Early-time dynamics of Bose gases quenched into the strongly interacting regime

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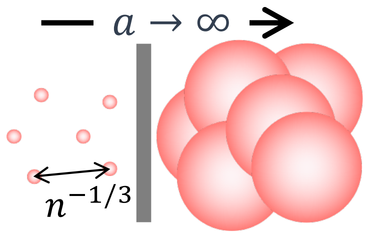
IFIMAC

Tunable interactions (Feshbach resonance)

→ Tune quantum (many-body) correlations

1. Strongly interacting Bose gases
   - → beyond MF corrections
   - → existence, stability, scaling,...

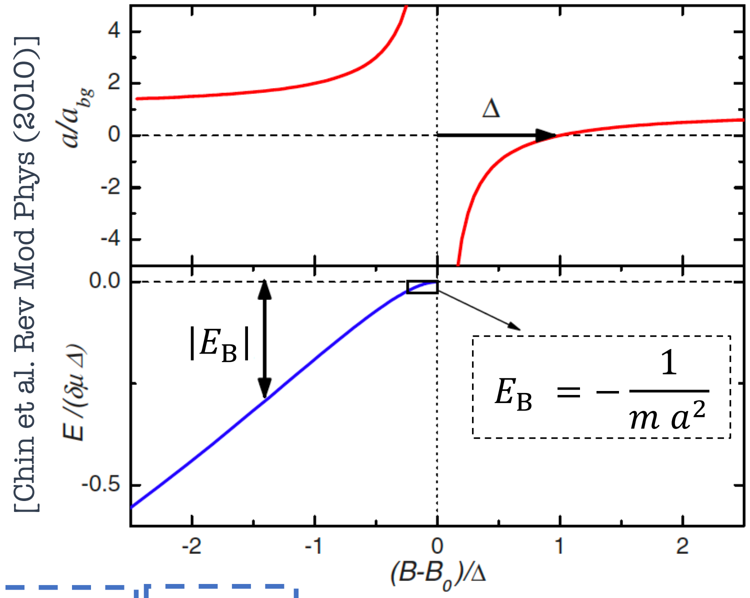


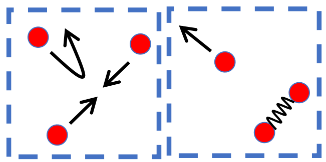
2. Unitary regime $a \to \infty$
   - ▷ Universality (n scales only)?
   - → 3-body phenomena & Efimov physics
   - → non-universality

$$N_{\mathbf{k}} \simeq \frac{C_2}{k^4} + C_3 \frac{F(R^* k)}{k^5} + \ldots$$

[Braaten et al. PRL (2011)]

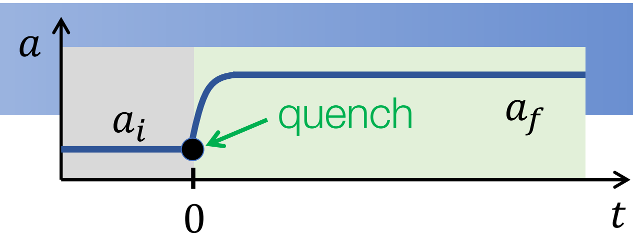
   - → metastability
   - → three-body recombinations: losses & heating





3. Accessing the unitary regime by interaction quenches

Tunable interactions (Feshbach resonance)



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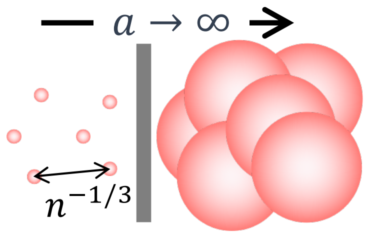

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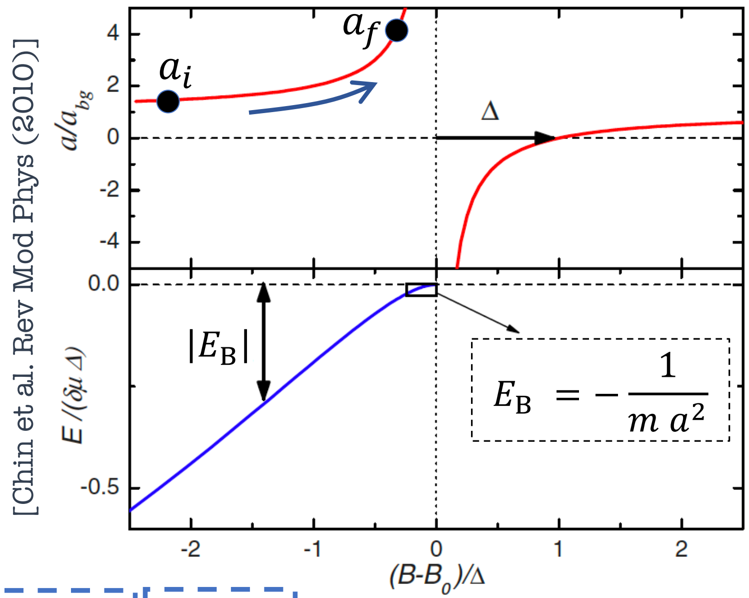
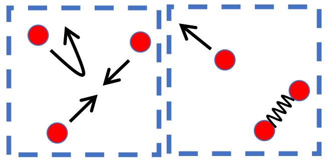
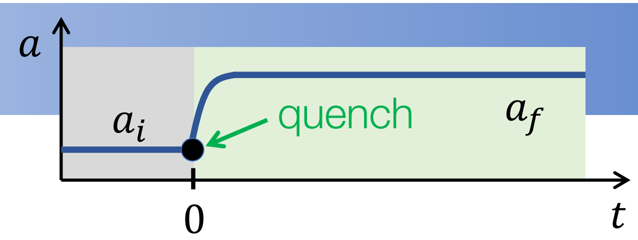


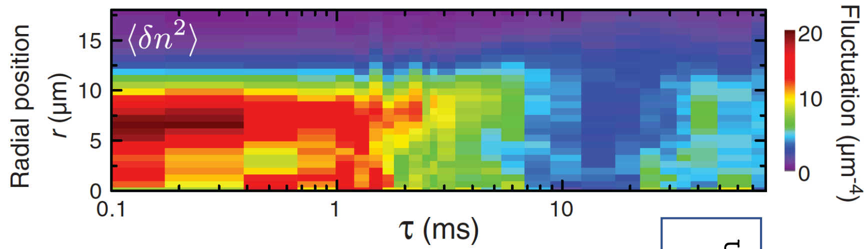


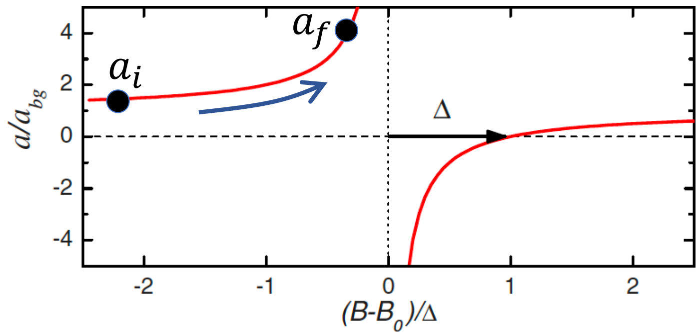
3. Accessing the unitary regime by interaction quenches

