

Introduction to experiments in ultracold atomic gases – part 2

Introduction to BEC & superfluidity

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UAM, May 2012

Memo: Dilute & ultracold

1. metastable equilibrium:
dilute gas

$$\tau_3 \ll \tau_2$$

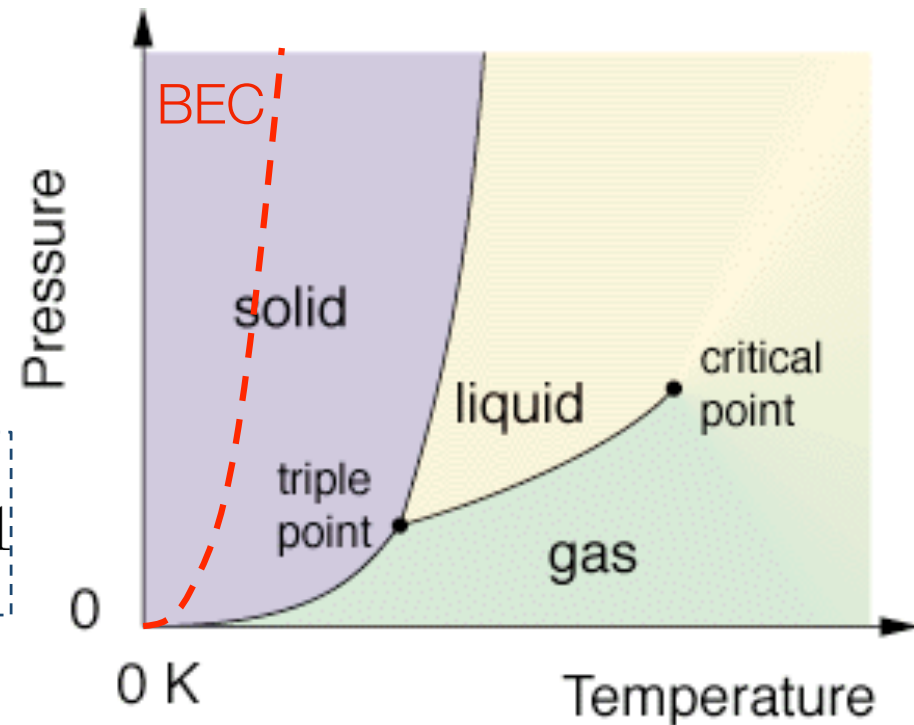
$$n \sim 10^{13} - 10^{15} \text{cm}^{-3}$$

2. quantum degeneracy:
cold gas

$$\rho = n\lambda_T^3 = n \left(\frac{2\pi\hbar^2}{mk_B T} \right)^{3/2} > 1$$

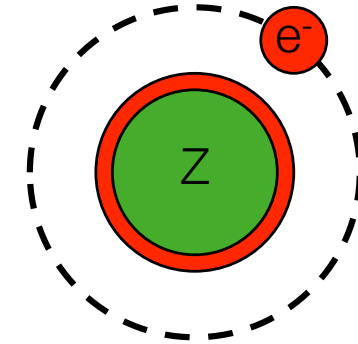
$$T \sim 100 \text{nK} - \mu\text{K}$$

3. Trapped gas



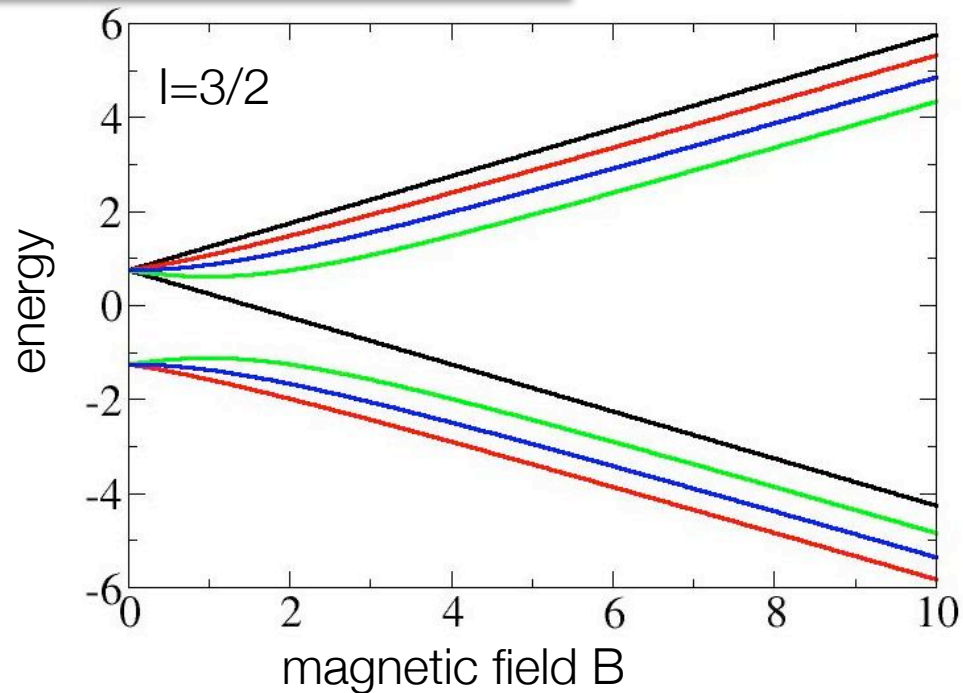
Memo: Alkali atoms & magnetic trapping

bosons	^{85}Rb	$l=5/2$
	^{87}Rb	$l=3/2$
	^{23}Na	$l=3/2$
	^7Li	$l=3/2$
fermions	^{40}K	$l=4$
	^6Li	$l=1$



Atoms in an inhomogeneous field experience a spatially-varying potential

$$E_\alpha \simeq \text{const} - \mu_\alpha B$$



$$V(\mathbf{r}) = \frac{1}{2}\bar{\omega}^2 r^2$$

A diagram of a parabolic potential well. The vertical axis is labeled z and the horizontal axes are labeled x and y . The potential is represented by a cone-like shape opening upwards, with a dashed line indicating the back part of the cone.

Cooling to BEC

⇒ Typical multistage cooling process

- Gas temperature is reduced by a factor 10^9 !!!
- in each step the ground state population increases by 10^6 !!

$$\rho = n\lambda_T^3 = n \left(\frac{2\pi\hbar^2}{mkT} \right)^{3/2}$$

	Temperature	Density (cm^{-3})	Phase-space density
Oven	500 K	10^{14}	10^{-13}
Laser cooling	$50 \mu\text{K}$	10^{11}	10^{-6}
Evaporative cooling	500 nK	10^{14}	1
BEC			10^7

⇒ Several steps of laser cooling are applied before the cloud is transferred into a magnetic trap

⇒ Last cooling step to reach a BEC is the evaporative cooling technique

Laser cooling & Optical traps

- The interaction of the atoms with laser fields provides another possibility of confinement, as well as laser cooling

$$U(\mathbf{r}) = -\frac{1}{2}\alpha(\omega)\overline{E^2(\mathbf{r}, t)}$$

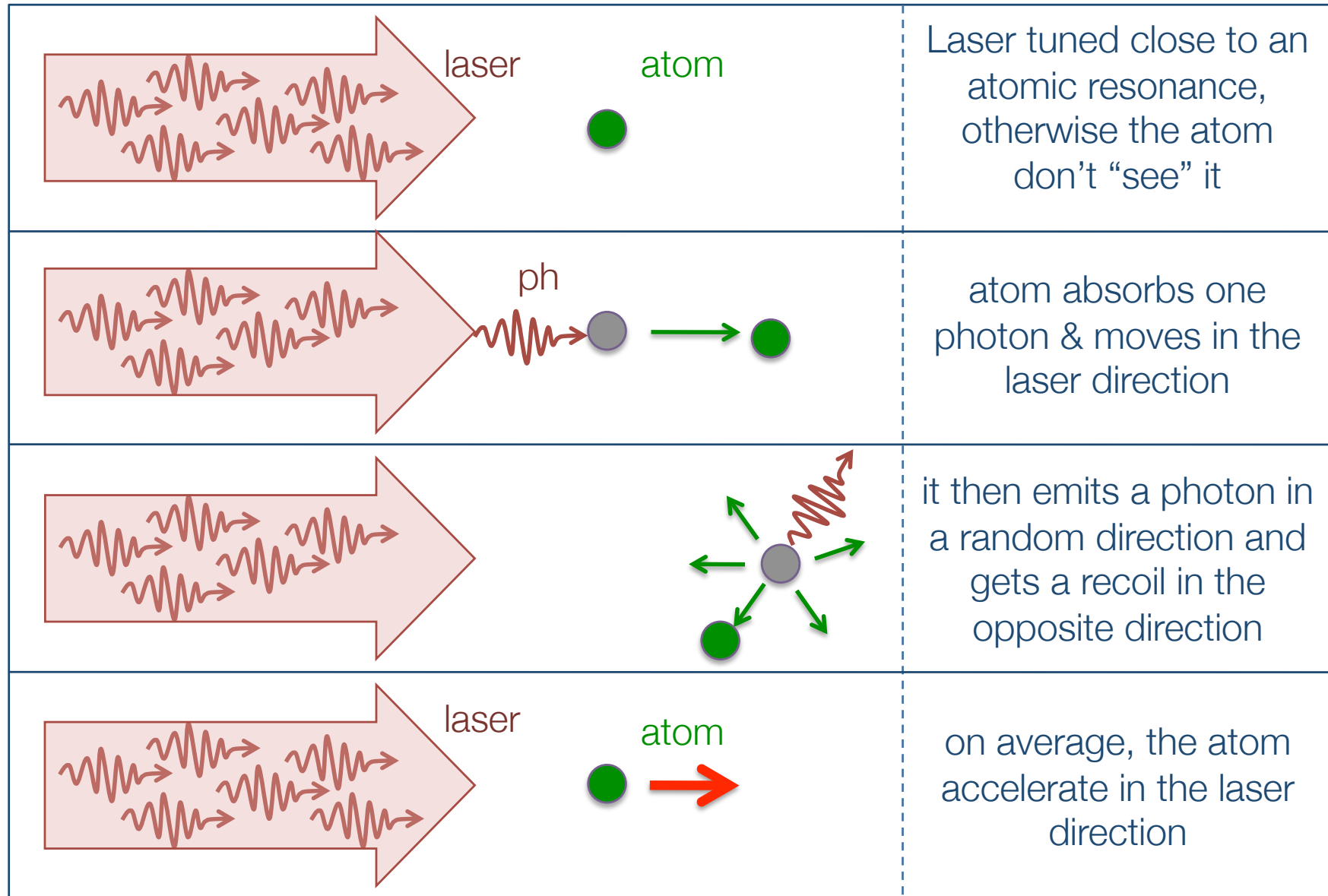
dynamic polarisability time average

- If the intensity of the electric field varies with the position, the atoms are subjected to a force $-\nabla U(\mathbf{r})$

