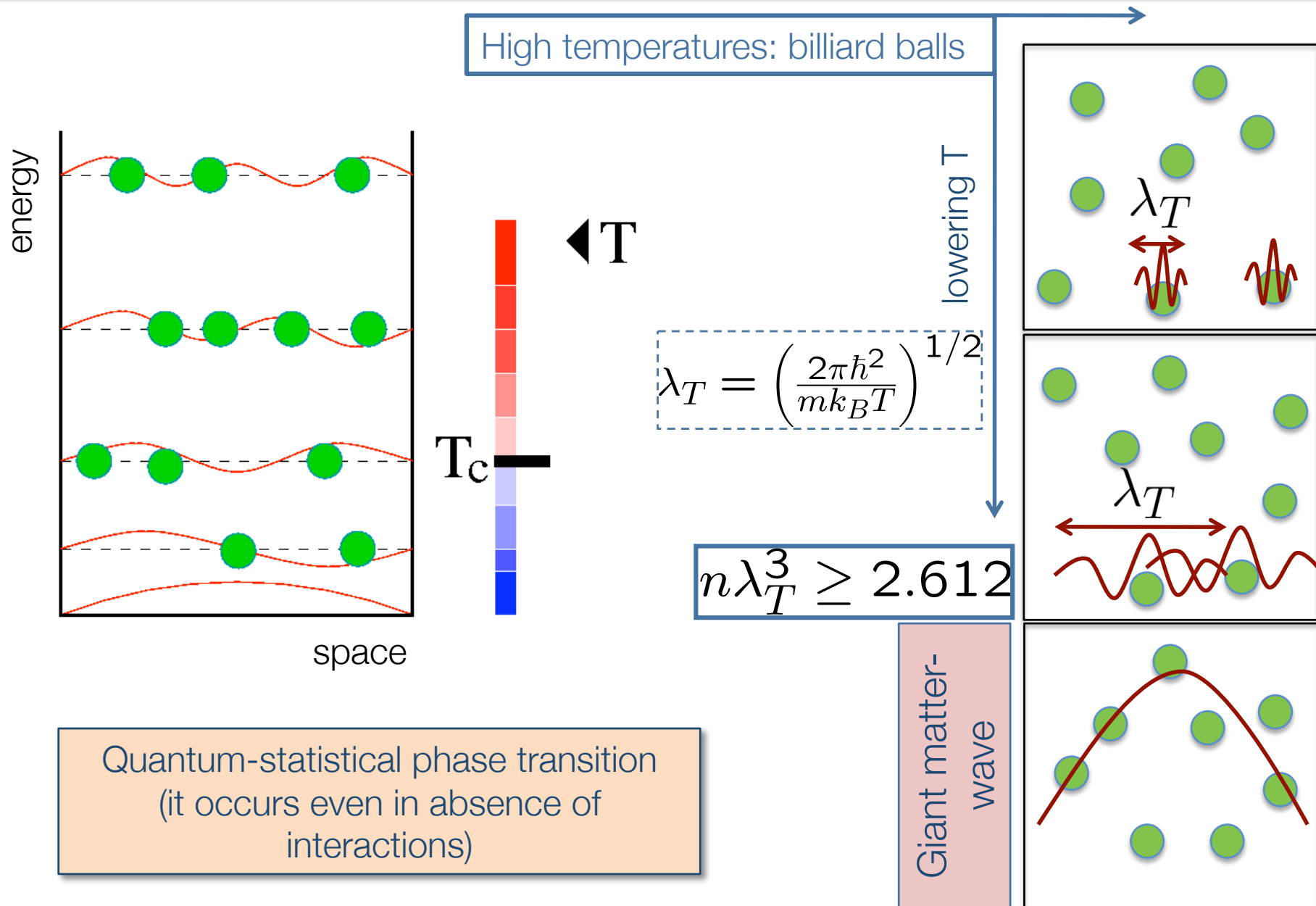
Satyendranath Bose Albert Einstein

“...A separation is affected; some part condenses, the rest remains a ‘saturated ideal gas’...”

“...condensation without attractive forces...”

[Link to animation](#)

Macroscopic occupation of the ground state



BEC

NON INTERACTING IDEAL BOSE GAS

⇒ paradigm of quantum statistical mechanics

- indistinguishability
- wave nature of particles
- thermal equilibrium

⇒ macroscopic quantum phenomena

- macroscopic wavefunction
(many-body ground state wf is the product of N identical single-particle ground-state wfs)
- similarly for weak interactions
(quantum depletion = 1% for alkali condensates)

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IDEAL BOSE GAS

⇒ paradigm of quantum statistical mechanics

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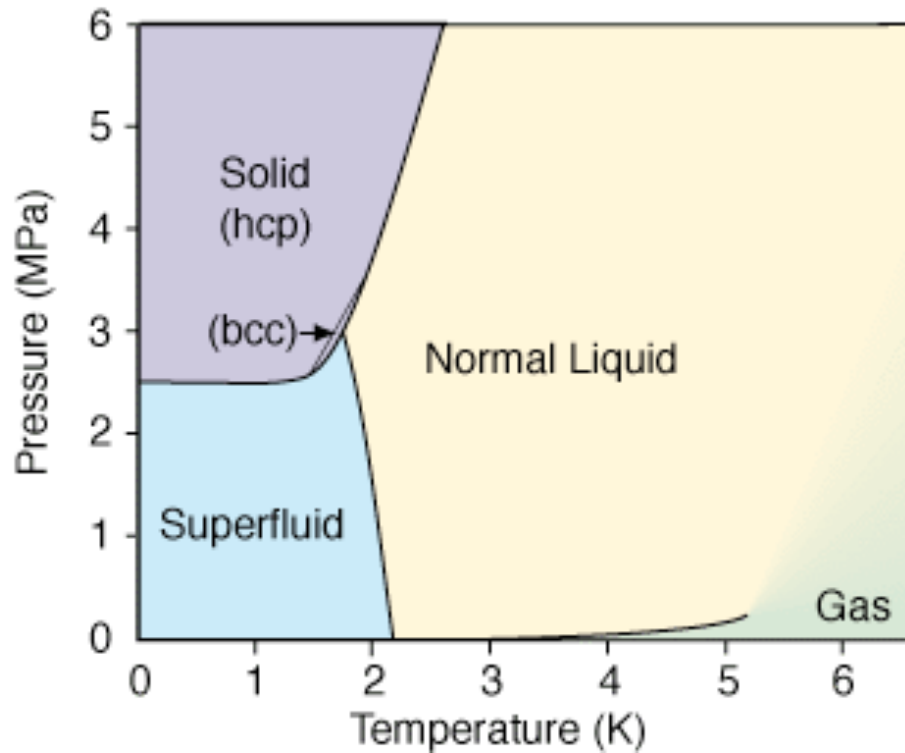
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BEC: from 1925 to 1995

- 1938 Discovery of superfluidity in liquid helium ^4He (Allen & Misener; Kapitza).

