

Theory of Microwave-Assisted Supercurrent in Quantum Point Contacts

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We present a microscopic theory of the effect of a microwave field on the supercurrent through a quantum point contact of arbitrary transmission. Our theory predicts that (i) for low temperatures and weak fields, the supercurrent is suppressed at certain values of the superconducting phase, (ii) at strong fields, the current-phase relation is strongly modified and the current can even reverse its sign, and (iii) at finite temperatures, the microwave field can enhance the critical current of the junction. Apart from their fundamental interest, our findings are also important for the description of experiments that aim at the manipulation of the quantum state of atomic point contacts.

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The dc Josephson effect is one of the most striking examples of macroscopic quantum coherence. In the context of superconductivity this phenomenon manifests as the flow of a dissipationless dc current through a junction in the absence of any voltage [1,2]. Since its discovery in the early 1960s, this effect has been observed in a large variety of weak links such as tunnel junctions [3], microbridges [4], atomic contacts [5], carbon nanotubes [6], semiconductor nanowires [7], and graphene [8]. In spite of their intrinsic differences, the dc Josephson effect in these systems can be described in a unified manner. It has been shown that for constrictions shorter than the superconducting coherent length, the Josephson current is carried exclusively by a single pair of Andreev bound states (ABSs) [9,10]. In the simplest case of a single-channel contact of transmission τ , these states appear at energies $E_A^\pm(\varphi, \tau) = \pm\Delta[1 - \tau\sin^2(\varphi/2)]^{1/2}$, where Δ is the superconducting gap and φ is the phase difference between the order parameters on both sides (see Fig. 1). These states carry opposite supercurrents $I_A^\pm(\varphi) = (2e/\hbar)\partial E_A^\pm/\partial\varphi$, which are weighted by the occupation of the ABSs. A more complex weak link, like the ones mentioned above, can be viewed as a collection of independent conduction channels, characterized by a set of transmission coefficients. The supercurrent through it is given by the sum of the contributions from the individual channels [10].

This unified microscopic picture of the dc Josephson effect has been recently confirmed experimentally in the context of atomic contacts [11], where the current-phase relationship has been directly measured. These experiments mainly probed the ground Andreev state and, as it is stated in Ref. [11], it would be of great interest to also probe the excited state, for instance, through microwave spectroscopy. This leads us to the central question of the present work, how does a microwave radiation modify the

supercurrent of a single-channel quantum point contact (QPC)? Apart from its fundamental interest for the field of mesoscopic superconductivity, this question is also of great relevance for the proposals of using the ABSs of a QPC as the two states of a quantum bit [12–14], whose quantum state can be probed by means of current measurements.

Surprisingly, there is no complete answer to the question posed above. The theoretical analysis of the microwave-assisted supercurrent in point contacts has either been addressed within phenomenological approximations or in the limiting case of very weak fields [15–17]. In this Letter, we present a microscopic theory of the effect of a microwave field on the supercurrent of a single-channel quantum point contact valid for arbitrary range of parameters. Our

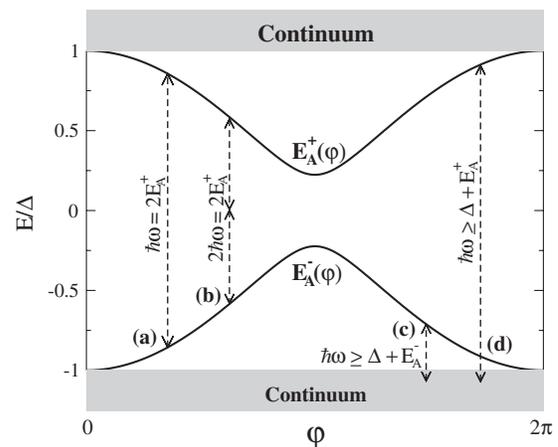


FIG. 1. Dispersion relation of the ABSs in a single-channel QPC with transmission $\tau = 0.95$. The vertical dashed lines indicate one- and two-photon transitions between the ABSs (a),(b), and transitions between the continuum and ABSs (c),(d). We have chosen $E = 0$ at the Fermi energy.

theory based on the Keldysh technique predicts the following novel effects: (i) at low temperatures, the supercurrent can be strongly suppressed at certain values of the phase due to resonant microwave-induced transitions between the two ABSs (processes of type *a* and *b* in Fig. 1); (ii) as the radiation power increases, the supercurrent-phase relation is strongly modified and it can even reverse its sign; (iii) at finite temperatures, the radiation can induce the transition of quasiparticles from the continuum to the lower ABS leading to an enhancement of the critical current as compared to the case in the absence of microwaves (process of type *c* in Fig. 1). We also compare our results to a two-level model (TLM), where the QPC is described exclusively in terms of the ABSs and show that effects (ii) and (iii) fall out of the scope of TLM. This is especially relevant for the quantum computing applications.