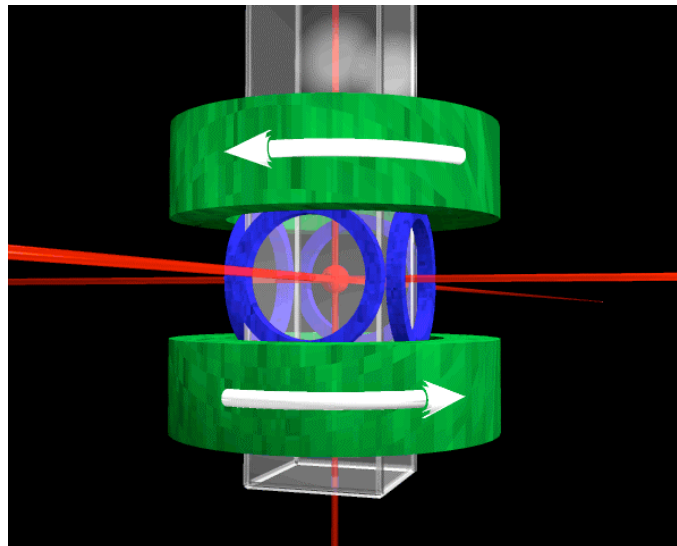
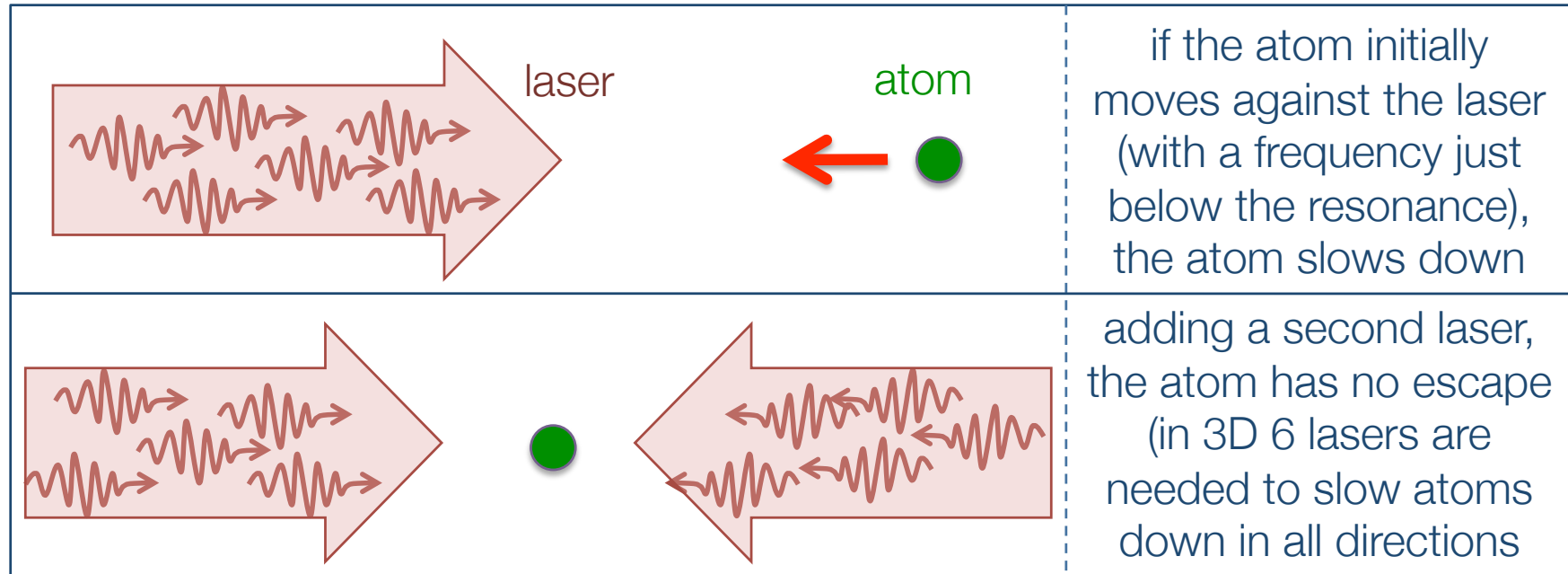
- Attractive:** if the laser is red-detuned (an electronic transition in the atom)
- Repulsive:** if the laser is blue-detuned

Nobel prize 1997 (Chu, Cohen-Tannoudji, Phillips): “for development of methods to cool and trap atoms with laser light”

Laser cooling



Laser cooling



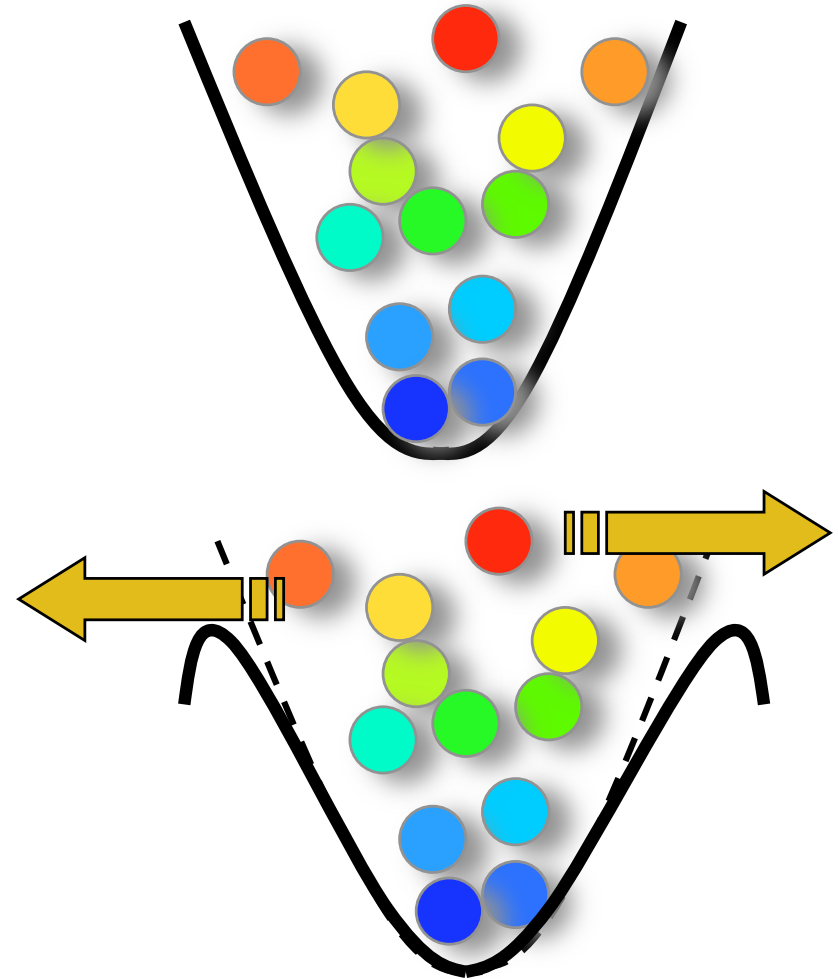
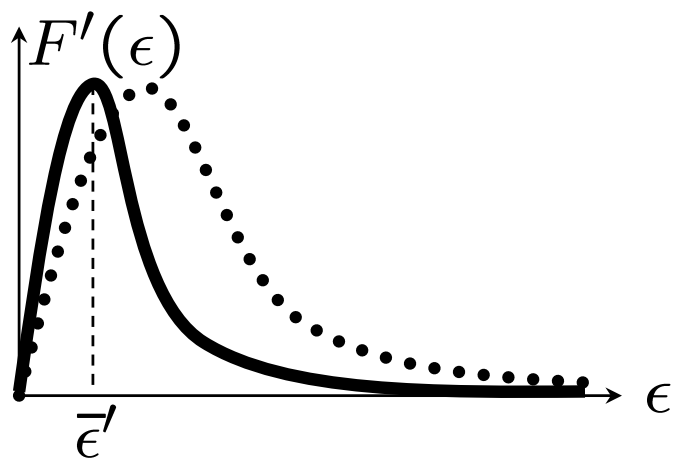
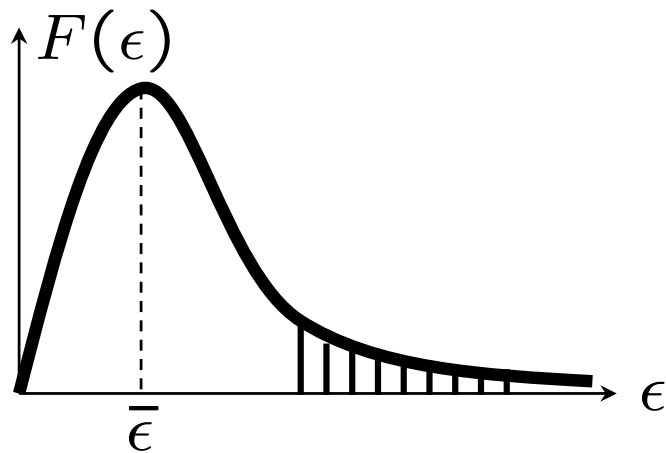
[Link to animation 1](#)

[Link to animation 2](#)

[Link to animation 3](#)

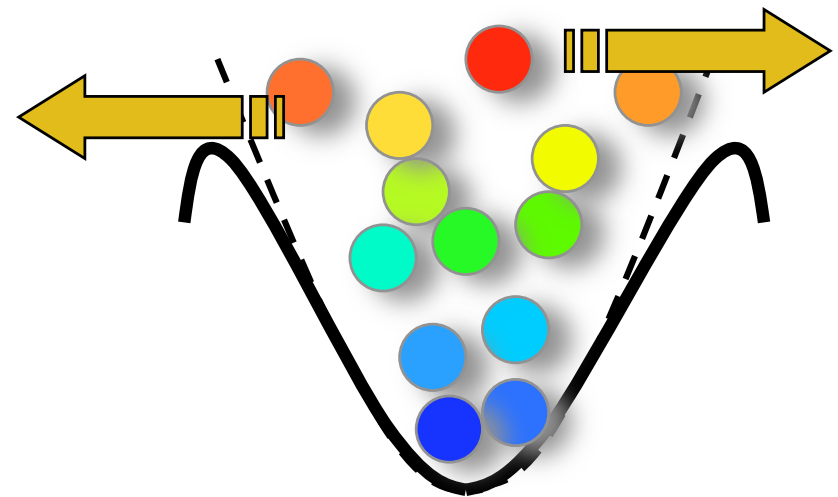
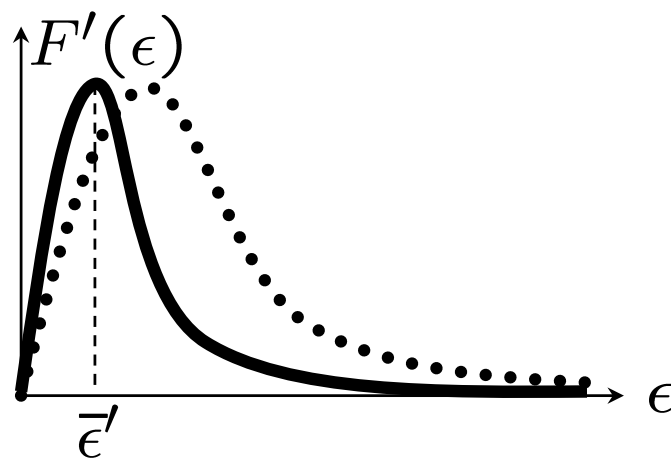
Evaporative cooling

1. Remove from the trap the high-energy tail of the thermal distribution
2. Remaining atoms rethermalise to a lower temperature (i.e., high energy tail is repopulated by collisions)