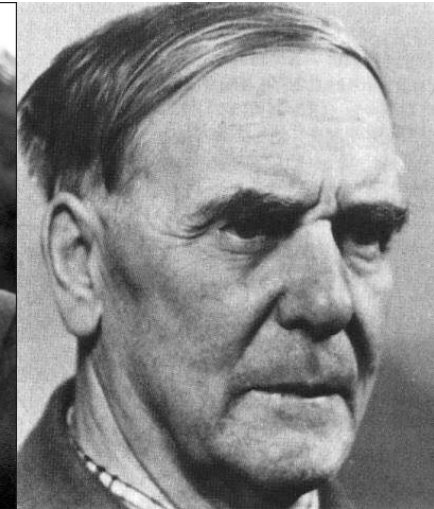
What's the relation between BEC & superfluidity?



Peter L. Kapitza



Jack Allen



Don Misener

BEC: from 1925 to 1995

- 1938 Discovery of superfluidity in liquid helium ^4He (Allen & Misener; Kapitza).

What's the relation between BEC & superfluidity?

- Immediately after, London suggested the connection between the superfluidity of ^4He and BEC

⇒ first to bring out the idea of BEC displaying quantum behaviour on a macroscopic scale

- controversial issue for decades

The theory of superfluidity of Landau does not explicitly mention BEC!

- E.g., Landau's criterion

$$v_c = \min \frac{E_p}{p}$$

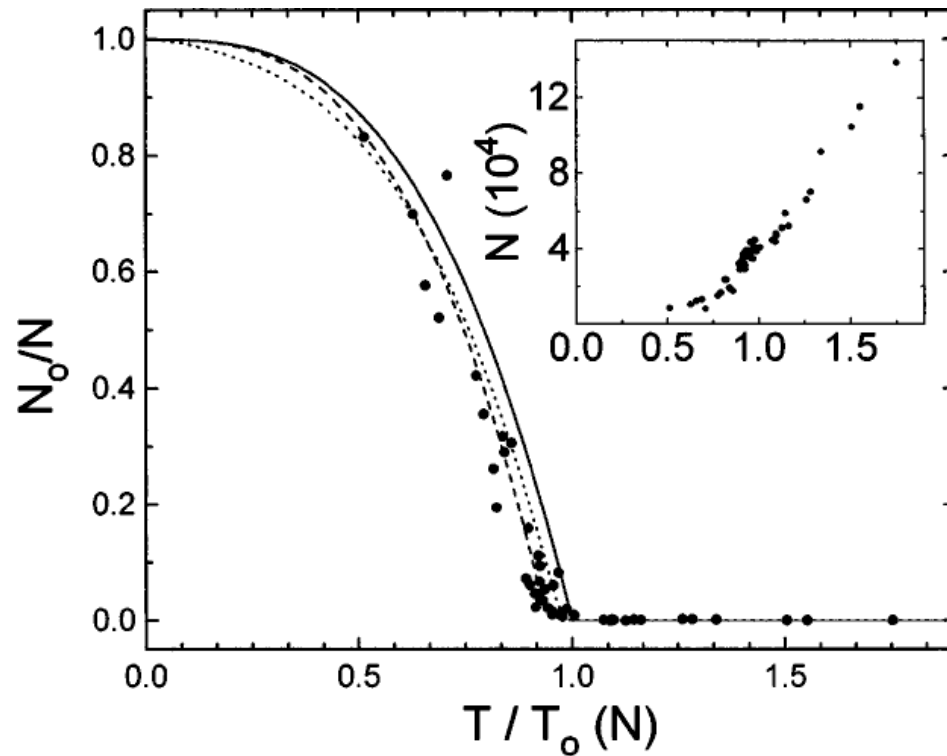


Fritz London

Condensate fraction

Ultracold gases of bosonic atoms

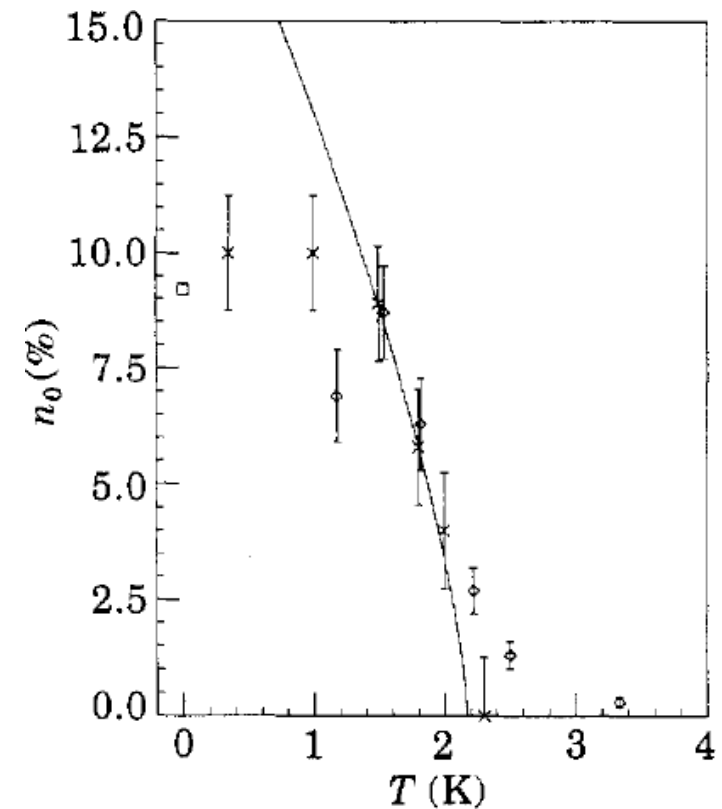
condensate fraction in a BEC of Rubidium ultracold atoms (rather good agreement with predictions for an ideal Bose gas model)



[J. R. Ensher *et al.*, PRL **77**, 4984 (1996)]

Superfluid ^4He

condensate fraction in superfluid ^4He at constant density



[W.M. Snow *et al.*, Europhys. Lett **19**, 403 (1992)]

BEC: from 1925 to 1995

- 1947 microscopic theory of interacting Bose gases (Bogoliubov)

$$E_p = \sqrt{\epsilon_p(\epsilon_p + 2gn)}$$

...provides the microscopic picture behind Landau's theory



Nikolai N. Bogoliubov

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...provides the microscopic picture behind Landau's theory

- 1951 off-diagonal long range order (Landau&Lifshitz; Penrose)
- and much more theoretical work...

QUEST TO REALISE A BEC:
Search of weakly interacting Bose gases



Nikolai N. Bogoliubov



Lev D. Landau

BEC

vs.

Superfluidity

NON INTERACTING
IDEAL BOSE GAS

⇒ paradigm of quantum statistical mechanics

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- thermal equilibrium

⇒ macroscopic quantum phenomena

- macroscopic wavefunction (many-body ground state wf is the product of N identical single-particle ground-state wfs)
- similarly for weak interactions (quantum depletion = 1% for alkali condensates)

IDEAL BECs ARE NOT SUPERFLUID

INTERACTING
BOSE GAS

⇒ mainly related to transport phenomena (flow without friction)

- is essential the form of the dispersion of the elementary excitations (Landau criterion)

$$E_p = \sqrt{\epsilon_p(\epsilon_p + 2gn)} \quad v_c = \min \frac{E_p}{p}$$

- interactions are essential (change the dispersion to phonon-like)
- superfluidity is possible even with few% of atoms in the ground state (see ^4He)

LINK BETWEEN BEC &
SUPERFLUIDITY: ORDER
PARAMETER and ODLRO

BEC

vs.