Shallow interaction quenches $na_{i,f}^3 \ll 1$



▷ Density-density correlation function oscillations



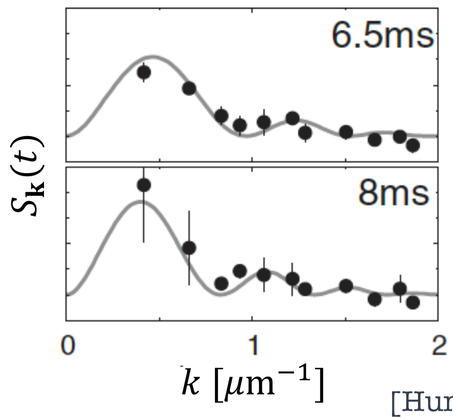


▷ Structure factor

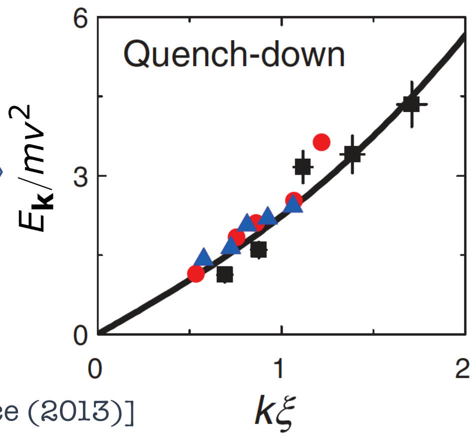
$$S_{\mathbf{k}}(t) \propto \langle \delta n_{-\mathbf{k}}(t) \delta n_{\mathbf{k}}(t) \rangle$$
$$= S_{\mathbf{k}}(0) + A_{\mathbf{k}} \sin^2(E_{\mathbf{k}} t)$$

→ analog of cosmological Sakharov oscillations

Fourier transform

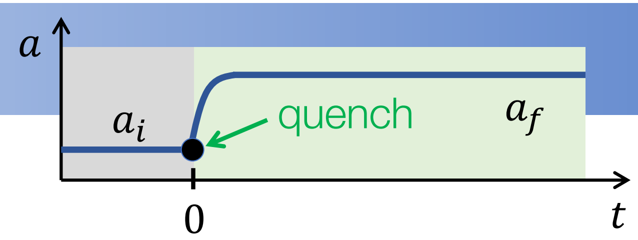


Bogoliubov spectrum

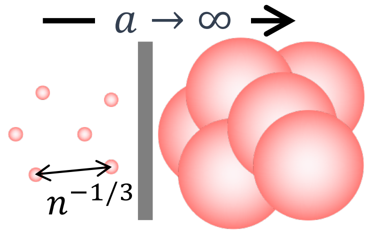


[Hung et al. Science (2013)]

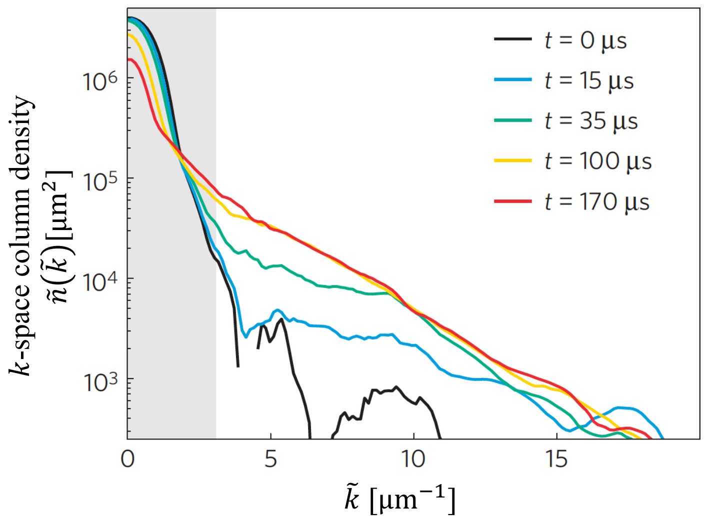
Deep quenches $na_i^3 \ll 1 \to na_f^3 \sim 1$

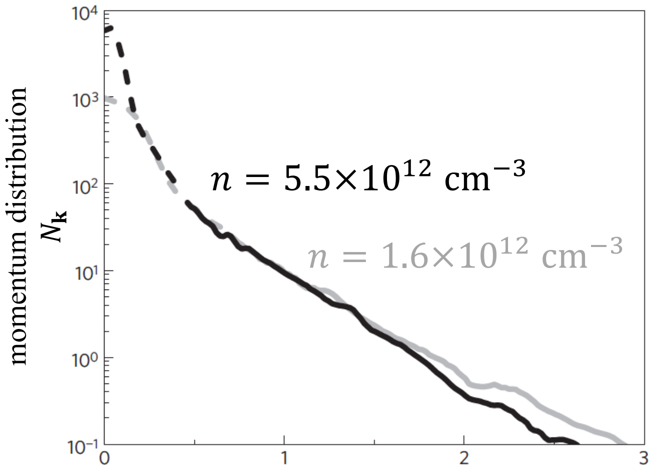


1. Role & nature of 3-body losses?
2. Universality? $k_n = (6\pi^2 n)^{\frac{1}{3}}$ $\epsilon_n = t_n^{-1} = \frac{k_n^2}{2m}$
3. [Boulder experiment]



- ▷ dynamics saturates to prethermal steady-state
- ▷ universal momentum distribution
  - → no time for 3-body correlations to build up





[Makotyn et al. Nature Phys (2014)]

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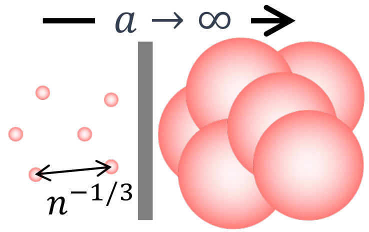
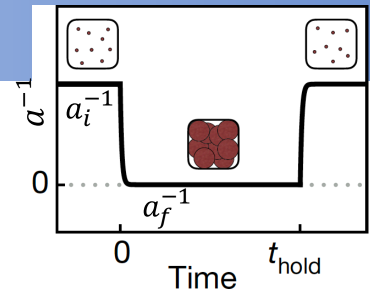
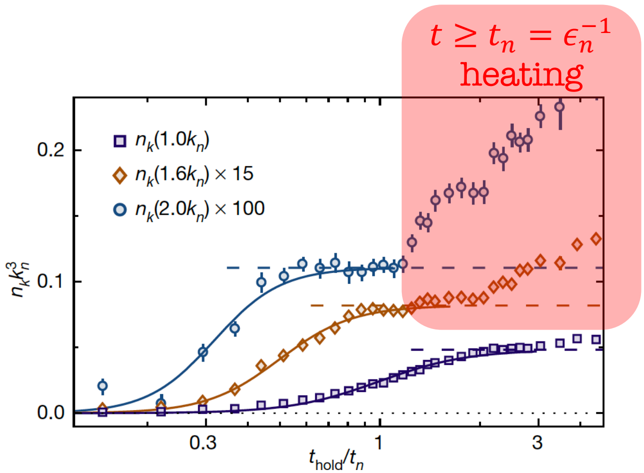
3. [Cambridge experiment]
   - ▷ universal pre-thermal post-quench dynamics





$k_n \simeq 7\ \mu m^{-1}$
$t_n = \epsilon_n^{-1} \simeq 30\mu s$



[Eigen et al. Nature (2018)]

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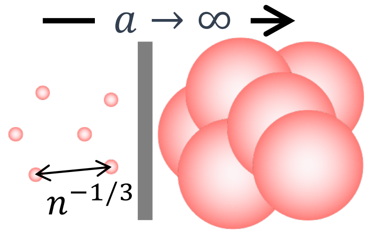