Evaporative cooling

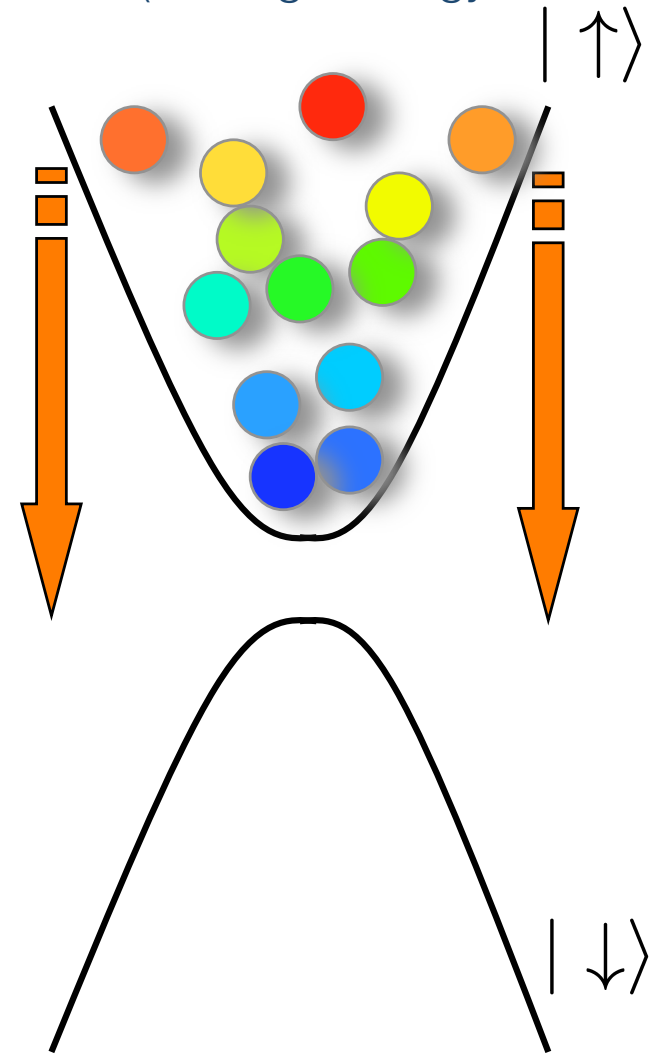
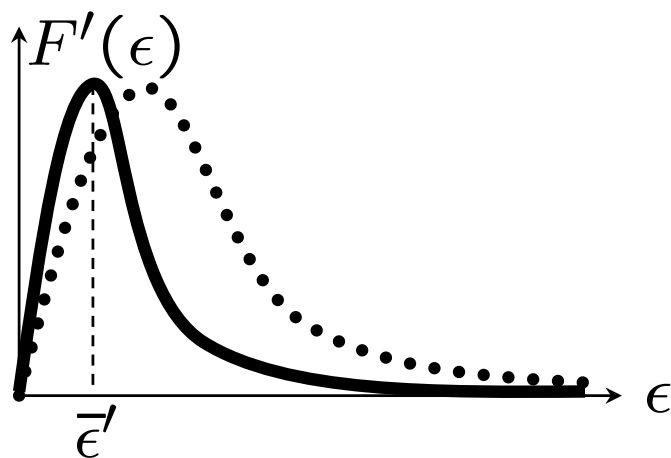
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Evaporative cooling

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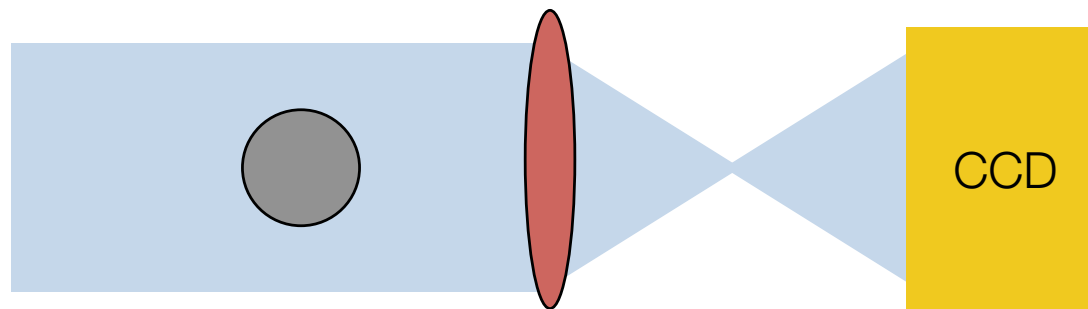
[Link to animation](#)



Probe the atomic cloud

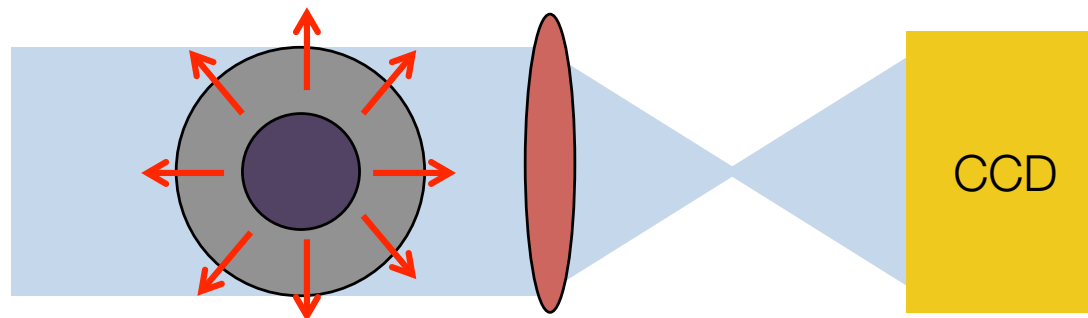
- ⇒ **Optical diagnostics:** atoms are illuminated with a laser beam and images of the shadow cast by the atoms are recorded on a CCD
- ⇒ Absorptive or dispersive imaging

1. In-situ imaging: space distribution $n(\mathbf{r})$

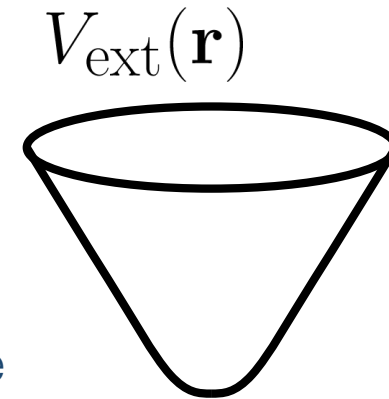


2. Time of flight: momentum distribution

$$\lim_{t\bar{\omega} \gg 1} n(\mathbf{r}, t) \propto n(\mathbf{p} = \frac{m\mathbf{r}}{t})$$



- ⇒ Probing consists in providing density distributions of the atomic cloud, either trapped or in ballistic expansion
- ⇒ All properties of the condensate and thermal cloud are inferred from these density distributions and the comparison with theoretical modeling
- ⇒ Distribution in space determined by the trap potential



1. High temperatures $T \geq T_c$: the distribution can be evaluated in the semi-classical approximation

$$n(\mathbf{r}) = n_T(\mathbf{r}) \simeq \lambda_T^{-3} g_{3/2}(e^{\beta[\mu - V_{\text{ext}}(\mathbf{r})]})$$

1. Low temperatures (pure condensates) $T \ll T_c$

$$n(r) = n_0(\mathbf{r}) \begin{cases} \text{Ideal gas} \\ \text{Tomas-Fermi limit} \end{cases}$$

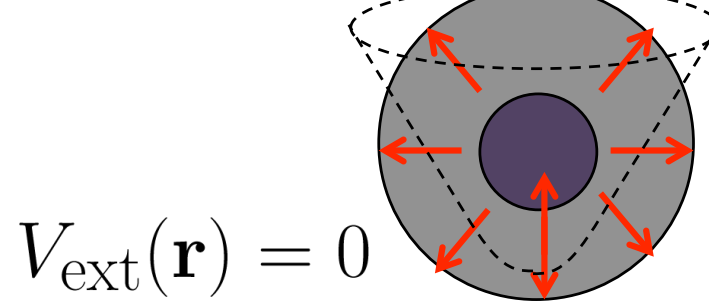
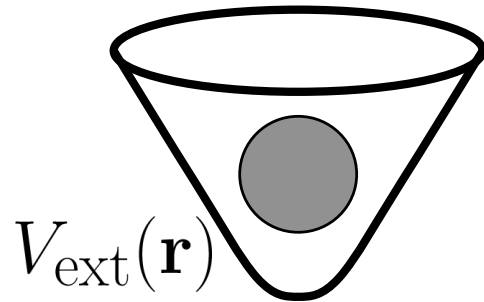
2. Intermediate regime: bimodal distribution $T \lesssim T_c$

$$n(r) = n_0(\mathbf{r}) + n_T(\mathbf{r})$$

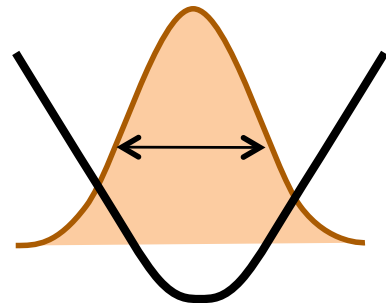
Time of flight measurements

Following lectures

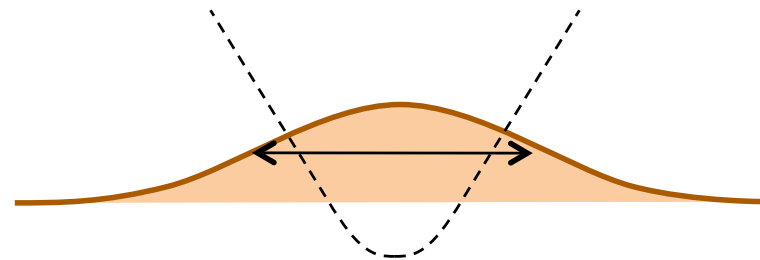
⇒ Much experimental information is obtained by switching off the confining trap and letting the gas free to expand



1. Let's consider the simple case of an ideal gas (Schrödinger equation)



$$p \sim \frac{mr}{t}$$



$$e^{-\frac{1}{2} \left(\frac{r}{a\omega} \right)^2}$$

$$e^{-\frac{1}{2} \left(\frac{pa\omega}{\hbar} \right)^2}$$

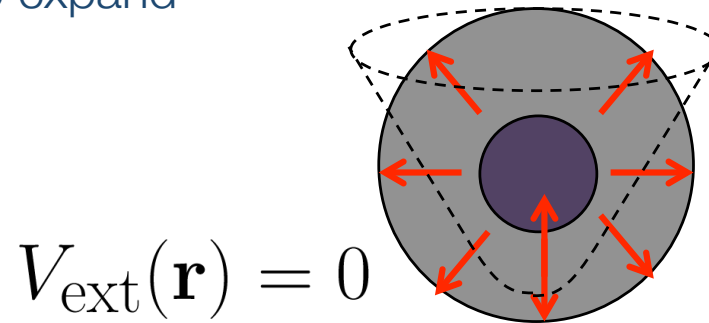
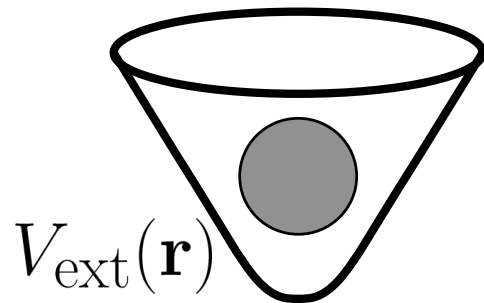
N.B. the phase evolves classically

$$\phi \sim \frac{mr^2}{2\hbar t}$$

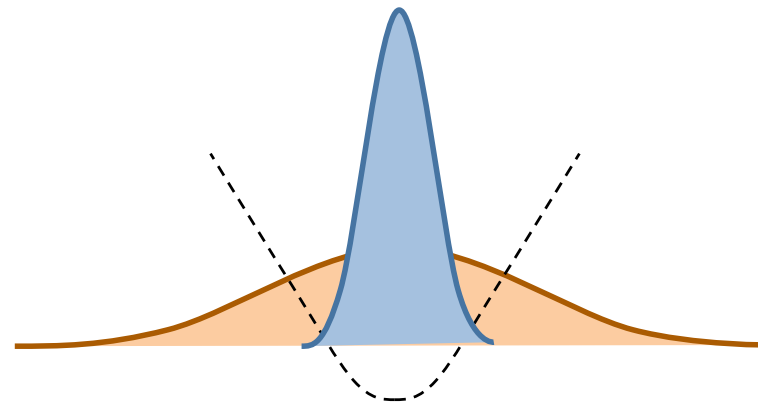
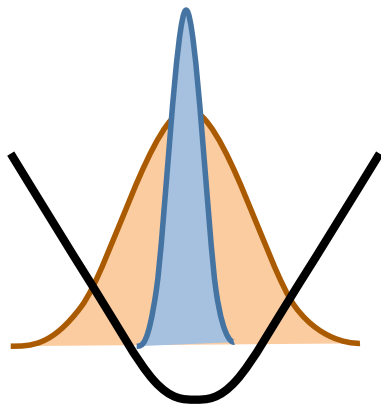
2. For the thermal component, we will explicitly show that

$$\lim_{t\bar{\omega} \gg 1} n(\mathbf{r}, t) \propto n(\mathbf{p} = \frac{m\mathbf{r}}{t})$$