We consider a QPC consisting of two identical superconducting electrodes (denoted as *L* and *R*) and linked by a single conduction channel of transmission τ . Our goal is to compute the supercurrent through this QPC when it is subjected to a monochromatic microwave field of frequency ω . We assume that the external radiation generates a time-dependent voltage $V(t) = V_0 \sin \omega t$ [3]. According to the Josephson relation, this voltage induces a time-dependent superconducting phase difference given by $\phi(t) = \varphi + 2\alpha \cos \omega t$, where φ is the dc part of the phase and $\alpha = eV_0/\hbar\omega$ is a parameter that measures the strength of the coupling to the electromagnetic field and that is proportional to the square root of the radiation power at the junction. Following Refs. [18,19], the current through a QPC with an arbitrary time-dependent voltage can be computed as $I(t) = (e/4\hbar) \text{Tr} \hat{\tau}_3 \hat{I}^K(t, t)$, where $\hat{\tau}_3$ is the third Pauli matrix and $\hat{I}^K(t, t)$ is the Keldysh component of the current matrix given by

$$\check{I}(t, t') = 2\tau[\check{G}_L, \check{G}_R] \circ [4 - \tau(2 - \{\check{G}_L, \check{G}_R\}_\circ)]^{-1}(t, t'). \quad (1)$$

Here the symbol $\check{}$ represents 4×4 matrices in Keldysh-Nambu space and the symbol \circ denotes the convolution over intermediate time arguments. Moreover, $\check{G}_{L(R)}$ are the quasiclassical Green functions for the left and right electrodes, which can be expressed as $\check{G}_j(t, t') = e^{i\phi_j(t)\hat{\tau}_3/2} \check{g}_j(t-t') e^{-i\phi_j(t')\hat{\tau}_3/2}$. Here, $\check{g}(t) = \int (dE/2\pi) e^{-iEt/\hbar} \check{g}(E)$ is the equilibrium Green function of the leads and $\phi_j(t)$ is the time-dependent phase of the *j* superconductor, $j = L, R$, i.e., $\phi_L(t) = -\phi_R(t) = \phi(t)/2$. The retarded (*R*), advanced (*A*), and Keldysh (*K*) components of $\check{g}(E)$ adopt the form $\hat{g}^{R(A)}(E) = g^{R(A)}(E)\hat{\tau}_3 + f^{R(A)}(E)i\hat{\tau}_2$ and $\hat{g}^K(E) = [\hat{g}^R(E) - \hat{g}^A(E)] \tanh(E/2k_B T)$, where $g^{R(A)}(E) = E/[(E \pm i\eta)^2 - \Delta^2]^{1/2}$, $f^{R(A)}(E) = \Delta/[(E \pm i\eta)^2 - \Delta^2]^{1/2}$, and $\eta \rightarrow 0^+$.

It is easy to show that, due to the time dependence of the phase, the lead Green functions $\check{G}_{L(R)}$, and any product of them, admit the following Fourier expansion:

$\check{G}(t, t') = \sum_{m=-\infty}^{\infty} e^{im\omega t'} \int \frac{dE}{2\pi} e^{-iE(t-t')/\hbar} \check{G}_{0m}(E)$, where $\check{G}_{nm}(E) \equiv \check{G}(E + n\hbar\omega, E + m\hbar\omega)$ are the corresponding Fourier components in energy space. Thus, \check{I} in Eq. (1) can be written as a product of matrices in energy space. In particular, its Keldysh component is given by

$$\hat{I}_{nm}^K = \sum_l [\hat{A}_{nl}^R \hat{X}_{lm}^K + \hat{A}_{nl}^K \hat{X}_{lm}^A]. \quad (2)$$