Superfluidity

NON INTERACTING
IDEAL BOSE GAS

INTERACTING
BOSE GAS

Ideal Bose-Einstein condensates are not superfluid, but also there are superfluid systems that do not display Bose-Einstein condensation (e.g., 2D)

IDEAL BECs ARE NOT SUPERFLUID

LINK BETWEEN BEC &
SUPERFLUIDITY: ORDER
PARAMETER and ODLRO

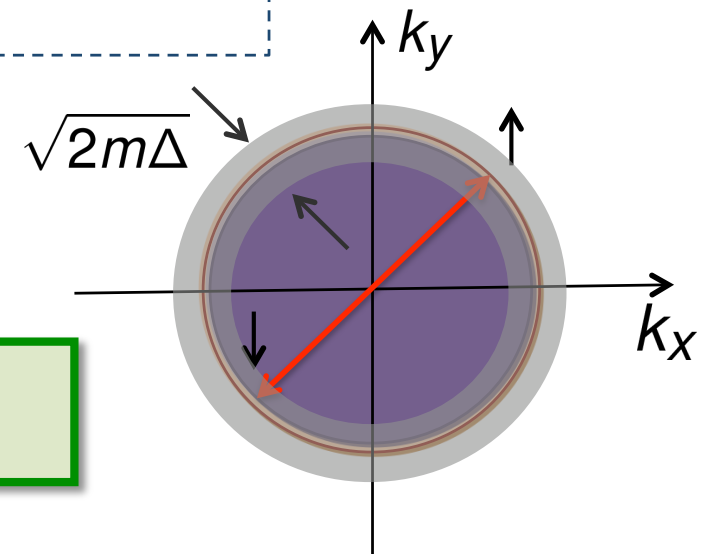
BEC in other systems

- BEC is involved in several macroscopic quantum phenomena (even if some systems are not ideal Bose gases):

macroscopic
occupation of a single
quantum state

- ⇒ ^4He (but is a strongly interacting system)
- ⇒ superconductors (BEC of Cooper pairs)
(the BEC-BCS crossover)
- ⇒ ^3He (also fermions)
- ⇒ lasers (but out of equilibrium: requires inversion of the population)
- ⇒ ...

QUEST TO REALISE A BEC:
Search of weakly interacting Bose gases



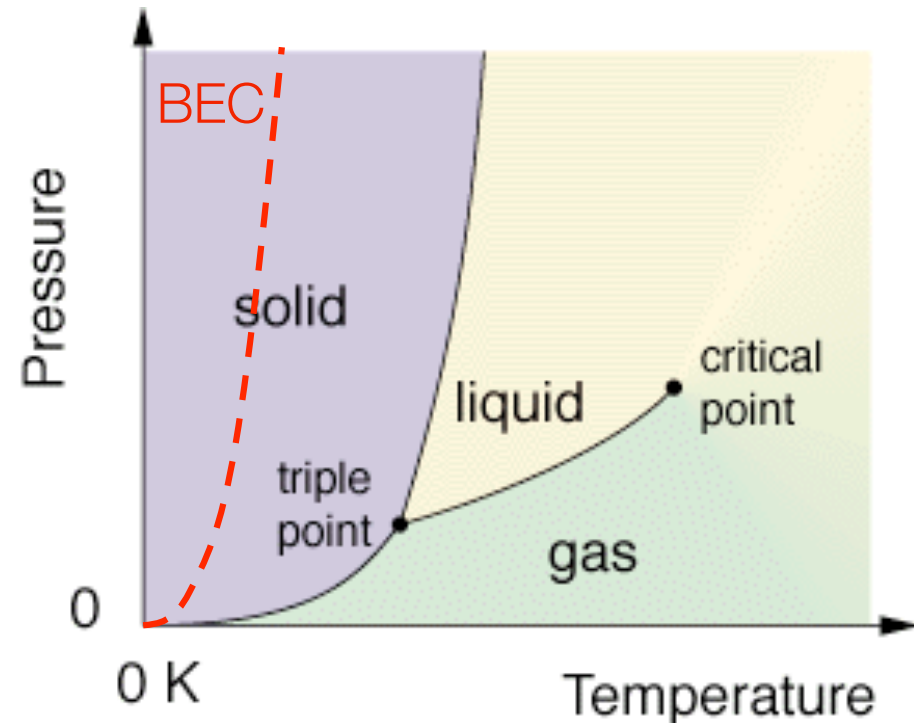
Searching for weakly interacting Bose gases

Why so hard?

⇒ At very low T most substances are solid (or liquid) & interaction becomes large

- The pressure versus temperature phase transition line for BEC falls into the region where the equilibrium phase is a solid

$$P = \frac{2E}{3V} \simeq \left(\frac{m}{2\pi}\right)^{3/2} (k_B T)^{5/2}$$



A BEC (in the true chemical thermal equilibrium) is unstable

Why dilute and why ultracold

1. metastable equilibrium

$$\tau_3 \ll \tau_2$$

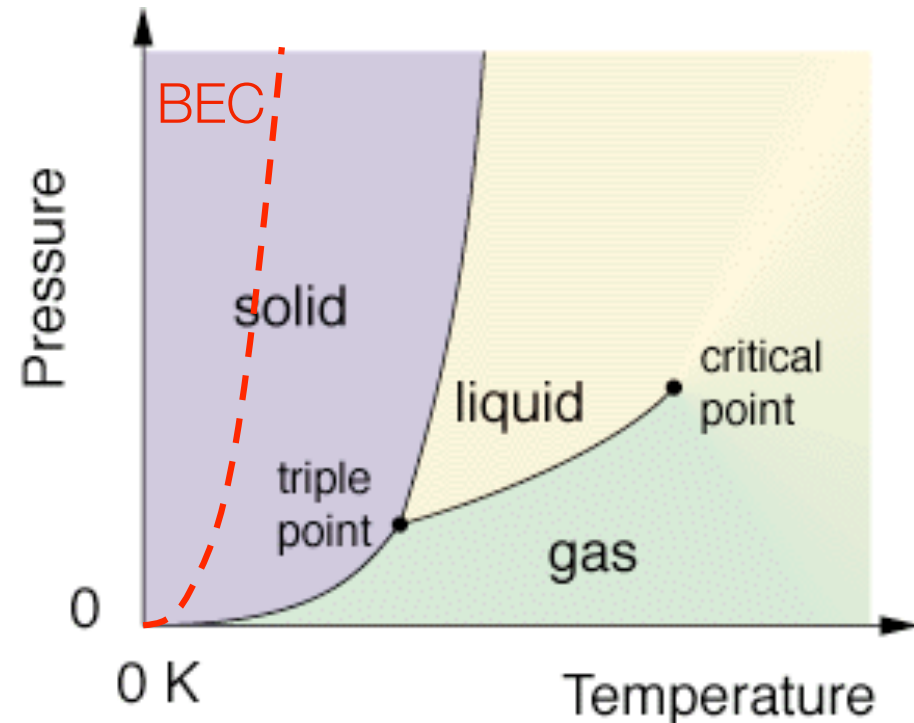
(3-body recombination rate \ll
2-body scattering rate)