⇒ Much experimental information is obtained by switching off the confining trap and letting the gas free to expand

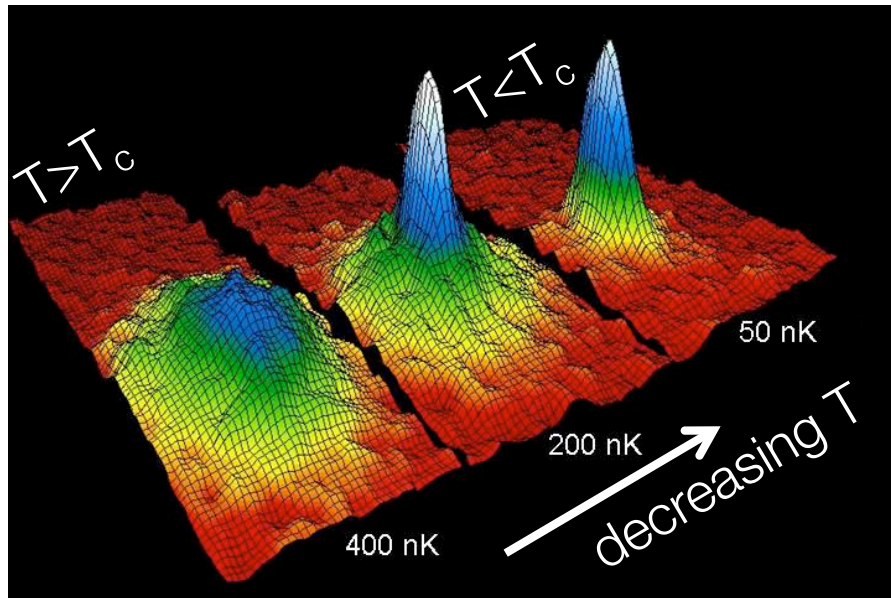


3. Bimodal distribution $n(r) = n_0(\mathbf{r}) + n_T(\mathbf{r})$



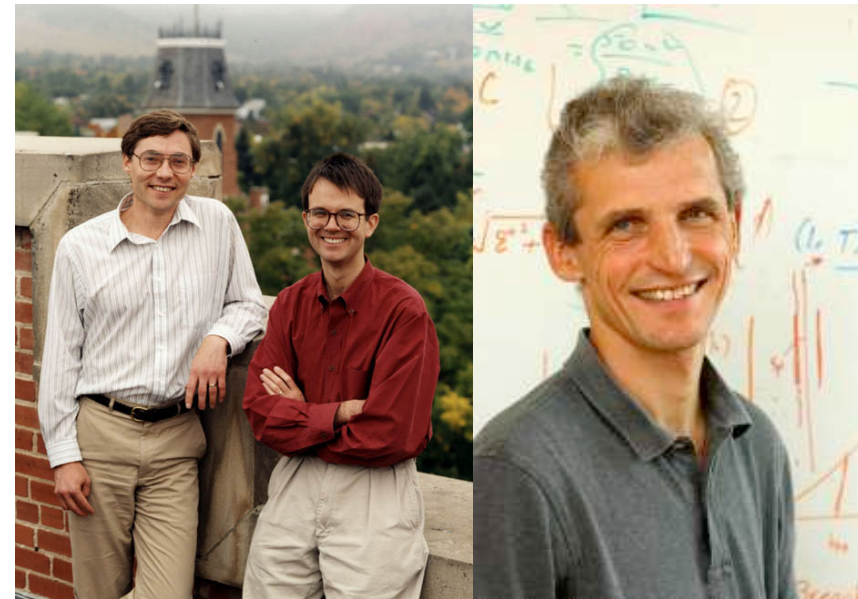
First realisation of a BEC in ultracold atoms

- 1995 BEC in alkali atoms (^{87}Rb , ^{23}Na , ^7Li , ...)



Coollest system in the universe!

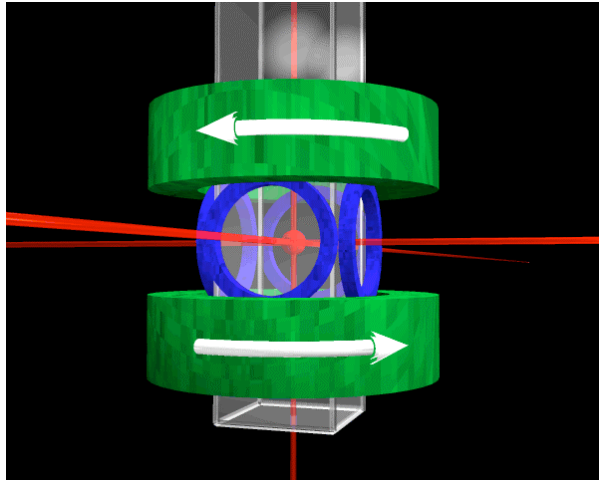
Nobel prize (2001)



Carl Wieman & Eric Cornell Wolfgang Ketterle

$$T \sim 500\text{nK} - \mu\text{K}$$
$$n \sim 10^{11} - 10^{13}\text{cm}^{-3}$$

First BEC in ^{87}Rb (Boulder, June 1995)



[M. H. Anderson *et al.* Science **269**, 198 (1995)]

oven $T \simeq 300\text{K}$
laser cooling $T \simeq 90\mu\text{K}$
 $n \simeq 2 \cdot 10^{10} \text{ cm}^{-3}$

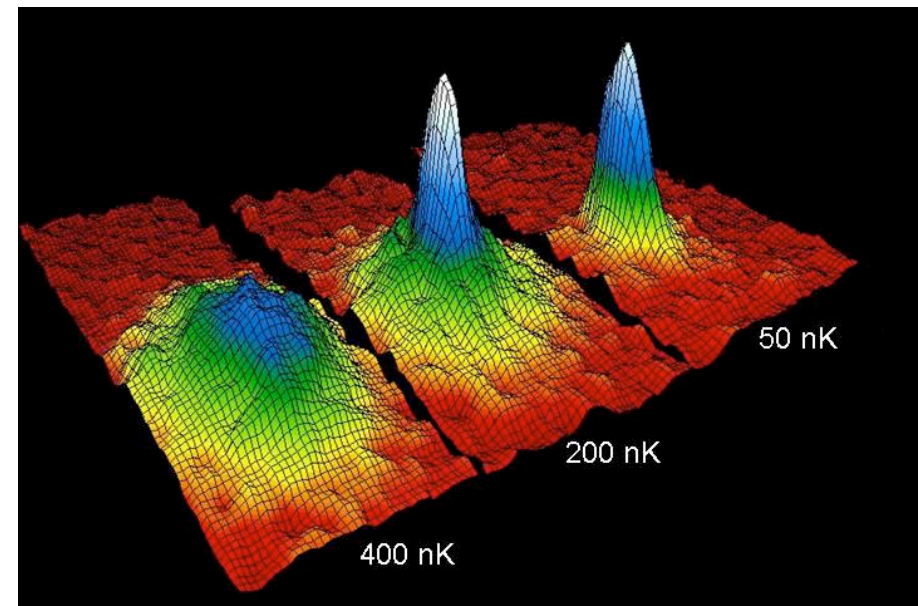
$$|F = 2, m_F = 2\rangle$$

\Rightarrow evaporative cooling in TOP trap

$$T_c \simeq 170\text{nK}$$

$$n \simeq 2.6 \cdot 10^{12} \text{ cm}^{-3}$$

momentum distribution bimodality



expansion & probe

BEC in ^{23}Na (MIT, September 1995)

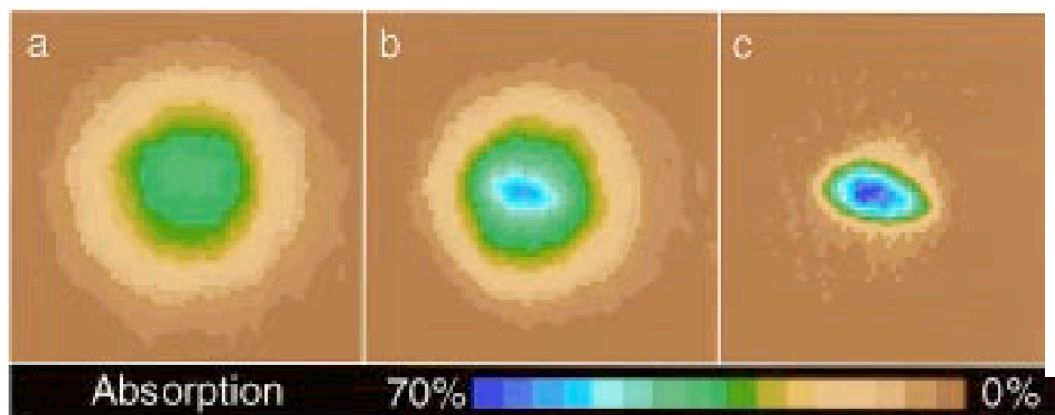
$$|F = 1, m_F = -1\rangle$$

⇒ evaporative cooling in optical plug trap

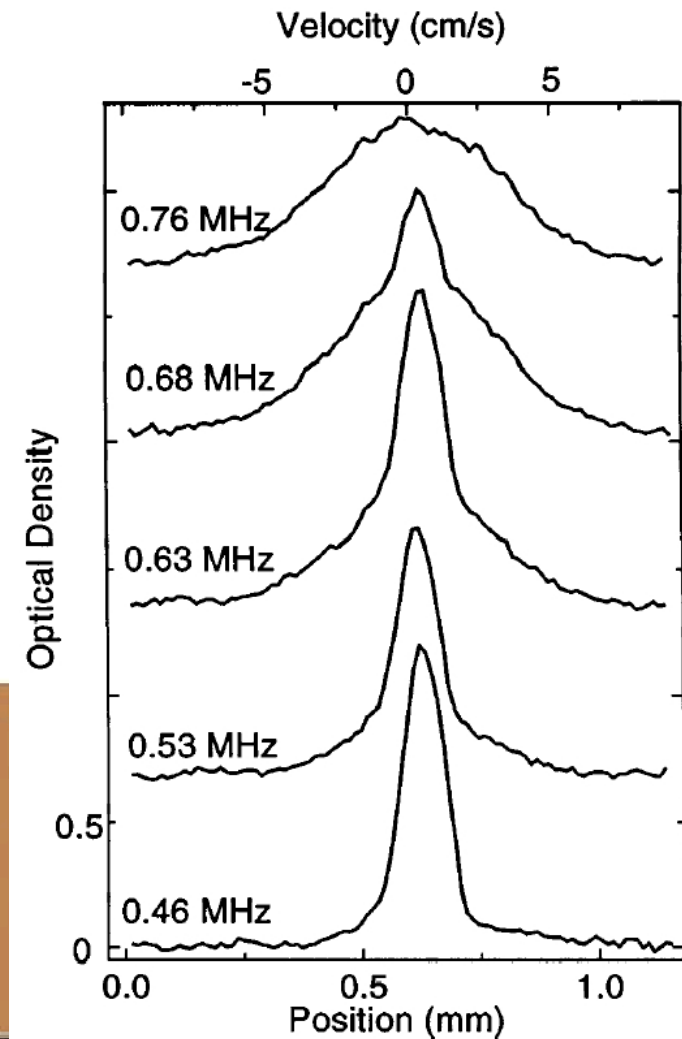
$$T_c \simeq 2.0 \mu\text{K}$$

$$n \simeq 1.5 \cdot 10^{14} \text{ cm}^{-3}$$

- Bimodal distribution
- Non-isotropic velocity distribution



[K. B. Davis *et al.* PRL 75, 3969 (1995)]