Here, we have defined the matrices $\check{A}_{nm} \equiv 2\tau[\check{G}_L, \check{G}_R]_{nm}$ and $\check{X}_{nm} = [4\check{I} - \tau(2 - \{\check{G}_L, \check{G}_R\})]_{nm}^{-1}$, which can be determined from the Fourier components of $\check{G}_{L(R)}$. Once the components of \hat{I}^K are obtained from Eq. (2), one can compute the current. We are only interested in the dc component, which reads

$$I(\varphi, \omega, \alpha) = \frac{e}{4\hbar} \int \frac{dE}{2\pi} \text{Tr} \hat{\tau}_3 \hat{I}_{00}^K(E, \varphi, \omega, \alpha). \quad (3)$$

The dc current can only be calculated analytically in certain limiting cases like in the absence of microwaves, in the tunnel regime, or for very weak fields. In general, Eq. (2) and the current have to be evaluated numerically.

In the absence of microwaves, the current from Eq. (3) can be written as a sum of the contributions of the two ABSs as $I_{\text{eq}}(\varphi) = I_A^- n_F(E_A^-) + I_A^+ n_F(E_A^+)$, $n_F(E)$ being the Fermi distribution function, which yields

$$I_{\text{eq}}(\varphi) = \frac{e\Delta^2}{2\hbar} \frac{\tau \sin \varphi}{E_A^+(\varphi)} \tanh\left(\frac{E_A^+(\varphi)}{2k_B T}\right). \quad (4)$$

In equilibrium, the maximum current is then obtained at temperature $T = 0$, when the lower ABS is fully occupied and the upper one is empty. In the presence of the microwave field the simplest approach is the so-called adiabatic approximation [3], which consists in replacing the stationary phase φ in Eq. (4) by the time-dependent phase $\phi(t)$. This leads to

$$I_{\text{ad}}(\varphi, \alpha) = \sum_{n=1}^{\infty} I_n J_0(2n\alpha) \sin(n\varphi), \quad (5)$$

where $I_n = (1/\pi) \int_0^{2\pi} d\varphi I_{\text{eq}}(\varphi) \sin(n\varphi)$ are the harmonics of the equilibrium current-phase relation and J_0 is the zero-order Bessel function of the first kind.

In Fig. 2(a) we show the zero-temperature current-phase relation (CPR) computed numerically from Eq. (3) for a highly transmissive channel with $\tau = 0.95$ and a weak field $\alpha = 0.1$ of frequency $\hbar\omega = 0.6\Delta$. For comparison we also show as a dashed line the result obtained with the adiabatic approximation of Eq. (5). The main difference is that the exact result shows a series of dips where the current is largely suppressed. These dips originate from microwave-induced transitions from the lower ABS to the upper one that enhance the population of the latter, diminishing the supercurrent (see processes *a* and *b* in Fig. 1). Such transitions can occur whenever the Andreev gap (distance between the ABSs) is equal to a multiple of the microwave

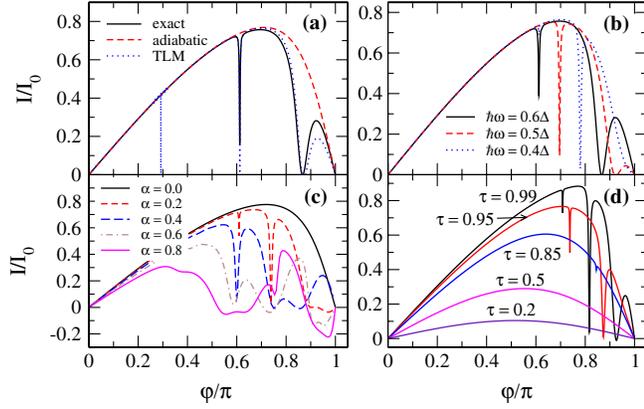


FIG. 2 (color online). Zero-temperature supercurrent, in units of $I_0 = e\Delta_0/\hbar$, as a function of the phase. (a) The solid line shows the exact result for $\tau = 0.95$, $\hbar\omega = 0.6\Delta$, and $\alpha = 0.1$, the dashed line corresponds to the approximation of Eq. (5), and the dotted line to the TLM. The rest of the panels show the exact result for (b) $\tau = 0.95$, $\alpha = 0.1$, and different frequencies; (c) $\tau = 0.95$, $\hbar\omega = 0.3\Delta$, and different α 's; (d) $\hbar\omega = 0.3\Delta$, $\alpha = 0.1$, and different τ 's.