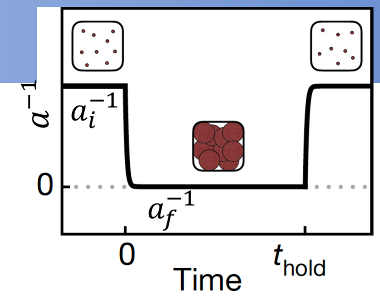

- ▷ universal pre-thermal post-quench dynamics
  - → universal scaling laws of momentum distribution growth time $\tau_{\text{gr}}(k)$
  - → quasiparticle excitations Bogoliubov-like

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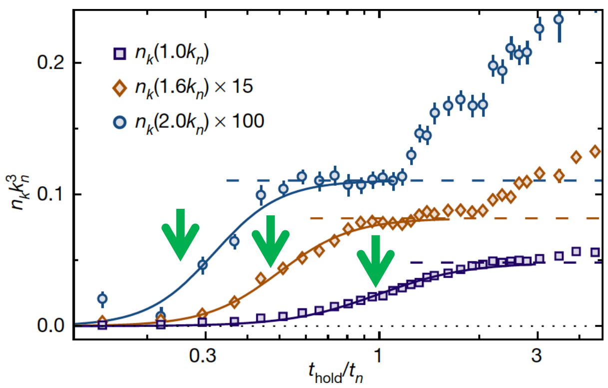

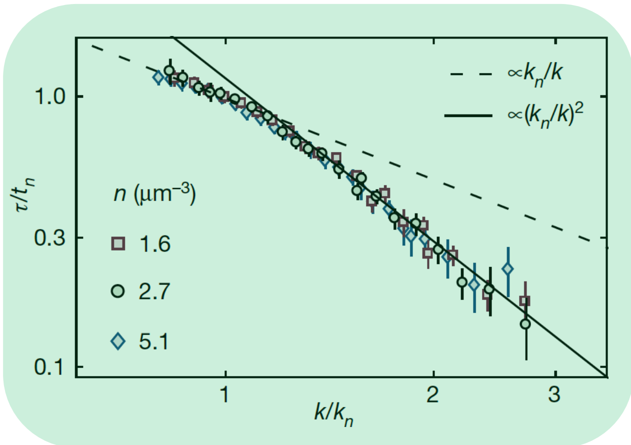

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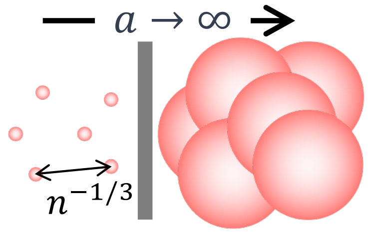

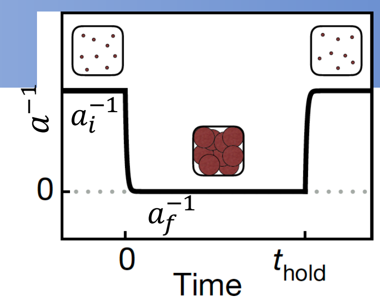

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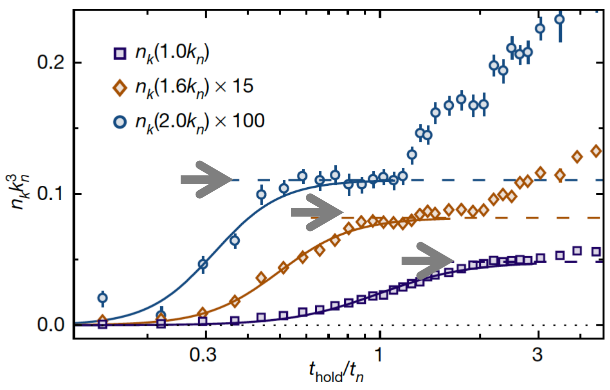
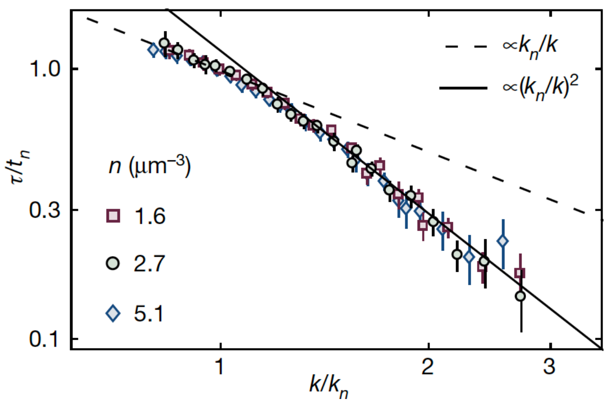
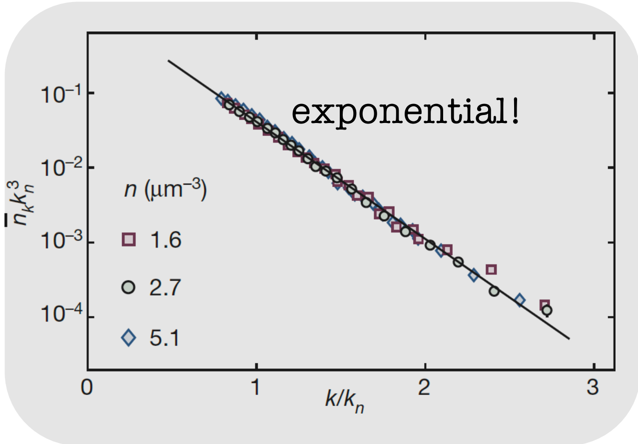


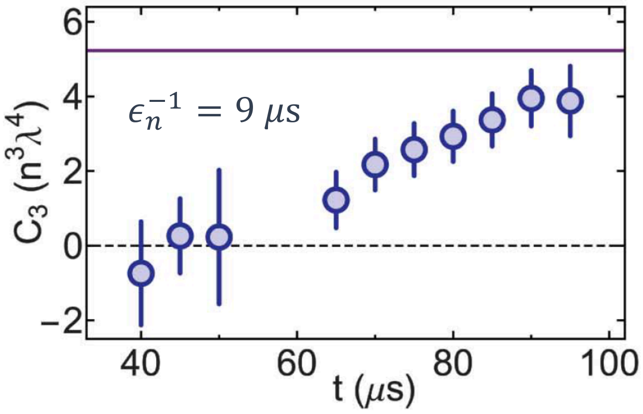




[Eigen et al. Nature (2018)]

What about 3-body & Efimov physics?

1. Degenerate gas exp [Eigen et al. Nature (2018)]
   → universal lossless dynamics up to $t \lesssim \epsilon_n^{-1}$
2. Thermal gas exp
   → $C_3(t)$ negligible up to $t \lesssim 5\epsilon_n^{-1}$



[Fletcher et al. Science (2017)]

▷ Efimov physics important only at later times of the dynamics
→ short-times dominated by short-distance pairwise processes

3. Our work (outline)
   - ▷ variational formalism including pairwise excitations out of the condensate
     → condensate depletion + correlations between non-condensed atoms
   - ▷ crossover from shallow to deep quenches
     → from coherent atom-molecule to atom-medium oscillations
   - ▷ universal scaling behaviour of the typical growth time of the momentum distribution