BEC in ^{23}Na (MIT, September 1995)

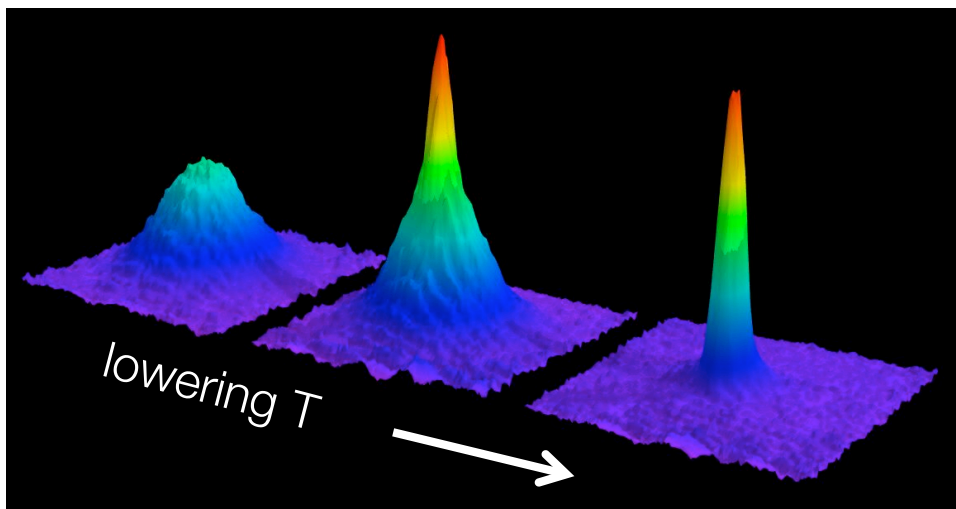
$$|F = 1, m_F = -1\rangle$$

⇒ evaporative cooling in optical plug trap

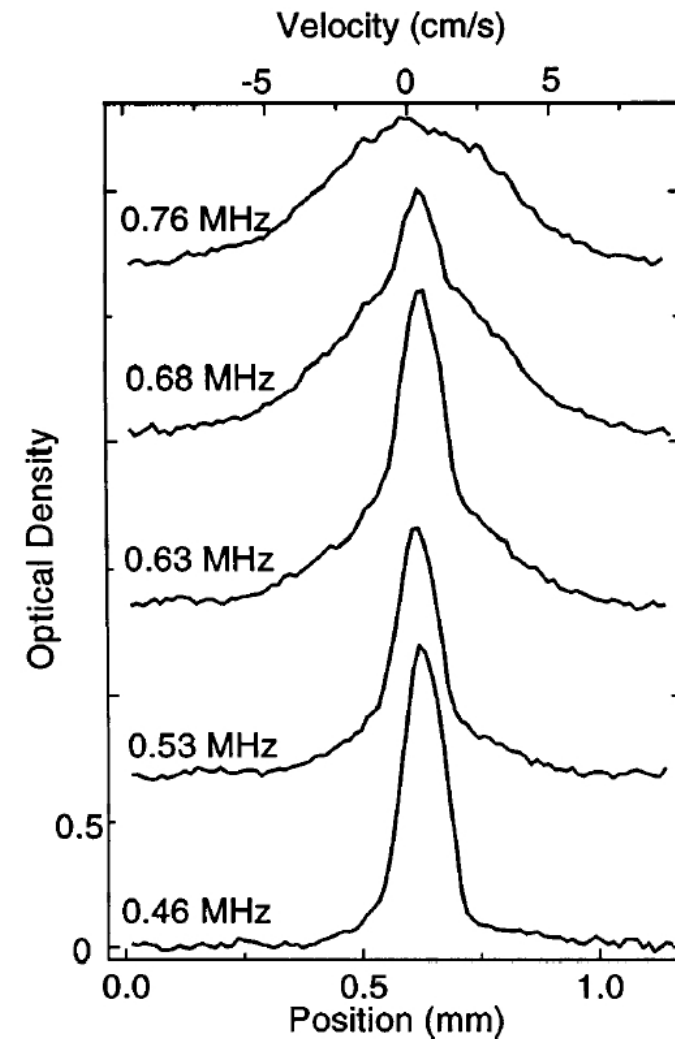
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[K. B. Davis *et al.* PRL 75, 3969 (1995)]



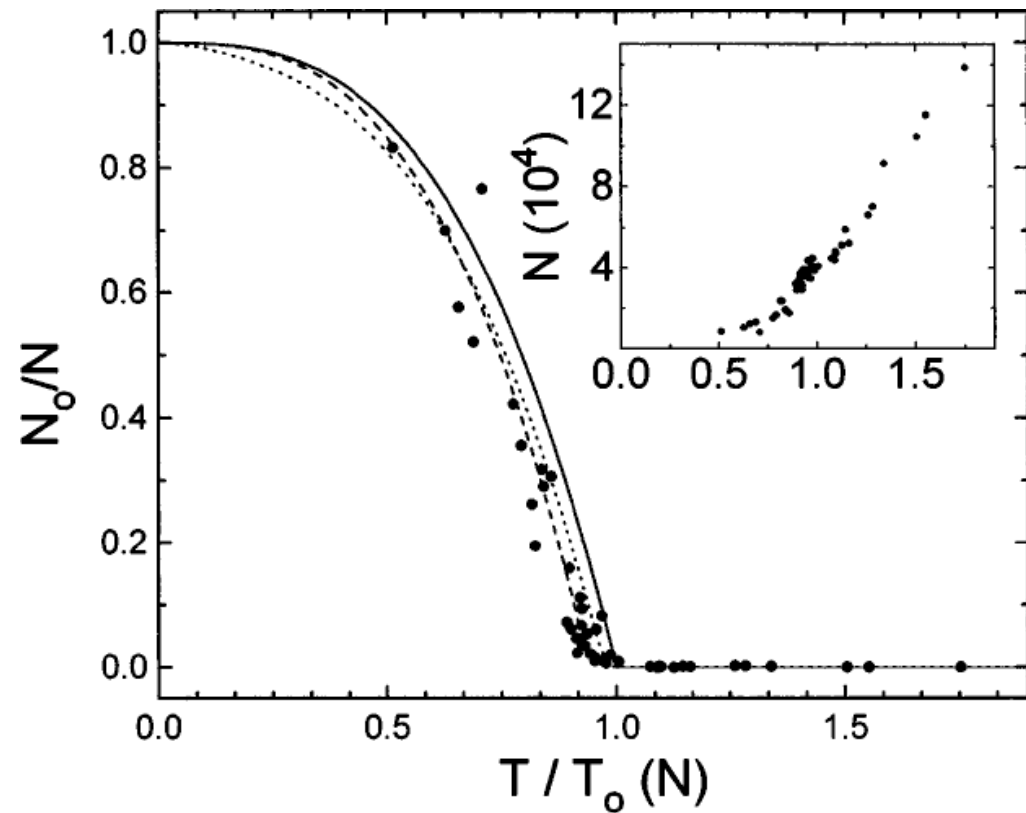
1. Temperature: determined by the shape of the spatial wings of the distribution (thermal cloud)
2. Chemical potential: given by the size of the condensate (Thomas-Fermi approximation)
3. Total number of atoms: integral of either the space or momentum distribution
4. Condensate fraction: bimodal distribution

Condensate fraction

⇒ condensate fraction in a BEC of Rubidium ultracold atoms
(rather good agreement with predictions for a **trapped** ideal Bose gas model)

$$k_B T_c = \frac{\hbar \omega \bar{N}^{1/3}}{[\zeta(3)]^{1/3}}$$

$$n_0 = 1 - \left(\frac{T}{T_c}\right)^3$$

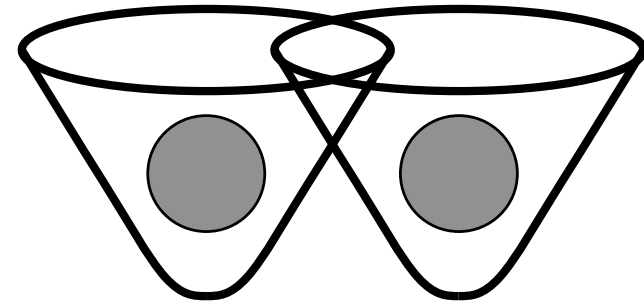


[J. R. Ensher *et al.*, PRL 77, 4984 (1996)]

Macroscopic phase coherence

Following lectures

Interference between two condensates

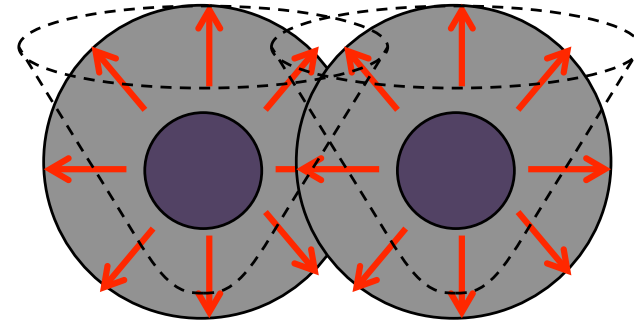


Macroscopic phase coherence

Following lectures

Interference between two condensates

(time of flight evolution)