frequency, i.e., $2E_A^+(\varphi) = n\hbar\omega$, where $n = 1, 2, \dots$ can be interpreted as the number of photons involved in the transition. For small values of α the ABSs remain almost unchanged; thus, the resonant processes take place at phases given by

$$\varphi_n = 2 \arcsin \sqrt{[1 - (n\hbar\omega/2\Delta)^2]/\tau}, \quad n = 1, 2, \dots \quad (6)$$

This expression accurately reproduces the positions of the dips in Fig. 2(a). The origin of the dips can be further confirmed by exploring the CPR for different frequencies, as we do in Fig. 2(b). One can see that by decreasing the frequency, the dip of order $n = 1$ moves to higher values of φ and disappears for $\hbar\omega \leq 0.4\Delta$ [cf. Eq. (6)].

One can gain further insight by analyzing this problem in terms of a TLM that describes the dynamics of our QPC in terms of the ABSs [13,20]. We consider the TLM of Ref. [13], whose effective Hamiltonian in the instantaneous basis of ABSs reads

$$\hat{H}_A(t) = E_A^+(\phi(t))\hat{\sigma}_z - \frac{r\tau\Delta^2\sin^2[\phi(t)/2]}{4[E_A^+(\phi(t))]^2}\hbar\dot{\phi}(t)\hat{\sigma}_y, \quad (7)$$

where $\hat{\sigma}_{y,z}$ are Pauli matrices, $r = \sqrt{1 - \tau}$, and $\dot{\phi}(t) = \partial\phi(t)/\partial t$. We have computed the CPR from this model using a Floquet approach [21]. In Fig. 2(a) we show a comparison of the results of this TLM with the exact results. There is an excellent agreement in a wide range of phases and, in particular, the TLM is able to reproduce the current dips. A discrepancy occurs at phases close to π , which is understandable as the model assumes that $\hbar\dot{\phi}(t) \sim \alpha\omega \ll 2E_A^+$, while for $\varphi \sim \pi$ and high τ , the ABSs are very close to each other and the assumption no

longer holds. Using a rotating-wave-type approximation we also obtain from the TLM the following analytical expression for the dc current at the first two resonances:

$$I(\varphi, \omega, \alpha) \simeq \frac{2eE_A'}{\hbar} \left(1 - \frac{\gamma_1^2}{\delta_1^2 + \gamma_1^2}\right) \left(1 - \frac{\gamma_2^2}{\delta_2^2 + \gamma_2^2}\right). \quad (8)$$

Here and below, E_A' and E_A'' are the first and second derivatives of E_A^+ with respect to the phase. The detunings $\delta_{1,2}$ are given by $\delta_1 = E_A^+(\varphi) - \hbar\omega/2 + \alpha^2 E_A''(\varphi) + 3r^2\alpha^2\hbar\omega(\Delta^2 - E_A^+(\varphi)^2)/(32E_A^+(\varphi)^4)$, and $\delta_2 = E_A^+(\varphi) - \hbar\omega + \alpha^2 E_A''(\varphi) - r^2\alpha^2\hbar\omega(\Delta^2 - E_A^+(\varphi)^2)/(12E_A^+(\varphi)^4)$, and include shifts in the ABS energies caused by the microwave field. The resonance widths are $\gamma_1 = r\alpha\hbar\omega(\Delta^2 - E_A^+(\varphi)^2)/(4E_A^+(\varphi)^2)$ and $\gamma_2 = r\alpha^2\hbar\omega E_A'(\varphi)(2\Delta^2 - E_A^+(\varphi)^2)/(2E_A^+(\varphi)^3)$, in general being proportional to the coupling strength, $\gamma_n \propto \alpha^n$. For small power, Eq. (8) is in a good agreement with the exact solution for the current-phase relation. We also find that within the TLM the current vanishes completely at the resonances (given by the condition $\delta_n = 0$), as a consequence of the fact that at these points the long-time average populations on the ABSs are equal, so that the currents carried by the two states cancel exactly [22]. Transitions from the continuum, however, make the current at the resonances finite and dependent on the frequency and the excitation energies. This is seen, for example, in the exact result for the second dip shown in Fig. 2(a) [23].