- Towards the formation of a solid:
of the three atoms colliding, two
form a molecule and the third one
can carry away the residual energy

$$\tau_3 \sim n^3 \ll \tau_2 \sim n^2$$

possible at very low densities

- Still 2-body interactions can guarantee kinetic thermal equilibrium



Why dilute and why ultracold

1. metastable equilibrium: requires the gas to be diluted

$$n \sim 10^{13} - 10^{15} \text{cm}^{-3}$$

⇒ the gas has a finite lifetime of the order between seconds and minutes, after which it becomes a solid

2. quantum degeneracy: requires the gas to be ultracold

$$T \sim 100 \text{nK} - \mu\text{K}$$

$$n\lambda_T^3 = n \left(\frac{2\pi\hbar^2}{mk_B T} \right)^{3/2} > 2.61$$

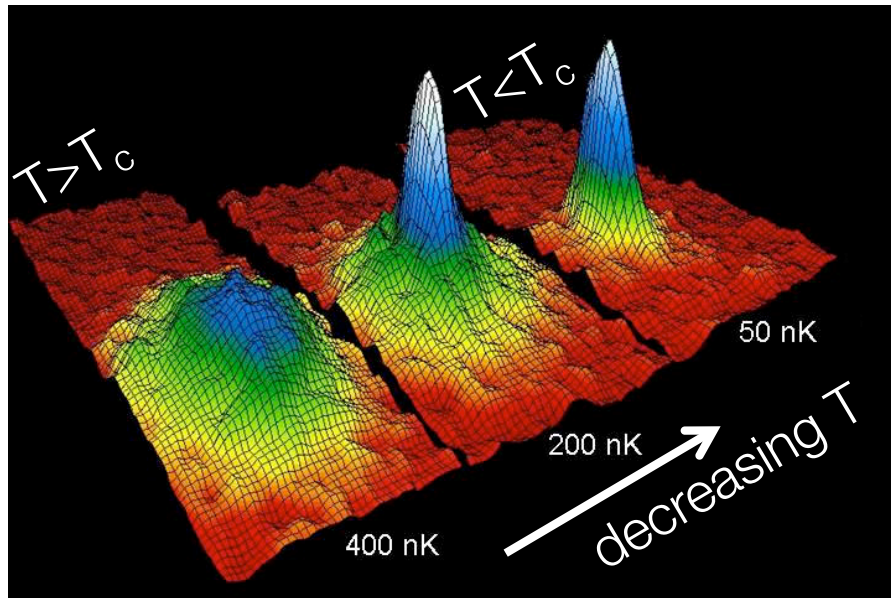
3. Trapping (atoms must be thermally isolated from all material walls)

⇒ Necessity of very sophisticated cooling and trapping techniques

From 1925 to 1995:
it took 70 years to realise a BEC in dilute atomic gases

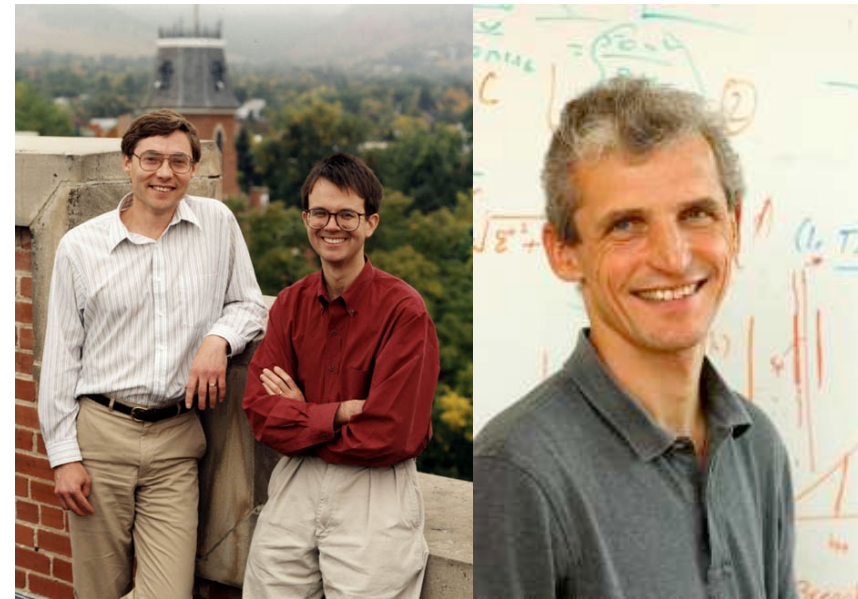
BEC: from 1925 to 1995

- 1995 BEC in alkali atoms (^{87}Rb , ^{23}Na , ^7Li , ...)



Coollest system in the universe!

Nobel prize (2001)



Carl Wieman & Eric Cornell Wolfgang Ketterle

$$T \sim 500\text{nK} - \mu\text{K}$$
$$n \sim 10^{11} - 10^{13}\text{cm}^{-3}$$

Hierarchy of energy and length scales

⇒ Simplifies the description of BECs

$$n \sim 10^{14} \text{cm}^{-3}$$

Energy Scale E	$= \hbar^2/2ml^2$	Length Scale		
limiting temperature for s-wave scattering	1 mK	scattering length	$a = l/2\pi$	= 3 nm
BEC transition temperature T_c	$2 \mu\text{K}$	separation between atoms	$n^{-1/3} = l/\sqrt{\pi}(2.612)^{1/3}$	= 200 nm
temperature T	$1 \mu\text{K}$	thermal de Broglie wavelength	$\lambda_{dB} = l/\sqrt{\pi}$	= 300 nm
mean field energy μ	300 nK	healing length	$\xi = l/2\pi$	= 200 nm
harmonic oscillator level spacing $\hbar\omega$	0.5 nK	oscillator length ($\omega \simeq 2\pi \cdot 10\text{Hz}$)	$a_{HO} = l/\sqrt{2}\pi$	= $6.5 \mu\text{m}$

New window into the quantum world

⇒ At first, interest was to realise a BEC as close as possible to the ideal case

⇒ rapid development since the achievement of the first BEC (1995) has been breathtaking

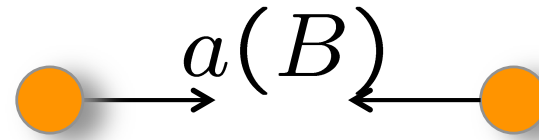
⇒ Ultracold atoms became ideal model systems for a host of phenomena:

- Diluteness = absence of not well understood interactions
- Control
- Manipulation
- Precise probe
 - ✓ Tune the interaction strength (e.g., Feshbach resonances)
 - ✓ Bosons, fermions, mixtures
 - ✓ Simulate crystals (optical lattices)
 - ✓ Reduced dimensions (2D, 1D, 0D)
 - ✓ Disorder
 - ✓ ...