Macroscopic phase coherence

Following lectures

Interference between two condensates

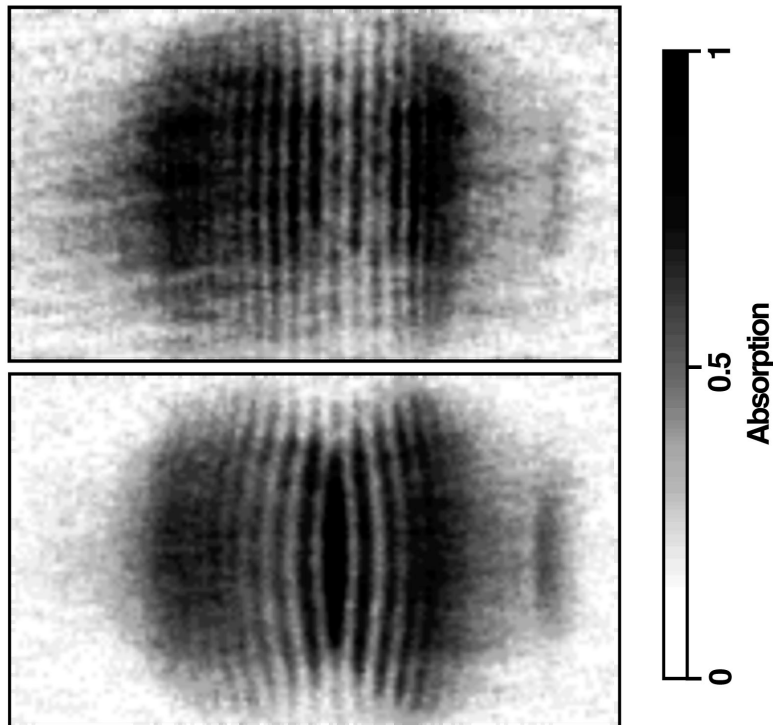
$$t = 40\text{ms}$$

$$d \simeq 40\mu\text{m}$$

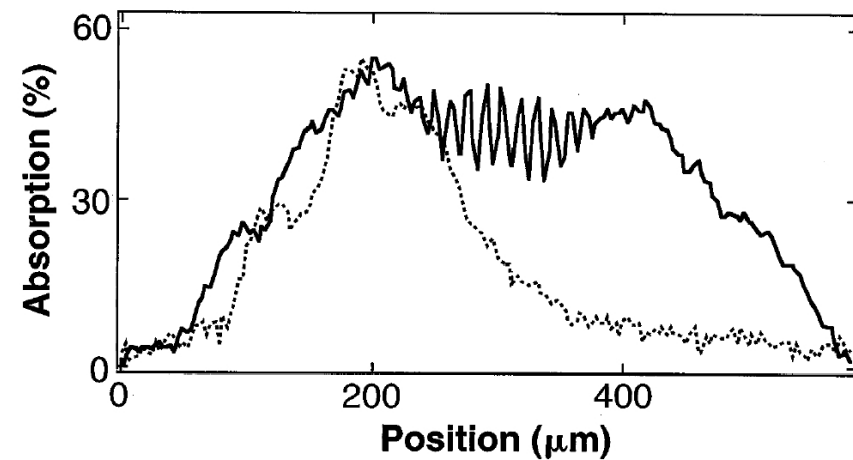
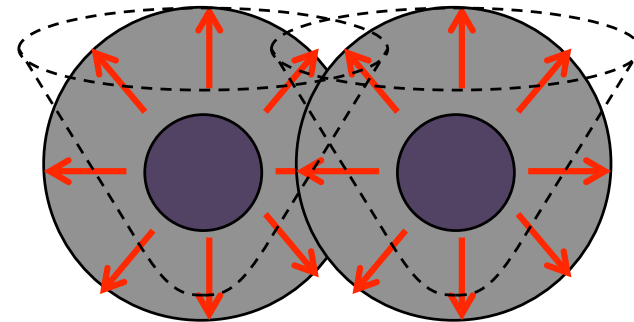
$$\lambda \simeq 20\mu\text{m}$$

$$\lambda = \frac{ht}{md}$$

[M. R. Andrews *et al.* Science **275**, 637 (1997)]



(time of flight evolution)



Macroscopic phase coherence

Following lectures

Interference between two condensates

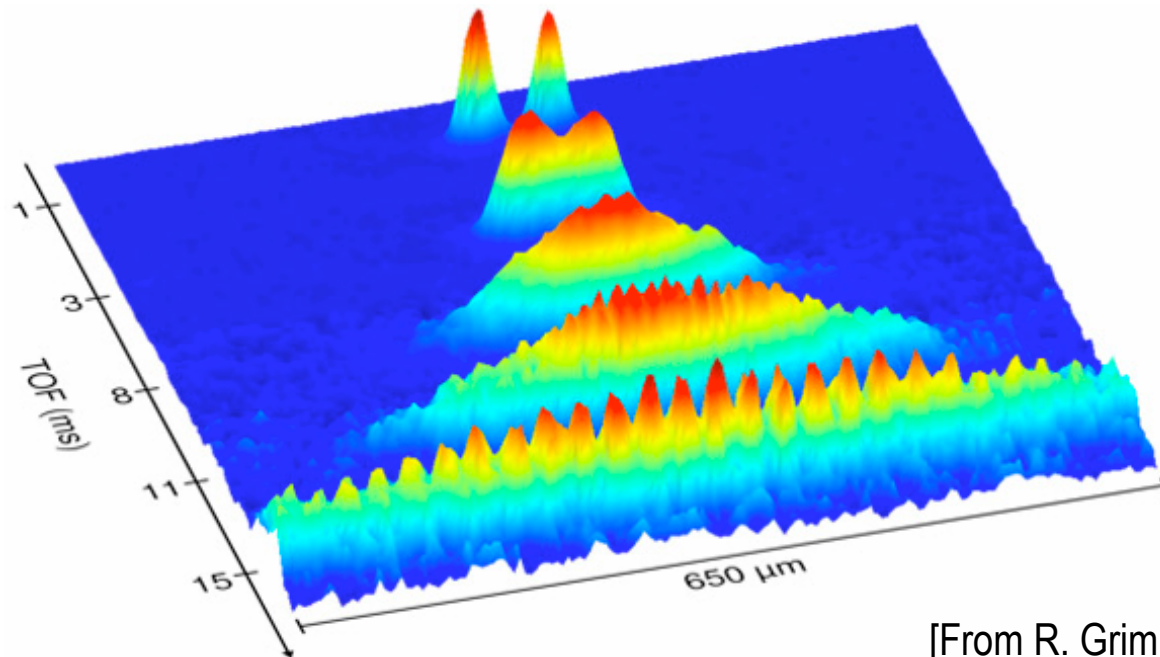
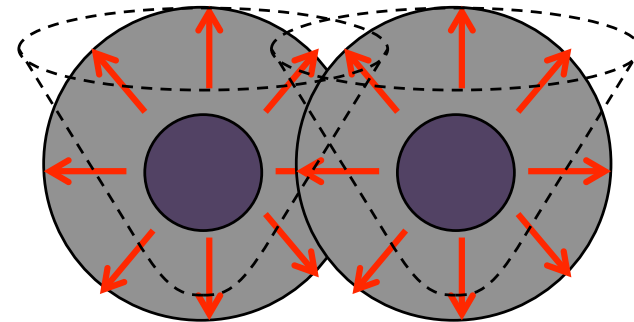
$$t = 40\text{ms}$$

$$d \simeq 40\mu\text{m}$$

$$\lambda \simeq 20\mu\text{m}$$

$$\lambda = \frac{ht}{md}$$

(time of flight evolution)

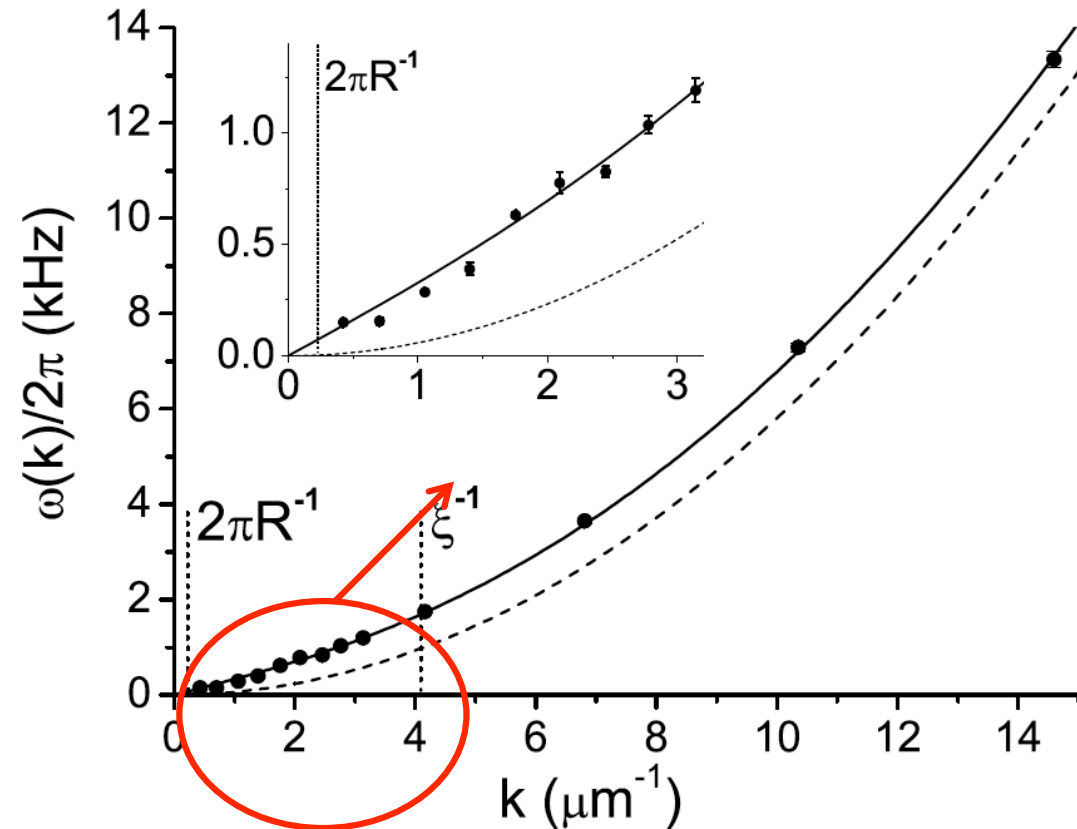


[From R. Grimm's group]

BEC & superfluidity

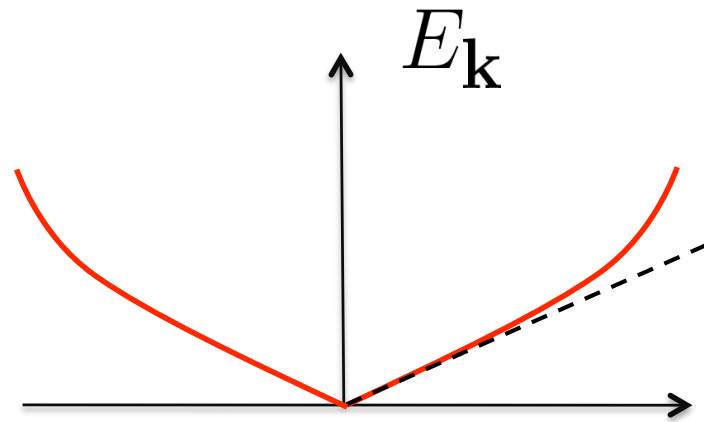
Spectrum of excitations

$$E_p = \sqrt{\epsilon_p(\epsilon_p + 2gn)}$$



[Steinhauer *et al.* PRL (2002)]