Model & results

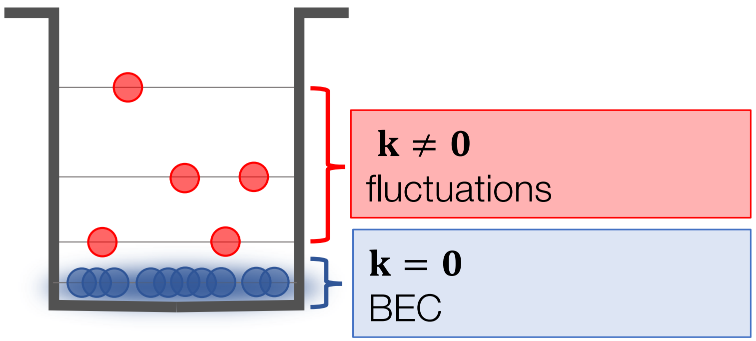
Modelling a Feshbach resonance

1. Short-range pseudopotential
$$\hat{H} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} \hat{a}^\dagger_{\mathbf{k}} \hat{a}_{\mathbf{k}} + \frac{U_\Lambda}{2V} \sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}} \hat{a}^\dagger_{\mathbf{k}_1+\mathbf{q}} \hat{a}^\dagger_{\mathbf{k}_2-\mathbf{q}} \hat{a}_{\mathbf{k}_2} \hat{a}_{\mathbf{k}_1}$$
   - ▷ s-wave scattering length
$$\frac{m}{4\pi a} = \frac{1}{U_\Lambda} + \frac{1}{V} \sum_{\mathbf{k}}^{\Lambda} \frac{1}{2\epsilon_{\mathbf{k}}}$$
   - ▷ molecular bound state ($a > 0$) for contact potential ($r_0 \sim \Lambda^{-1} \to 0$)
$$E_{\mathrm{B}} = -\frac{1}{ma^2}$$

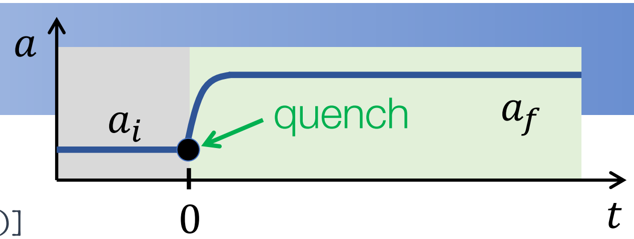
2. BEC ($\hat{a}_0 \delta_{\mathbf{k},\mathbf{0}}$) + excited states ($\hat{a}_{\mathbf{k}\neq\mathbf{0}}$)

   → quantum depletion not negligible

$$\hat{H} = \hat{H}_0 + \hat{H}_2 + \hat{H}_3 + \hat{H}_4$$
$$\hat{H}_2 \mapsto \hat{a}^\dagger_{\mathbf{k}} \hat{a}_{\mathbf{k}},\ \hat{a}^\dagger_{\mathbf{k}} \hat{a}^\dagger_{-\mathbf{k}},\ \hat{a}_{-\mathbf{k}} \hat{a}_{\mathbf{k}}$$
$$\hat{H}_3 \mapsto \hat{a}^\dagger_{\mathbf{k}-\mathbf{q}} \hat{a}^\dagger_{\mathbf{q}} \hat{a}_{\mathbf{k}},\ \hat{a}^\dagger_{\mathbf{k}} \hat{a}_{\mathbf{q}} \hat{a}_{\mathbf{k}-\mathbf{q}}$$
(Beliaev decay, Landau damping)
$$\hat{H}_4 \mapsto \hat{a}^\dagger_{\mathbf{k}_1+\mathbf{q}} \hat{a}^\dagger_{\mathbf{k}_2-\mathbf{q}} \hat{a}_{\mathbf{k}_2} \hat{a}_{\mathbf{k}_1}$$



Early-time dynamics after a quench $a_i \to a_f$



1. Pairwise excitations

[Nozieres & Saint James J Phys France (1982)]
[Yin & Radzihovsky PRA (2013), PRA (2016)]
[Sykes et al. PRA (2014)], [Corson & Bohn PRA (2015)]
[Colussi et al. PRA (2018)]

$$|\psi(t)\rangle \propto \exp\left(\sqrt{V}c_0(t)\hat{a}_0^\dagger + \frac{1}{2}\sum_{\mathbf{k}} g_{\mathbf{k}}(t)\hat{a}_{\mathbf{k}}^\dagger \hat{a}_{-\mathbf{k}}^\dagger\right)|0\rangle$$

$\rightarrow$ $\langle \hat{H}_2\rangle \neq 0, \; \langle \hat{H}_3\rangle = 0, \; \langle \hat{H}_4\rangle \neq 0$

$\rightarrow$ separation of time scales

$$N_0(t) = |\langle \hat{a}_0\rangle|^2 = V|c_0(t)|^2$$

$$N_{\mathbf{k}}(t) = \langle \hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}}\rangle = \frac{|g_{\mathbf{k}}(t)|^2}{1-|g_{\mathbf{k}}(t)|^2}$$

$$x_{\mathbf{k}}(t) = \langle \hat{a}_{\mathbf{k}} \hat{a}_{-\mathbf{k}}\rangle = \frac{g_{\mathbf{k}}(t)}{1-|g_{\mathbf{k}}(t)|^2}$$

momentum occupation numbers & pairing term

2. Equations of motion [Van Regemortel et al. PRA (2018)]

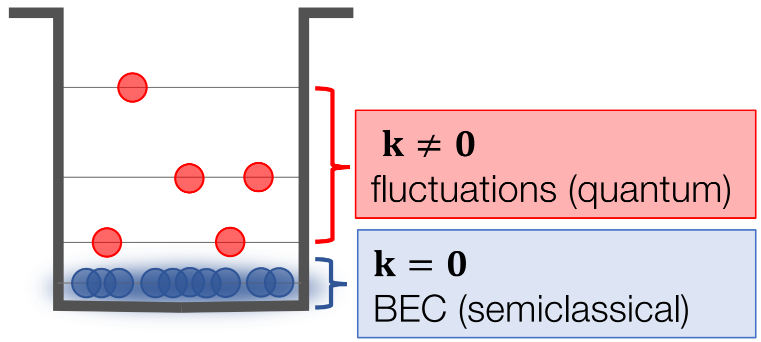
$$i\dot{c}_0 = F(c_0, g_{\mathbf{k}_i})$$

$$i\dot{g}_{\mathbf{k}_i} = G(c_0, g_{\mathbf{k}_j})$$

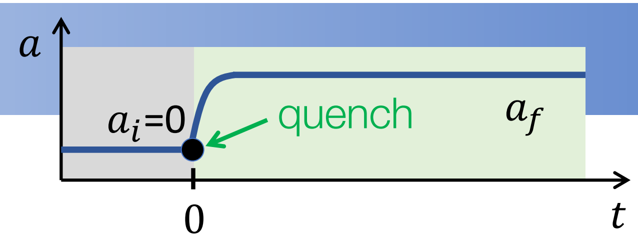
$\rightarrow$ initial conditions (a_i) + evolution (a_f)

$\rightarrow$ conserve total density $n = \underbrace{|c_0(t)|^2}_{n_0(t)} + \underbrace{\frac{1}{V}\sum_{\mathbf{k}} N_{\mathbf{k}}(t)}_{n_{ex}(t)}$

$\rightarrow$ equivalent to second order cumulant expansion or HFB



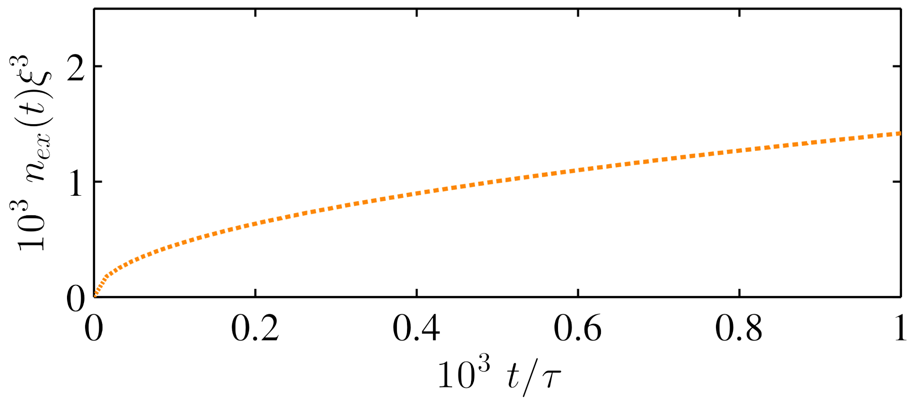
Length & time scales for early dynamics



▷ shallow quenches ($a_f k_n \ll 1$)