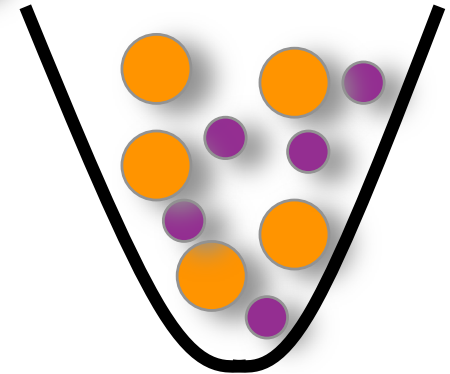
⇒ Special role of cold atom experiments: perform “quantum simulations” of condensed matter systems

Ultracold atoms today

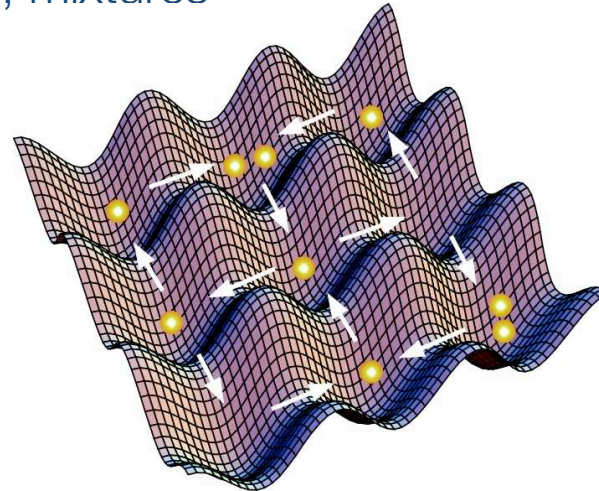
- Tune the interaction strength (Feshbach resonances)



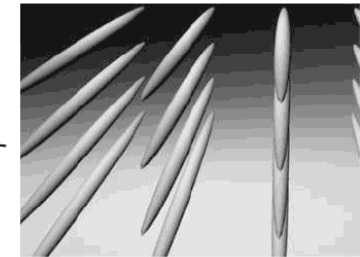
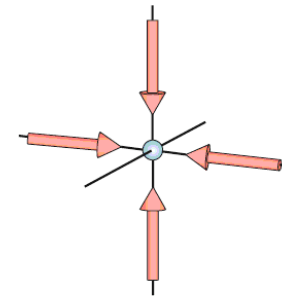
- Bosons, Fermions, mixtures



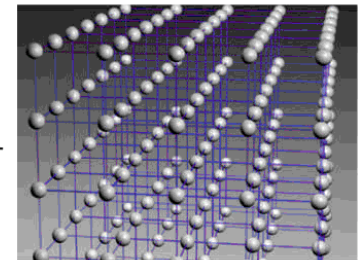
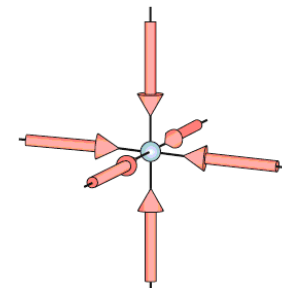
- Optical lattices



- Reduced dimensions (2D, 1D, 0D)



- Disorder

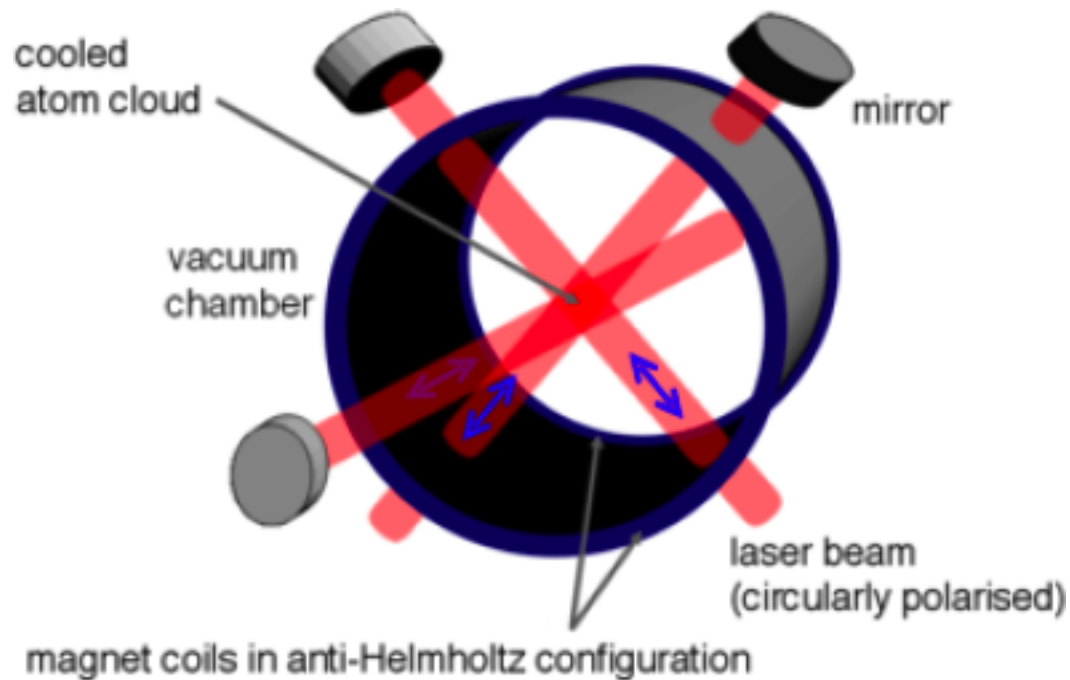
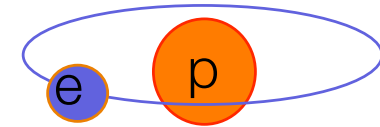


Simulate solid state systems
(with advantage of external control)

[Bloch *Nature Physics* (2005)]

Dilute ultracold atomic gases: Experiments

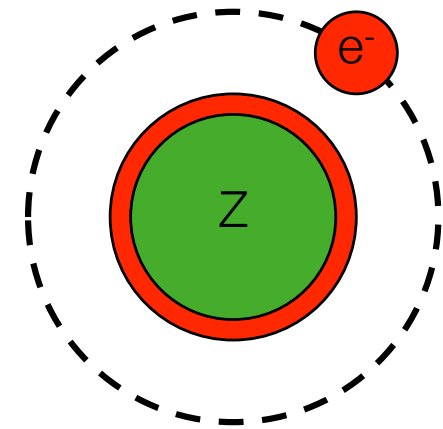
- 1959 spin-polarized (by a magnetic field) hydrogen proposed as a good candidate for a weakly interacting Bose gas
- '80 Developments in magnetic trapping, laser and evaporative cooling of alkali atoms



Group →	1	2	3
↓ Period			
1	1 H		
2	3 Li		
3	11 Na		
4	19 K		
5	37 Rb		
6	55 Cs		
7	87 Fr		

Alkali atoms

- ▶ Electronic spin $S=J=1/2$, nuclear spin I
- ▶ Z odd (neutral atoms = same number of electrons and protons)
- ▶ N determines the statistics
 - $A=Z+N$ odd for bosons (N even)
 - $A=Z+N$ even for fermions (N odd)