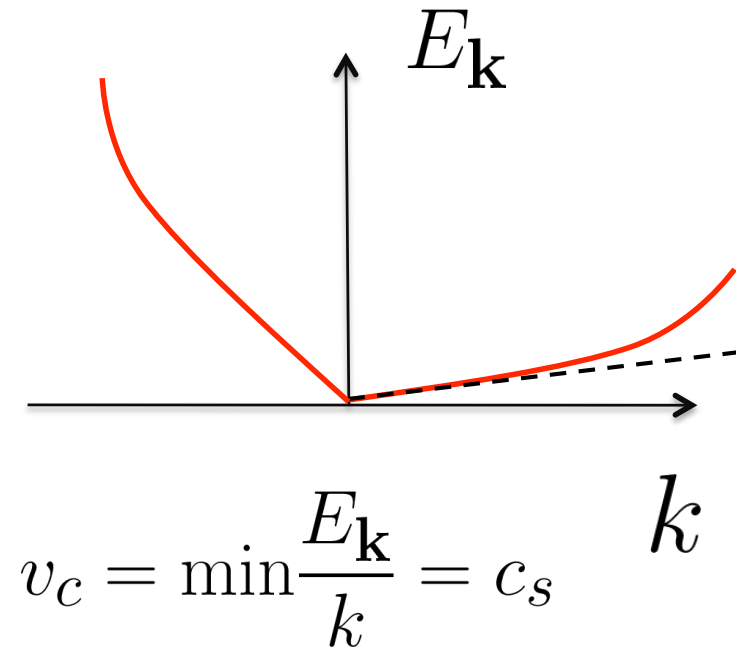
BEC & superfluidity



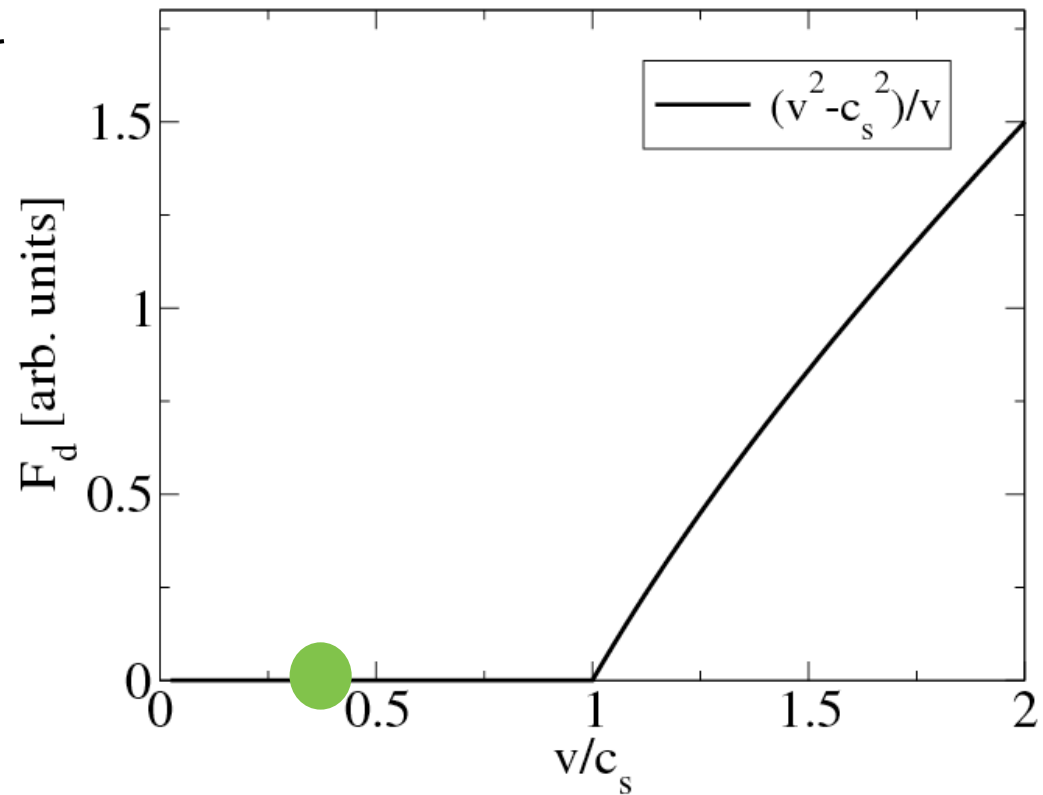
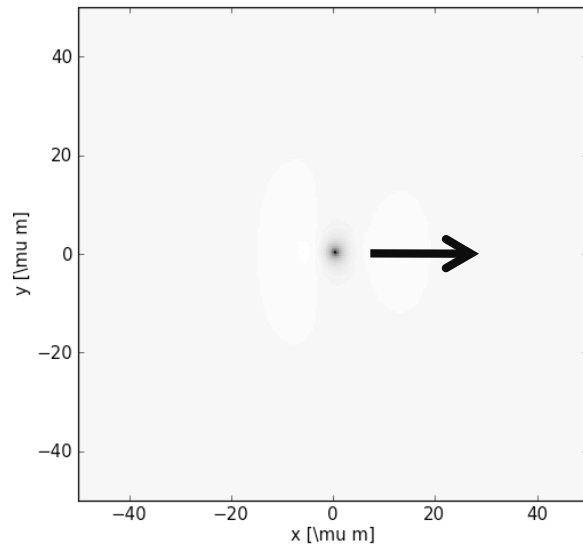
sound
velocity $c_s = \sqrt{\frac{gn}{m}}$

$$v_c = \min \frac{E_{\mathbf{k}}}{k} = c_s$$

BEC & superfluidity



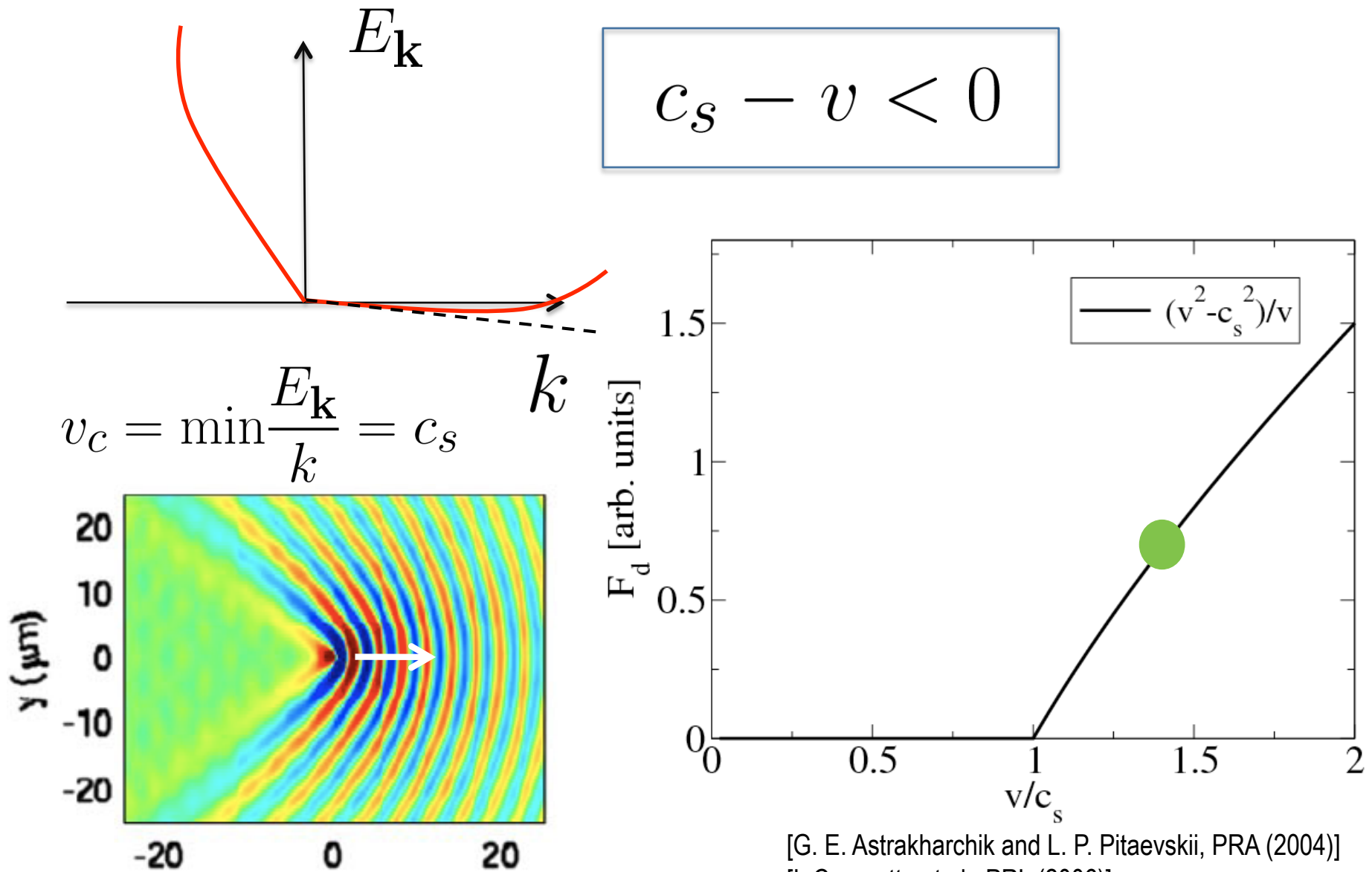
$$c_s - v > 0$$



[G. E. Astrakharchik and L. P. Pitaevskii, PRA (2004)]

[I. Carusotto et al., PRL (2006)]

BEC & superfluidity



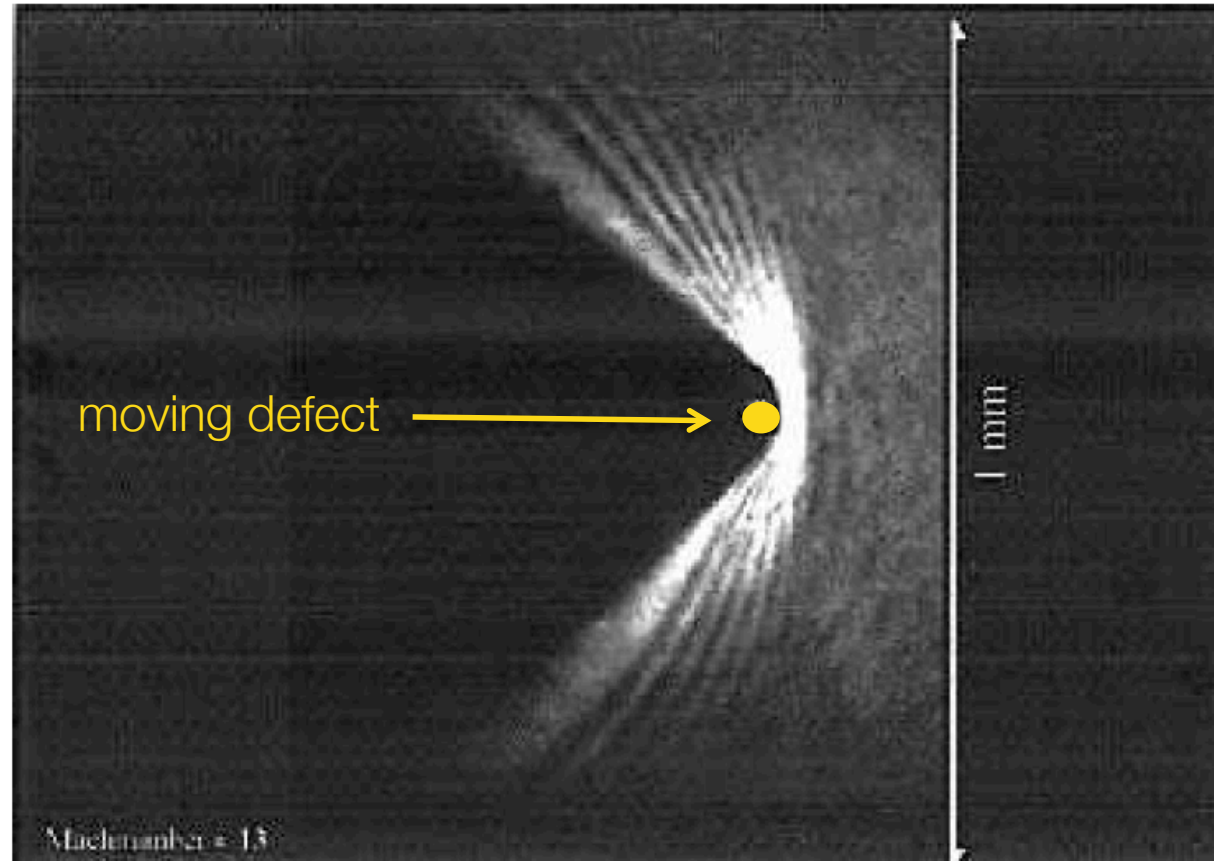
BEC & superfluidity

$$v < v_c$$



BEC & superfluidity

$$v > v_c$$



[from E. Cornell's group]