→ healing-length ξ & mean-field time τ

density of excited particles $n_{ex}(t)$



^{39}K gas at $n = 10^{12}\ \mathrm{cm}^{-3}$

	$ak_n = (6\pi^2 na^3)^{1/3}$	$\xi = (8\pi an)^{-1/2}$	$\tau = m/(4\pi an)$
shallow	2.1×10^{-2}	3 μm	9 ms

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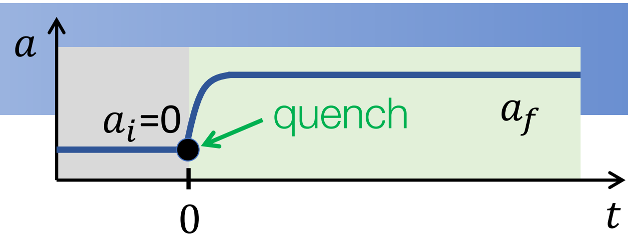

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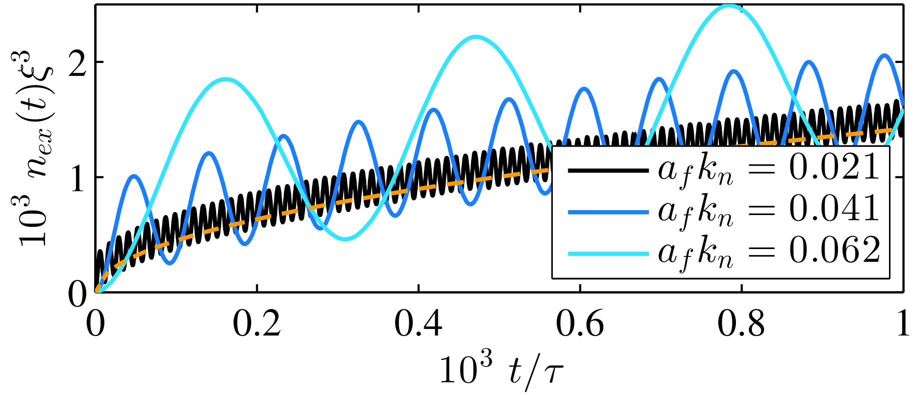

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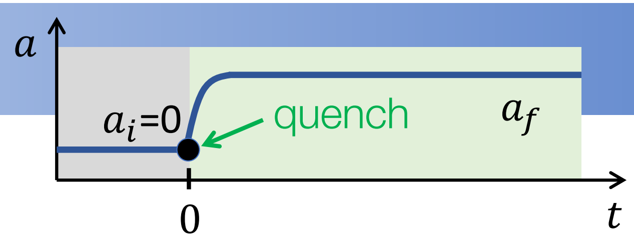
shallow	2.1×10^{-2}	3 μm	9 ms	20 ns

Length & time scales for early dynamics



- shallow quenches ($a_f k_n \ll 1$)
  - → healing-length ξ & mean-field time τ
- intermediate/deep quenches ($a_f k_n \sim 0.1$)
  - → universal atom-molecule coherent oscillations $T = 2\pi/|E_{\rm B}|$

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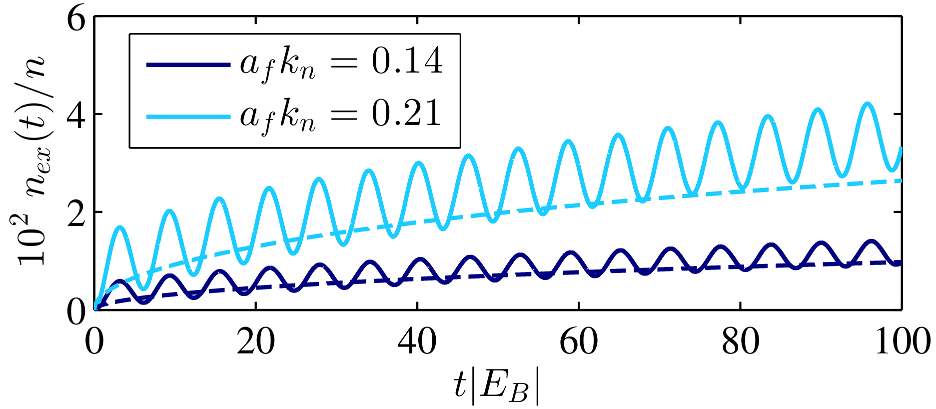

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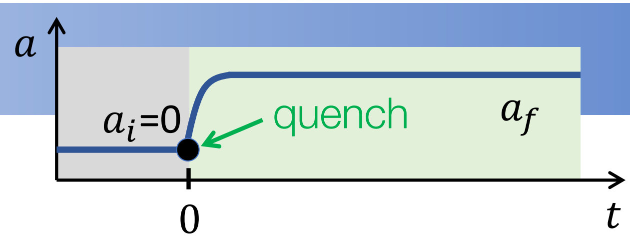
shallow	2.1×10^{-2}	3 μm	9 ms	20 ns	0.3 μm	80 μs
deep	2.1×10^{-1}	0.9 μm	0.9 ms	2 μs		

Length & time scales for early dynamics



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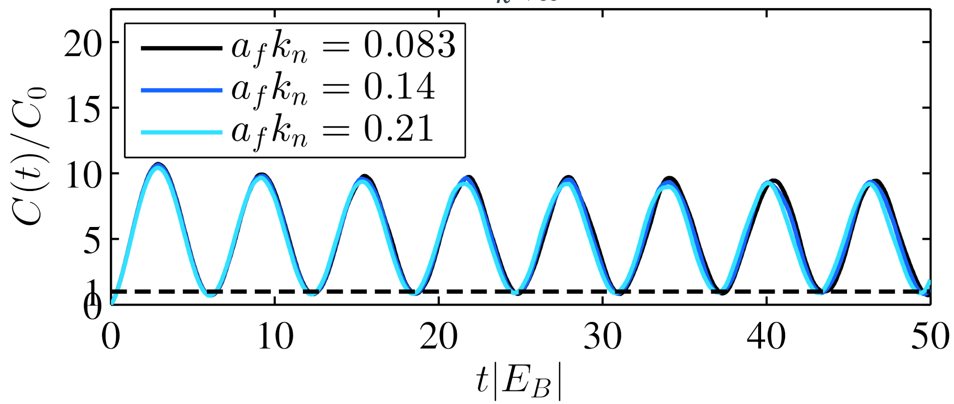
Tan's contact $C(t) = \lim_{k\to\infty} N_\mathbf{k}(t)$



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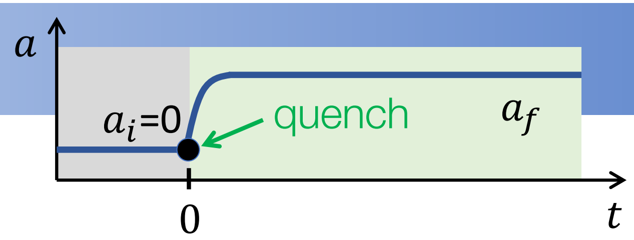
shallow	2.1×10^{-2}	3 μm	9 ms	20 ns		
deep	2.1×10^{-1}	0.9 μm	0.9 ms	2 μs	0.3 μm	80 μs