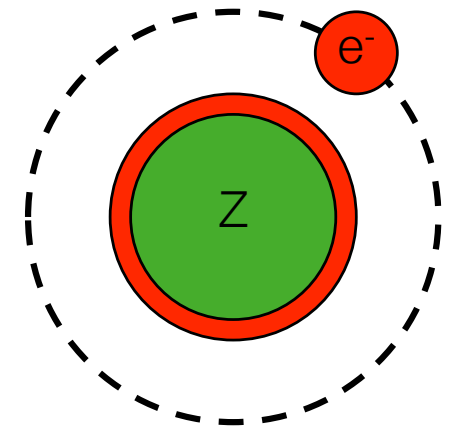
bosons	^{85}Rb	$I=5/2$
	^{87}Rb	$I=3/2$
	^{23}Na	$I=3/2$
fermions	^7Li	$I=3/2$
	^{40}K	$I=4$
	^6Li	$I=1$

At very low temperatures atoms are in their electronic ground state ($\ell = 0$).
The internal states are the hyperfine states

Hyperfine levels and Zeeman splitting

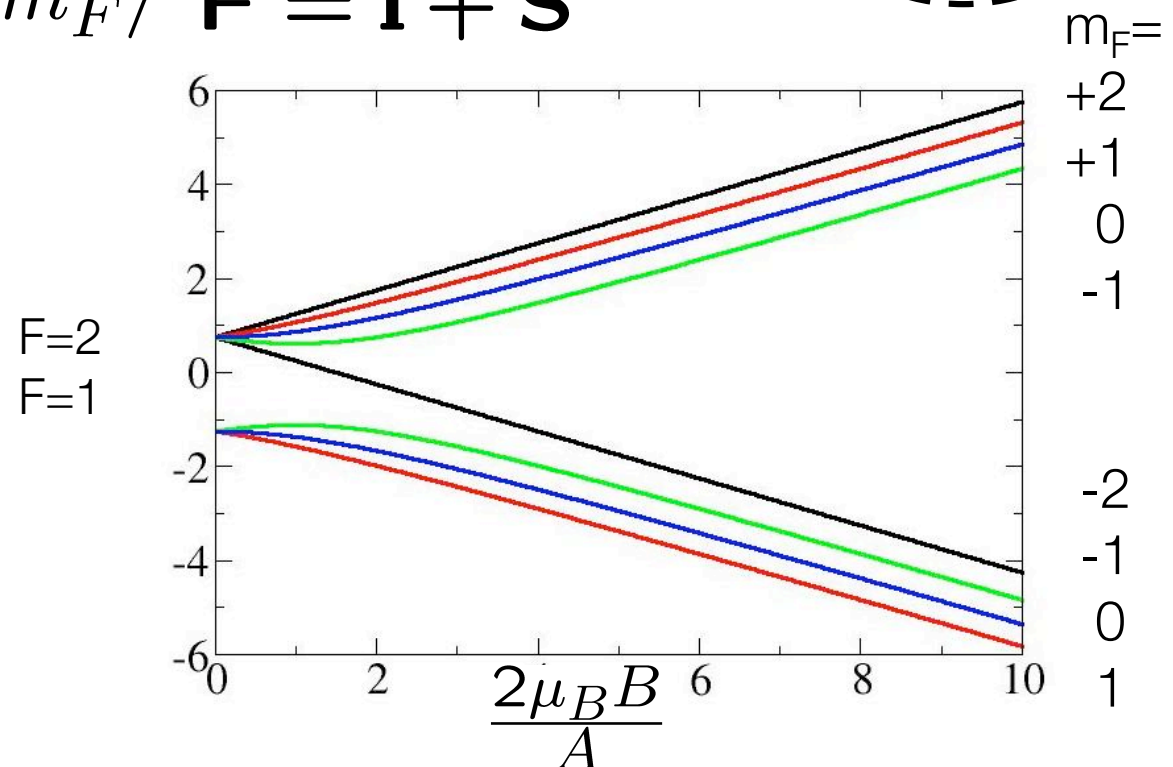
- ▶ Electronic spin $S=J=1/2$, nuclear spin I

$$\hat{H} = A\hat{\mathbf{I}} \cdot \hat{\mathbf{S}} + 2\mu_B B \hat{S}_z$$



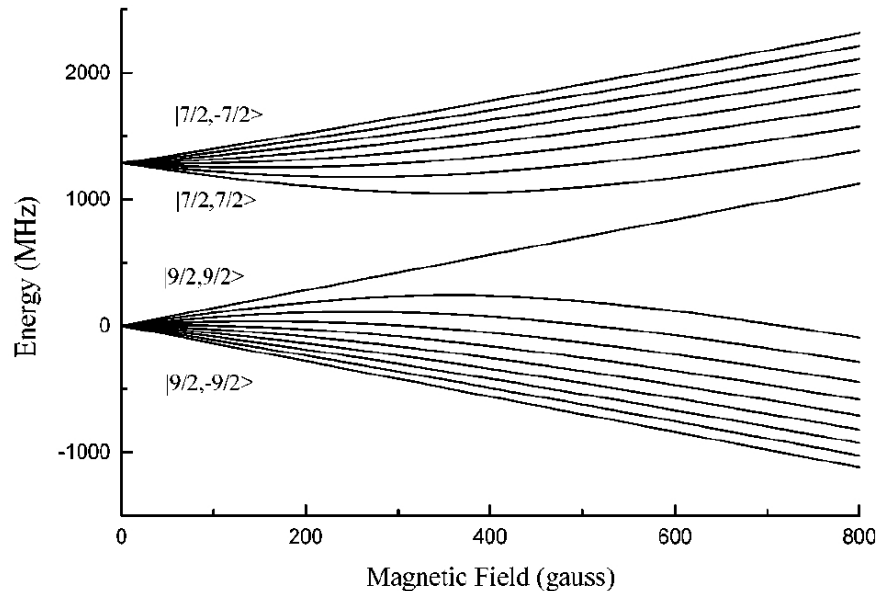
- ▶ Hyperfine levels $|F, m_F\rangle$ $\hat{\mathbf{F}} = \hat{\mathbf{I}} + \hat{\mathbf{S}}$

- ▶ $I=3/2$ (^{87}Rb , ^{23}Na , ^7Li)

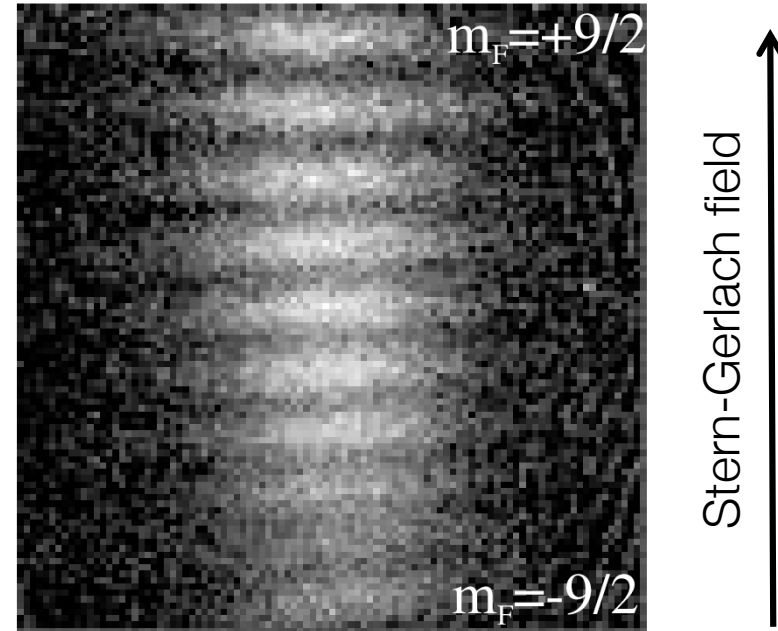


Hyperfine levels and Zeeman splitting

- $I=4$ (^{40}K)