Let us now discuss the dependence of the CPR on the radiation power. In Fig. 2(c) we show the CPR for $\tau = 0.95$, $\hbar\omega = 0.3\Delta$, and different values of α . As α increases, the CPR is drastically modified and the current is not only strongly suppressed around the phase values given by Eq. (6), but everywhere. Notice also that in certain regions, particularly at large phases, the current even reverses its sign. In this respect, such behavior differs however from so-called π junctions. It is also worth stressing that as α increases, we find larger deviations between the exact results and those of the TLM (not shown here) due to multiphoton processes connecting the ABSs and the continuum of states. Notice that in the context of atomic contacts the values of α considered in this work are experimentally achievable [24]. In Fig. 2(d) we show the influence of the transmission in the CPR for $\hbar\omega = 0.3\Delta$ and $\alpha = 0.1$. In this case, for $\tau \leq 0.9$ the CPR can be accurately described with the adiabatic approximation for all phases since the transitions between the ABSs are very unlikely.

We turn now to the analysis of the critical current I_C , i.e., the maximum value of the dc Josephson current. In Fig. 3(a) we show the critical current as a function of α for several temperatures. One clearly sees that the adiabatic approximation (dashed lines) only describes correctly the behavior of I_C when the frequency, power, and temperature are low enough so that the microwaves cannot induce transitions between the ABSs and between them and

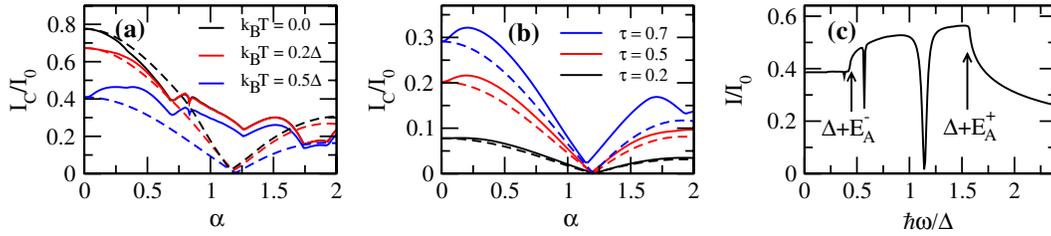


FIG. 3 (color online). Panels (a) and (b) show the critical current as a function of α for (a) $\hbar\omega = 0.3\Delta$, $\tau = 0.95$ and different values of T , and (b) $\hbar\omega = 0.3\Delta$, $k_B T = 0.5\Delta$, and different values of τ . In both panels the solid lines correspond to exact results and the dashed lines to Eq. (5). (c) Current as a function of the frequency for $\varphi = 2.0$, $k_B T = 0.5\Delta$, $\tau = 0.95$, and $\alpha = 0.1$.

the continuum. The most striking result in Fig. 3(a) is the enhancement of I_C with respect to the case $\alpha = 0$ as the temperature is raised. In Fig. 3(b) we show that this occurs in a wide range of transmission values. The enhancement of the critical current by a microwave field has been predicted and observed in superconducting microbridges [25,26] and in proximity effect structures [27,28]. In both cases the enhancement is due to a redistribution of the excitations induced by the field. In our case the underlying mechanism is similar. At finite temperature the lower ABS is not fully occupied and the microwave field can promote quasiparticles from the continuum to this state if $\hbar\omega \geq \Delta + E_A^-(\varphi)$, for values of φ close to that where the maximum takes place (see process *c* in Fig. 1). This naturally results in an enhancement of the supercurrent. We illustrate this argument in Fig. 3(c), where we show the frequency dependence of the supercurrent for a fixed value of the phase $\varphi_0 = 2.0$. As one can see, the current remains largely unaffected until $\hbar\omega$ reaches $\Delta + E_A^-(\varphi_0) \approx 0.46\Delta$, where it starts to increase. Then, as the frequency increases further one can observe the appearance of two dips, corresponding to the resonances at $\hbar\omega = E_A^+/2$ and E_A^+ . For larger values of $\hbar\omega$ one finally sees a decrease of the current at $\Delta + E_A^+(\varphi_0) \approx 1.54\Delta$ due to processes that promote quasiparticles from the continuum to the upper ABS (see process *d* in Fig. 1). Because of electron-hole symmetry, similar transitions exist between the ABS and the upper continuum. Obviously, the enhanced supercurrent cannot be explained in terms of any TLM since it involves the continuum of states.

In summary, we present here a microscopic theory of the microwave-assisted supercurrent in quantum point contacts. It predicts the appearance of a variety of novel phenomena that in general are out of the scope of simple approximations and two-level models. Our results are of relevance for many different types of weak links and, in particular, they can be quantitatively tested in the context of atomic contacts.

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