Length & time scales for early dynamics



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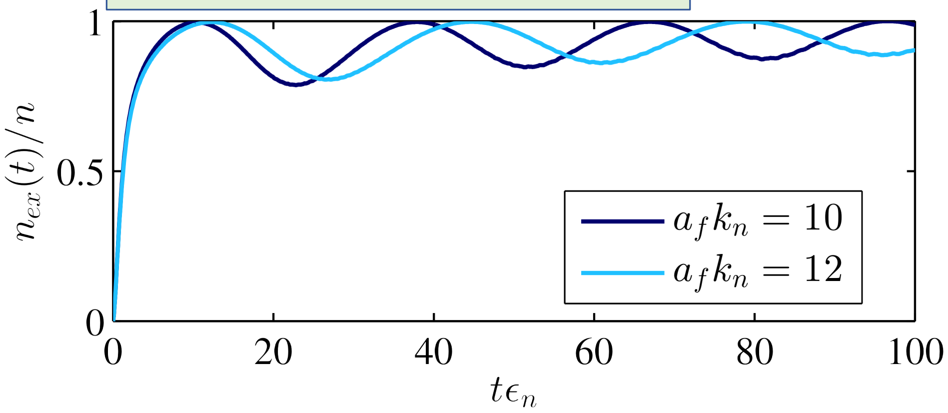
- intermediate/deep quenches ($a_f k_n \sim 0.1$)
  - → universal atom-molecule coherent oscillations $T = 2\pi/|E_\mathrm{B}|$
- quenches to unitarity ($a_f \to \infty$)
  - → coherent oscillations condensate-medium beyond the $t \lesssim \epsilon_n^{-1}$ limit (3-b, heating, loss)
  - → universal scaling of growth time τ_gr

density of excited particles $n_{ex}(t)$



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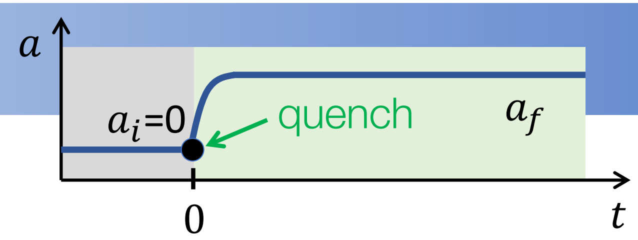
Length & time scales for early dynamics



- shallow quenches ($a_f k_n \ll 1$)
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- intermediate/deep quenches ($a_f k_n \sim 0.1$)
  - → universal atom-molecule coherent oscillations $T = 2\pi/|E_{\mathrm{B}}|$
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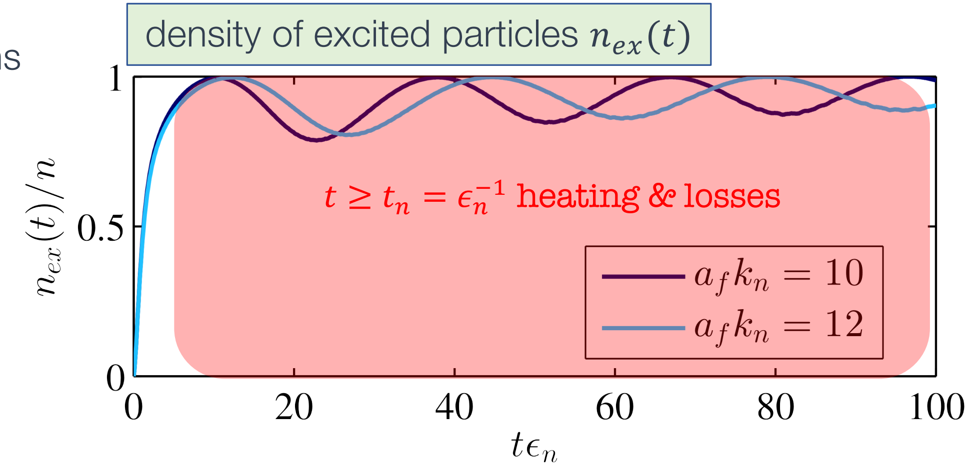
  - → universal scaling of growth time τ_{gr}



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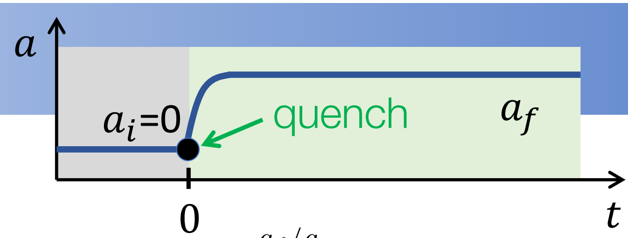

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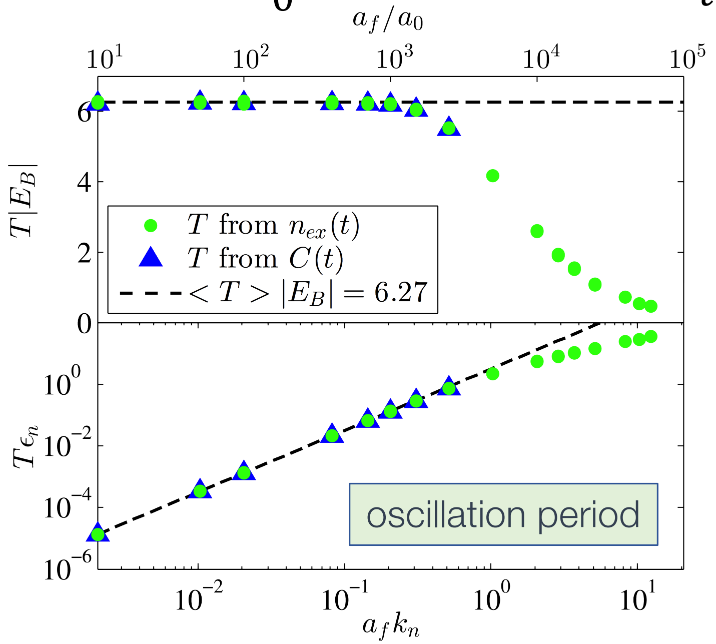

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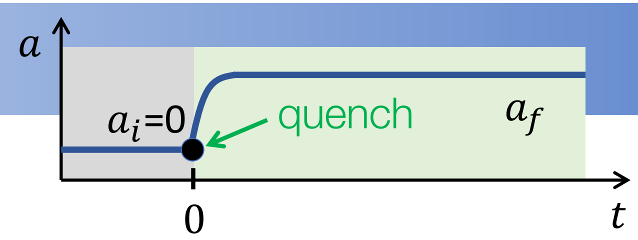