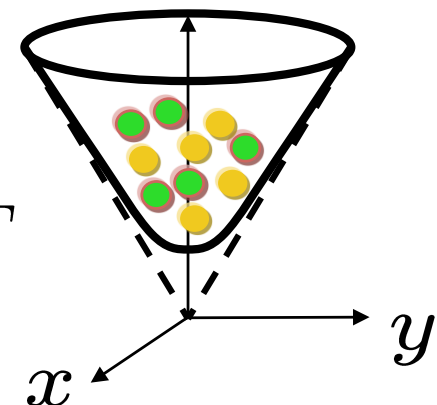
[J. L. Bohn *et al.* *PRA* **59**, 3660 (1999)]



[T. Loftus *et al.* *PRL* **88**, 173201 (2002)]

1. Control the populations of atoms in different hyperfine states
2. Magnetic trapping

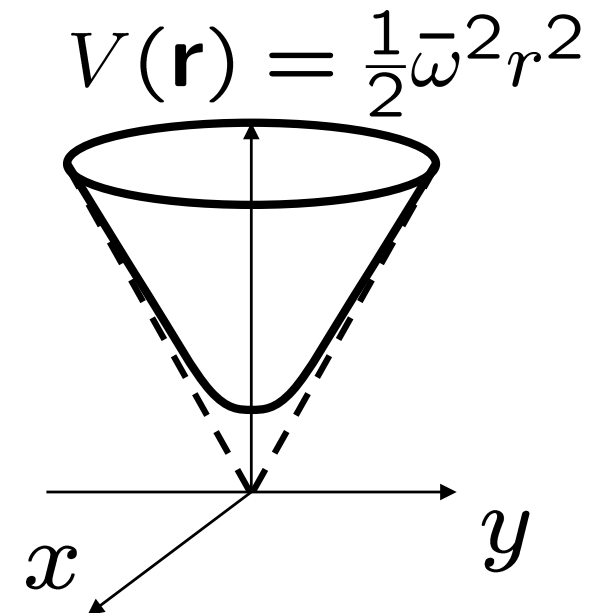
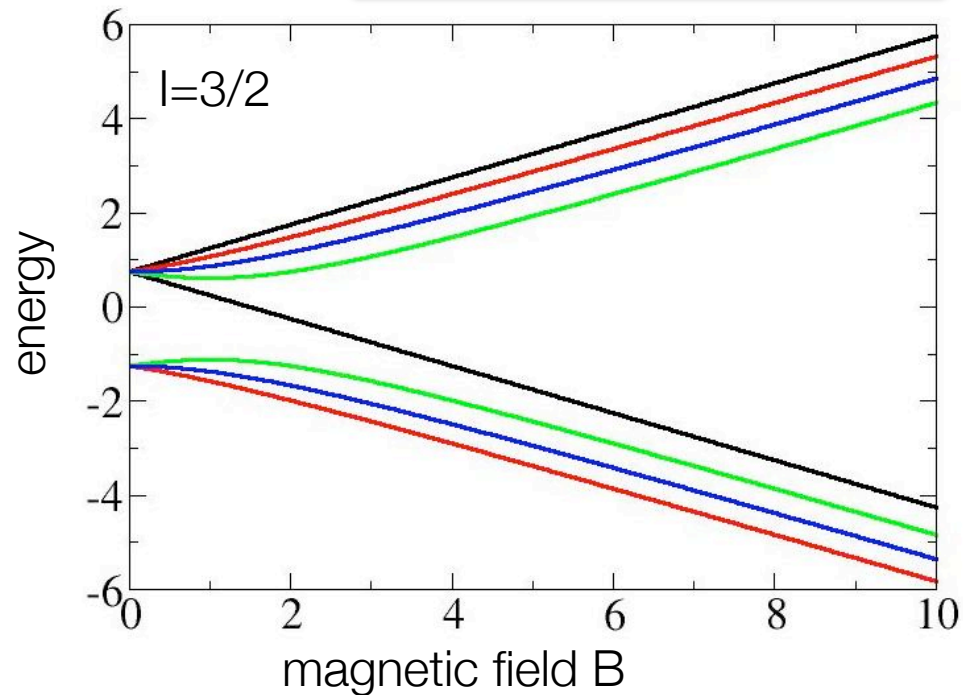
$$\Delta E_{\text{hyp}} \gg k_B T$$



Trapping the atoms magnetically

- $\mu_\alpha > 0$ high-field seeking states
- $\mu_\alpha < 0$ low-field seeking states

$$E_\alpha \simeq \text{const} - \mu_\alpha B$$



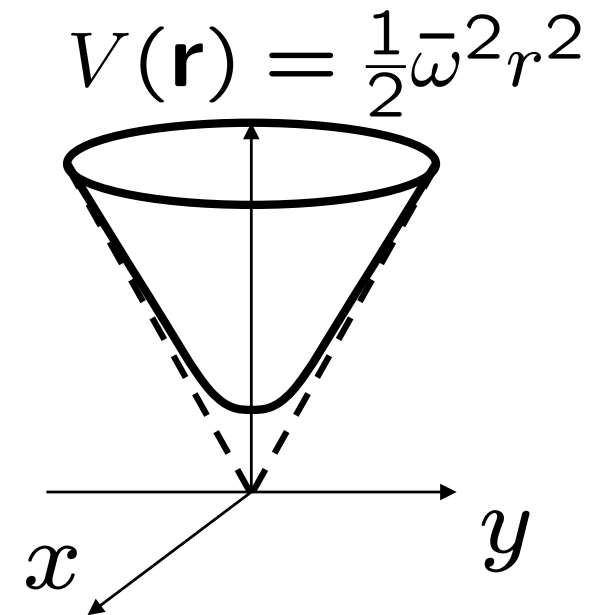
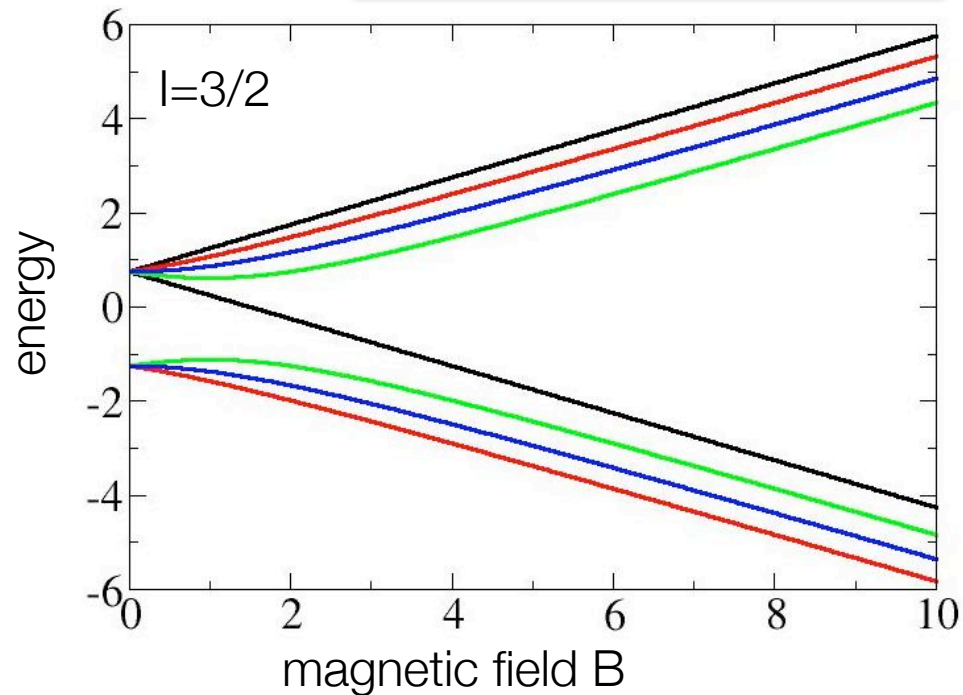
- Atoms moving slowly follow the direction of the local field adiabatically