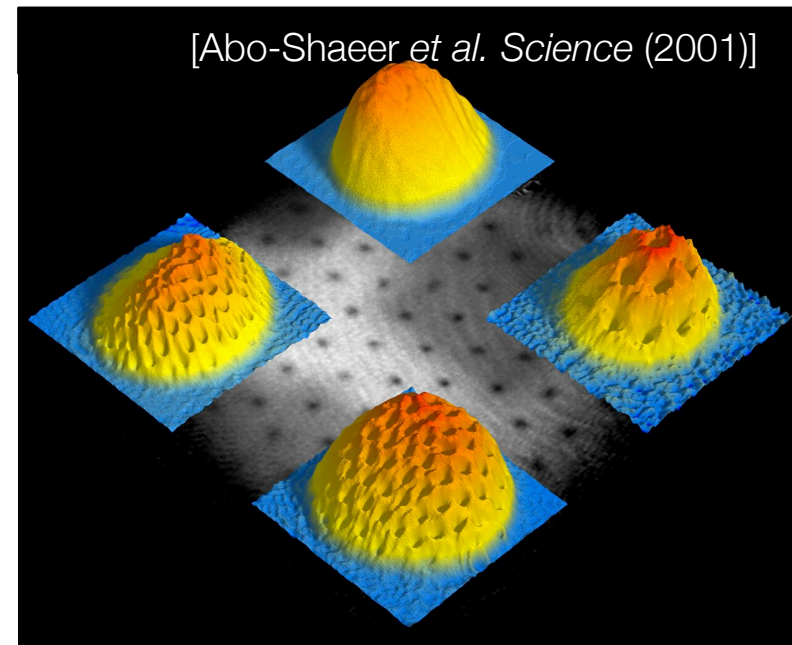
BEC & superfluidity

Quantised vortices in rotating condensates

$$\oint \mathbf{v}_s \cdot d\mathbf{l} = 2\pi \frac{\hbar}{m} (0, \pm 1, \dots)$$

ground state is flowless & vortices need external driving

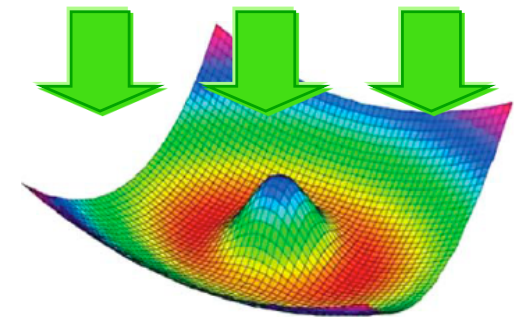
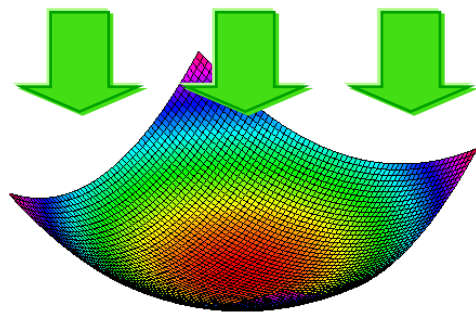
vortices are unstable solutions if rotation is halted



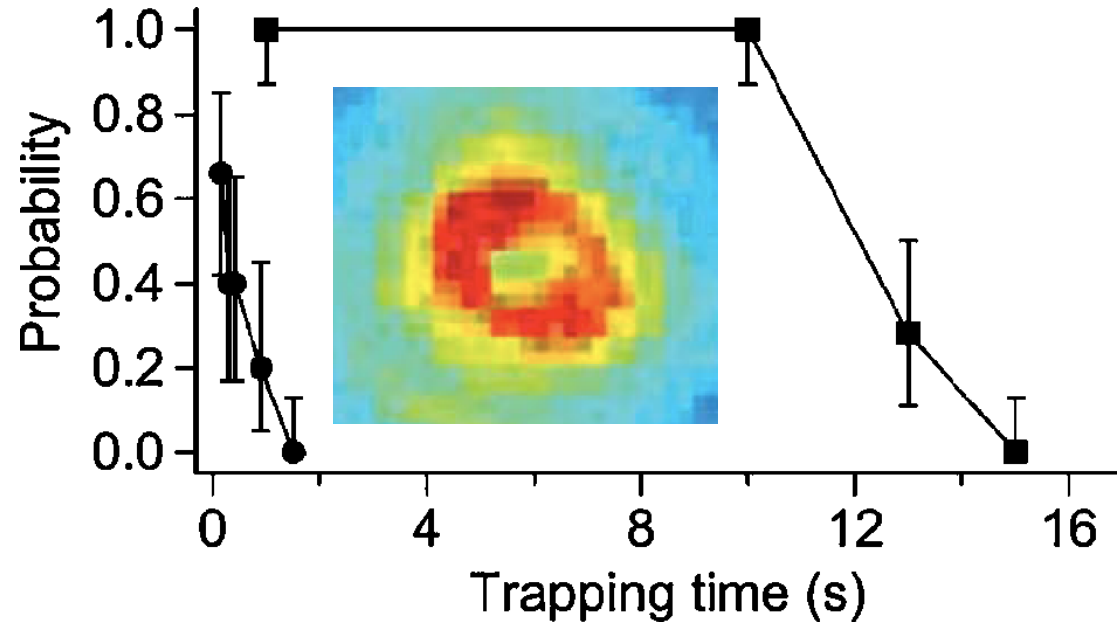
BEC & superfluidity

Metastable persistent flow

Gauss-Laguerre beam
(‘rotating drive’)

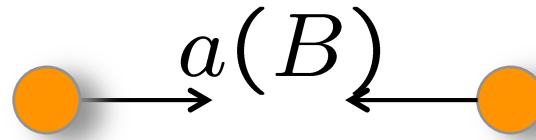


[Ryu *et al.* PRL (2007)]



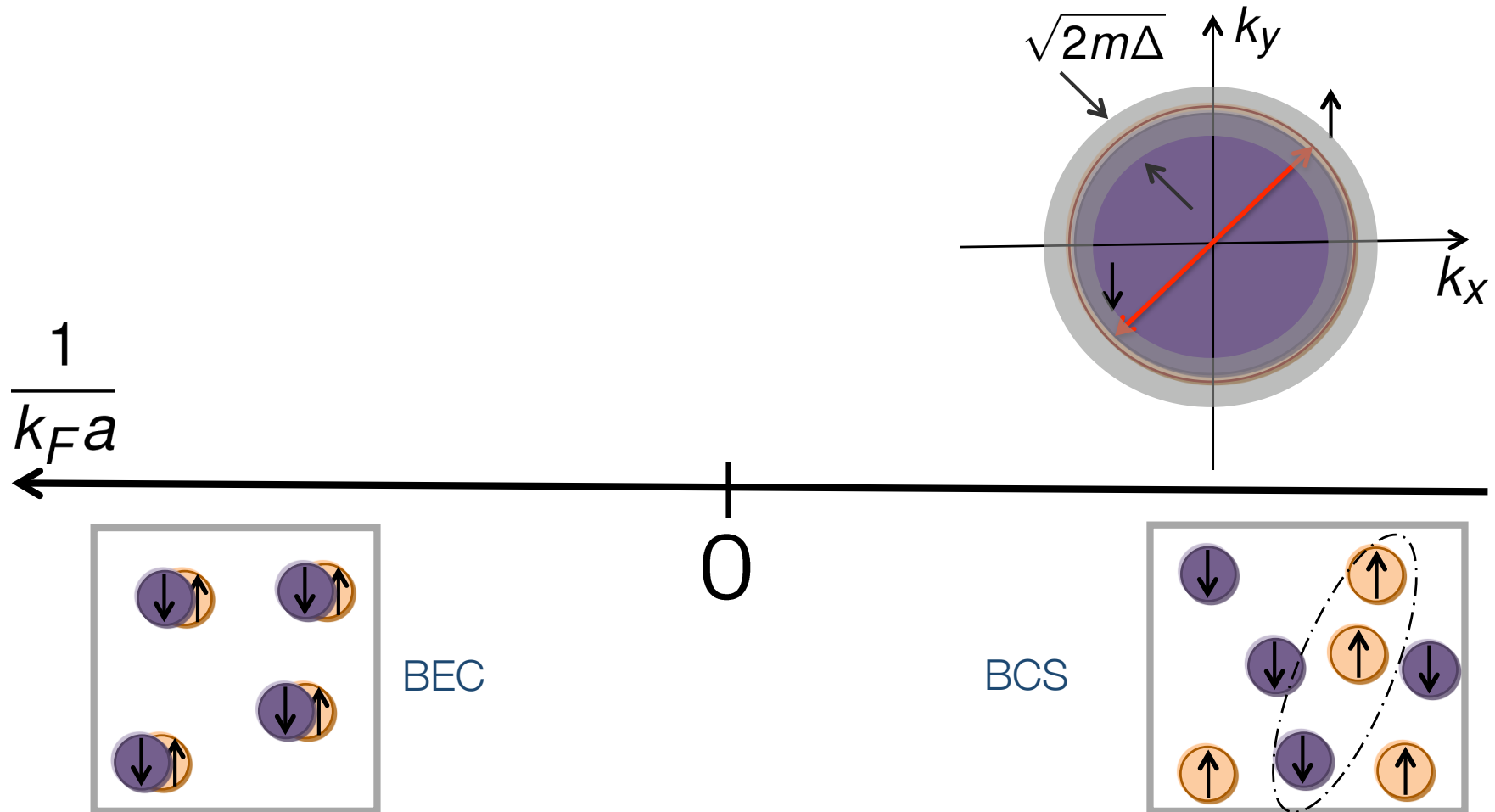
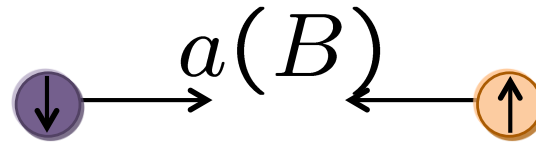
BEC-BCS crossover

- Tune the interaction strength (Feshbach resonances)



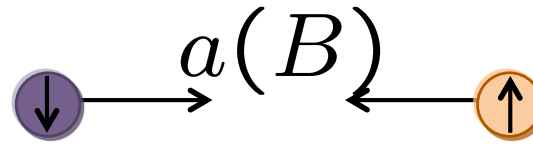
BEC-BCS crossover

- Tune the interaction strength (Feshbach resonances)



Imbalanced Fermi mixtures

- Tune the interaction strength (Feshbach resonances)



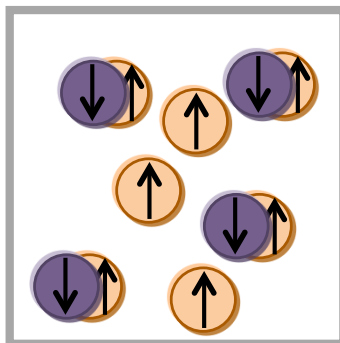
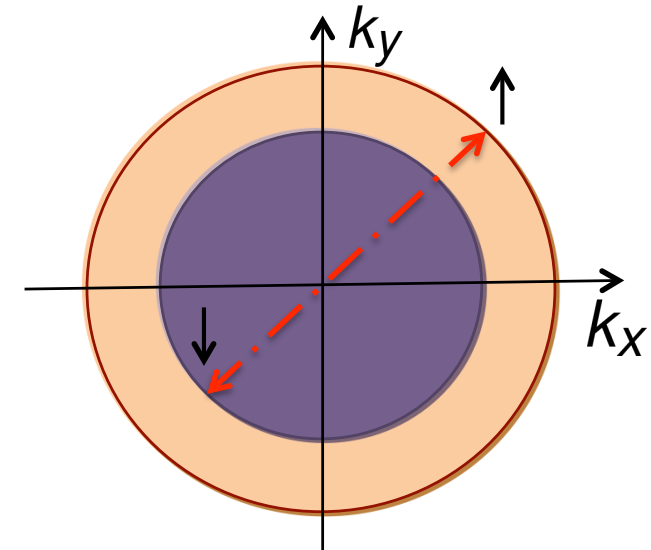
Can superfluidity persist in presence of a population imbalance?

Analogy with a superconductor in a magnetic Zeeman field

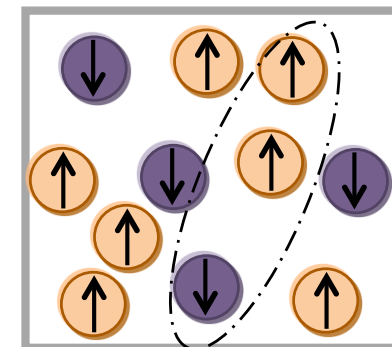
$\frac{1}{k_F a}$

$n_\uparrow - n_\downarrow$

0



BEC



BCS