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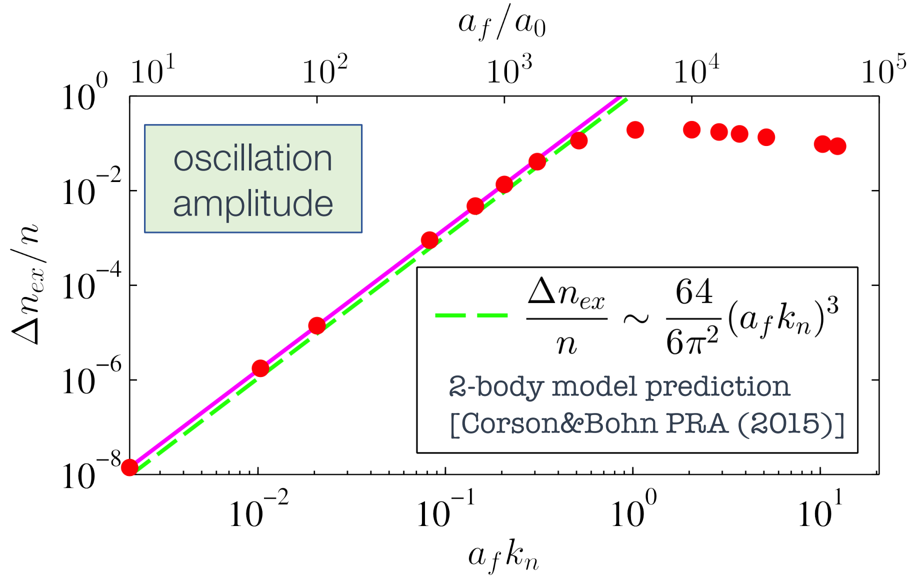

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Universal prethermal dynamics in the unitary regime

1. Normalised momentum distribution $\frac{1}{V}\sum_{\mathbf{k}} \tilde{N}_{\mathbf{k}}(t) = 1$

transient growth

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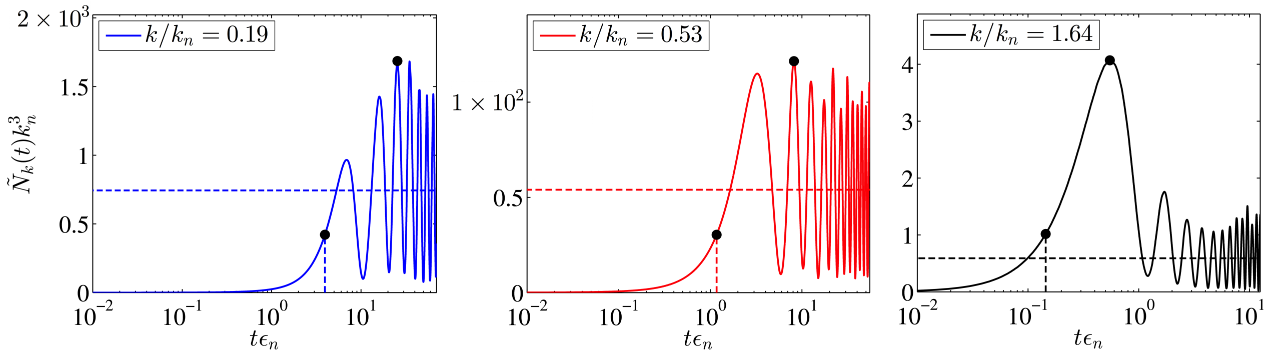
steady-state coherent oscillations (heating, loss)



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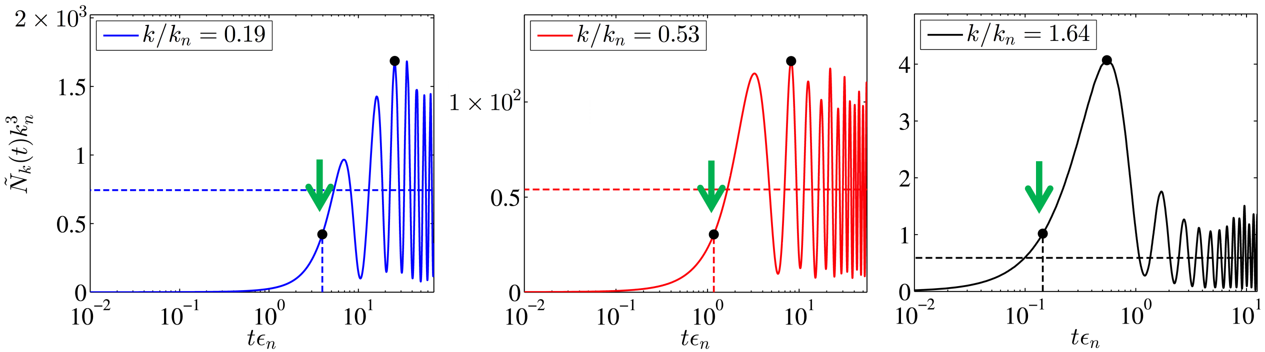


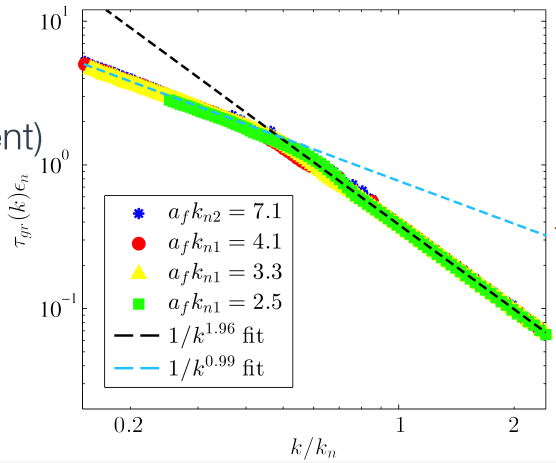
2. Growth time $\tau_{\mathrm{gr}}(k)$

▷ universal scaling (a_f & n independent)

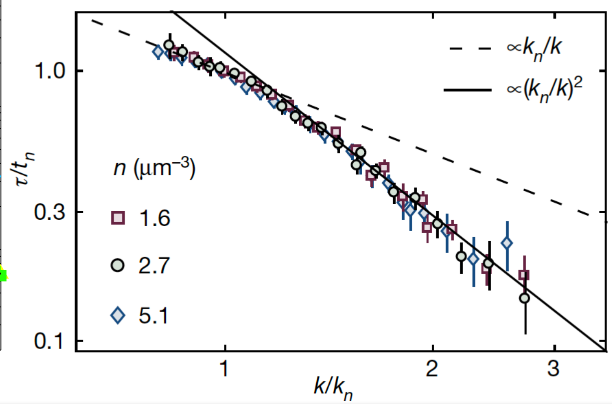
$$\tau_{gr}(k)\epsilon_n \underset{k<k_n}{\simeq} \frac{k_n}{k}$$

$$\tau_{gr}(k)\epsilon_n \underset{k>k_n}{\simeq} \left(\frac{k_n}{k}\right)^2$$

→ quasiparticle excitations Bogoliubov-like



[Eigen et al. Nature (2018)]