Atoms in an inhomogeneous field experience a spatially-varying potential

Trapping the atoms magnetically

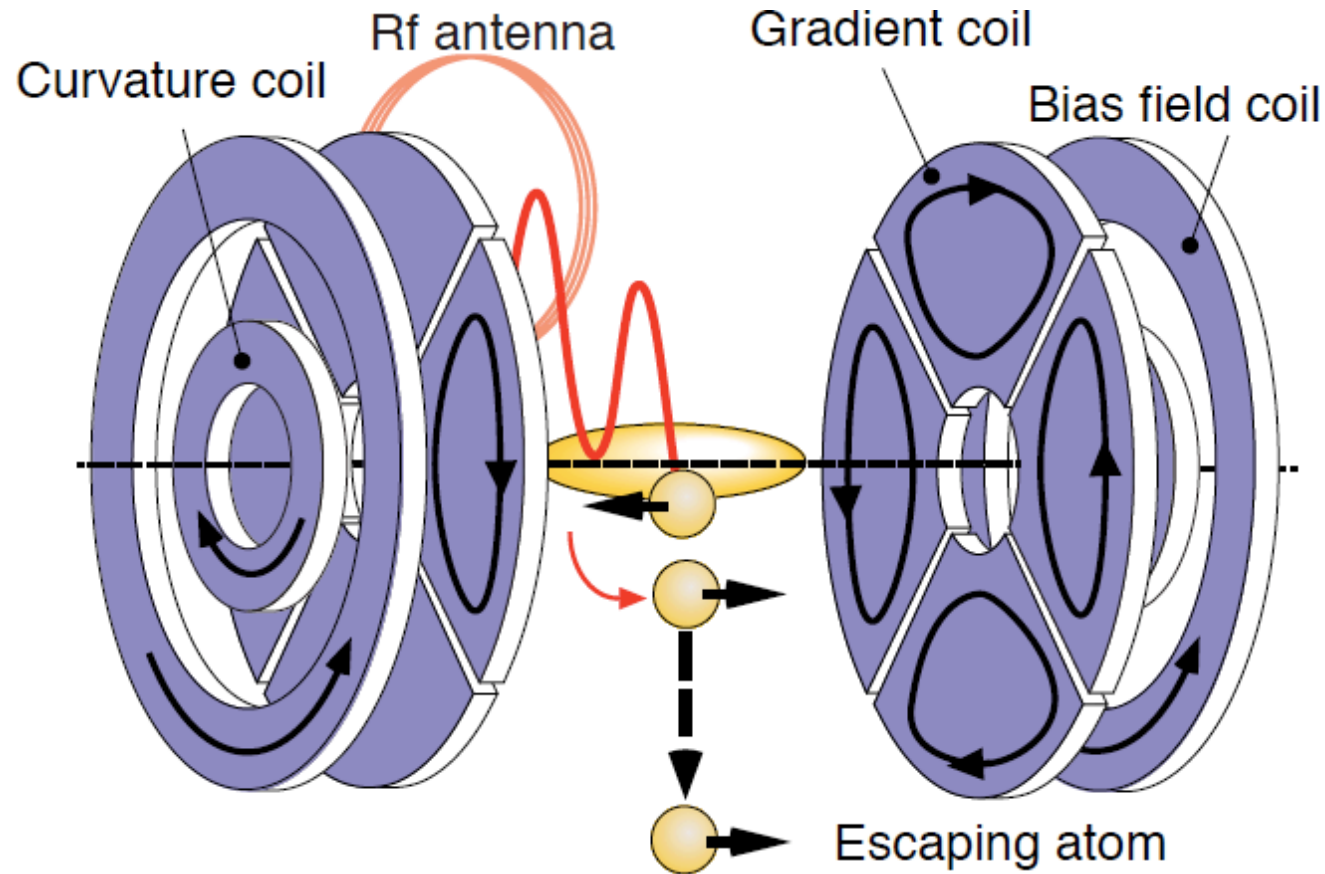
- $\mu_\alpha > 0$ high-field seeking states
- $\mu_\alpha < 0$ low-field seeking states

$$E_\alpha \simeq \text{const} - \mu_\alpha B$$



⇒ N.B. if the magnetic field is too small, atoms can flip their spin to a high-field seeking state and become untrapped (trap loss region)

Trapping the atoms magnetically



[W. Ketterle *et al.*, Varenna (1998)]

- ⇒ Full optical access
- ⇒ Coils can be placed outside a vacuum chamber

Optical traps

- The interaction of the atoms with laser fields provides another possibility of confinement (as well as laser cooling).
- Dipolar approximation

$$H = -\underset{\substack{\swarrow \\ \text{dipole}}}{\mathbf{d}} \cdot \underset{\substack{\searrow \\ \text{electric field}}}{\mathbf{E}(\mathbf{r}, t)}$$

- Second order (time-dependent) perturbation theory

$$U(\mathbf{r}) = -\underset{\substack{\swarrow \\ \text{dynamic polarisability}}}{\frac{1}{2}\alpha(\omega)} \overline{\underset{\substack{\searrow \\ \text{time average}}}{E^2(\mathbf{r}, t)}}$$

- If the intensity of the electric field varies with the position, the atoms are subjected to a force $-\nabla U(\mathbf{r})$

1. **Attractive:** if the laser is red-detuned (from an atomic resonance frequency)
2. **Repulsive:** if the laser is blue-detuned

Cooling to BEC

⇒ Typical multistage cooling process

- Gas temperature is reduced by a factor 10^9 !!!
- in each step the ground state population increases by 10^6 !!

$$\rho = n\lambda_T^3 = n \left(\frac{2\pi\hbar^2}{mkT} \right)^{3/2}$$

	Temperature	Density (cm ⁻³)	Phase-space density
Oven	500 K	10 ¹⁴	10 ⁻¹³
Laser cooling	50 μK	10 ¹¹	10 ⁻⁶
Evaporative cooling	500 nK	10 ¹⁴	1
BEC			10 ⁷

⇒ Several steps of laser cooling are applied before the cloud is transferred into a magnetic trap

⇒ Last cooling step to reach a BEC is the evaporative cooling technique