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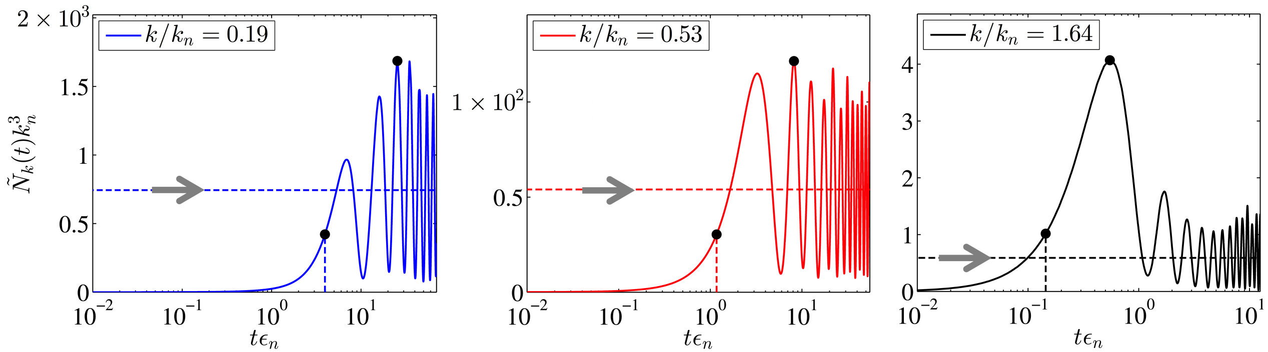
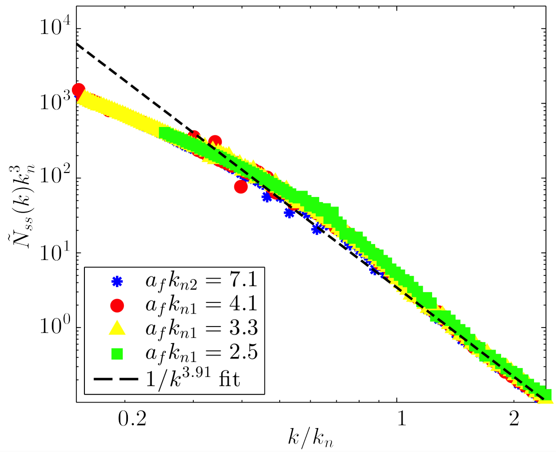


3. steady-state $\tilde{N}_{ss}(k)$

▷ power-law $\sim k^{-4}$ decay

→ vs exponential behavior

→ damping & dissipation



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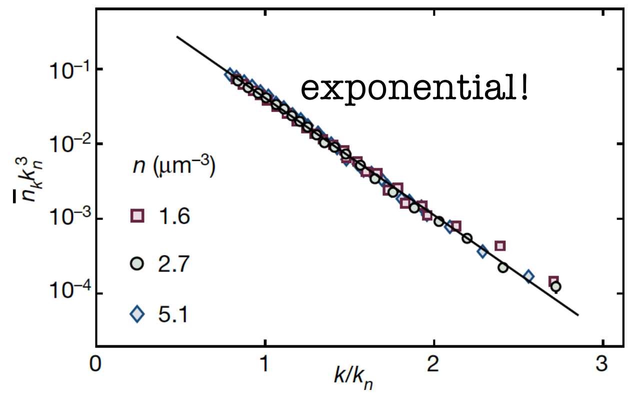


Early-time quench dynamics of strongly interacting Bose gases

Take-  message

- Very early-time quench dynamics into the unitary regime dominated by pairwise excitations out of the condensate

  → Universal scaling law of the growth time

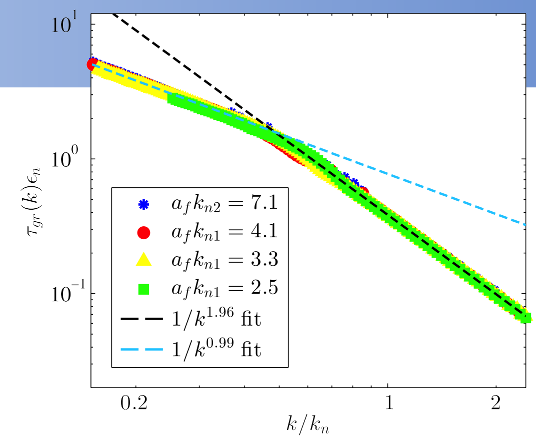
  → no 3-body effects for $t < \epsilon_n^{-1}$

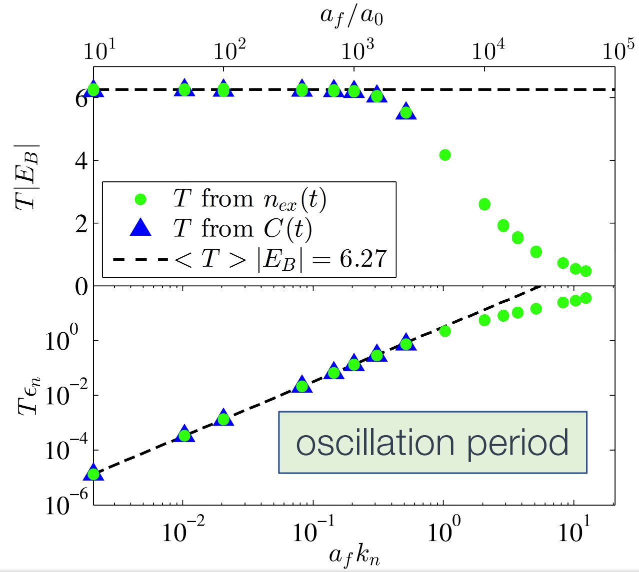
  → Bogoliubov modes at very short times (ϵ_n, k_n)

- Coherent atom-molecule oscillations

  → Universal $T = 2\pi/|E_\mathrm{B}|$ up to $a_f k_n \lesssim 0.21$

  → not negligible amplitude





Open questions & perspectives

1. Include 3-point correlations

   ▷ e.g., third-fourth order cumulant expansion
   [see S. Musolino's poster]

2. Excitation spectrum of the strongly interacting Bose gas

   → precursor of roton minimum

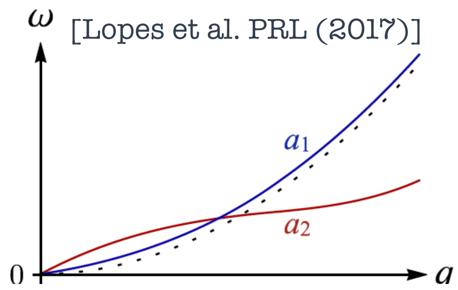
   → signatures in the dynamics of density-density correlations

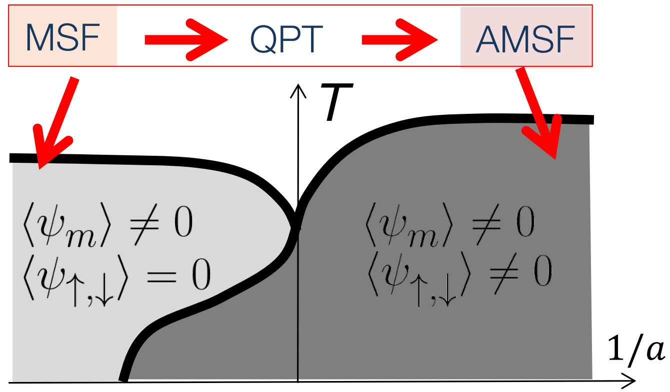
$$g^{(2)}(\mathbf{r},\mathbf{r}';t) = \langle\psi(t)|\hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r})\hat{\psi}^\dagger(\mathbf{r}')\hat{\psi}(\mathbf{r}')|\psi(t)\rangle$$

3. Quench dynamics for binary mixtures

   ▷ quenching across a quantum phase transition

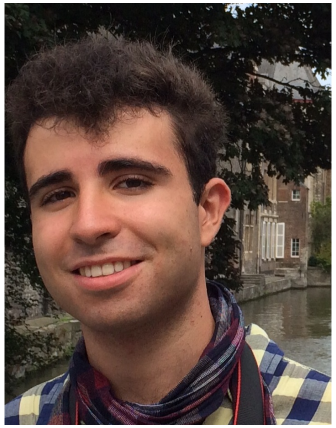
   → also in polaritons
   [see G. Bruun & J. Levinsen talks]





[Radzihovsky et al. PRL (2004)]
[Romans et al. PRL (2004)]
[Marchetti&Keeling PRL (2014)]

In collaboration with



Alberto Muñoz de las Heras
(now PhD student in Trento)

Meera Parish
(Monash University)

[*] A. Muñoz de las Heras, M. M. Parish, F. M. Marchetti, PRA **99** 023623 (2019)