Shot Noise and Multiple Andreev Reflections in *d*-Wave Superconductors

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We present a theoretical analysis of the shot noise in d-wave/d-wave contacts with arbitrary transparency, including the contribution of multiple Andreev reflections. The multiple charge quanta transferred in these processes are revealed as a huge enhancement of the noise-current ratio at low voltages, which survives for all crystal misorientations. We also show how different ingredients such as nonmagnetic impurities or a magnetic field produce very characteristic hallmarks in the shot noise, which can be used as a further test of the d-wave scenario in superconducting cuprates.

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In the past few years the intensive study of the nonequilibrium current fluctuations, known as shot noise, has provided a deeper understanding of the electronic transport in many different contexts [1]. The shot noise reveals aspects hidden in the usual conductance measurements such as the statistics and charge of the carriers, relevant energy scales, or transmission information. In the case of superconducting contacts, the noise has been mainly used for the analysis of the effective charges transferred in the different tunneling processes. This effective charge can be roughly defined as the noise-current ratio. A striking example is the recent observation in superconducting atomic-size contacts of effective charges much larger than unity attributed to multiple Andreev reflections [2] and in quantitative agreement with the theoretical expectations [3]. Unfortunately, the analysis of shot noise has been mainly restricted to conventional superconductors, and only a few theoretical works have recently addressed noise in *NIS* junctions with d-wave superconductors [4].

On the other hand, the origin and nature of the high temperature superconductivity in the cuprates is still an open problem. Different phase-sensitive experiments have provided strong indications that the order parameter in these materials has a dominant $d_{x^2-y^2}$ component [5,6]. However, these experiments have not definitively closed the debate about basic questions such as the universality of this symmetry or the existence of subdominant components [7,8]. In this sense, it is highly desirable to provide new tools which can further test the different scenarios. Thus, a natural question is what can the shot noise teach us in unconventional superconductivity? For instance, the shot noise provides fundamental information on the charge of the carriers, as was shown in complex situations such as in the fractional quantum Hall effect [9] or in superconducting point contacts [2]. On the other hand, the shot noise in a junction depends in a different way on the interface properties as compared with the current. In this sense, the study of this quantity can used as a cross-check for the different transport theories, and, in turn, it can be very valuable to solve the lack of consensus in the interpretation of the tunneling experiments in cuprate junctions [10].

In this Letter, we present the first theoretical analysis of the shot noise in d-wave/d-wave *SIS* junctions of arbitrary transparency. We show that the zero-frequency noise S may by far exceed the Poisson value $2eI$, where I is the current, due to the occurrence of multiple Andreev reflections. In particular, at high transparencies the effective charge q, defined as $q = S/2I$, exhibits a huge enhancement at low voltages ($q/e \gg 1$), which survives for all crystal misorientations. At low transparencies, contrary to the s-wave case, q is not quantized in units of the electron charge due to the averaging over the anisotropic gap. We also show that elastic scattering mechanisms such as bulk impurities may result in a strong reduction of the effective charge. Finally, we show how the Doppler shift of the Andreev bound states in the presence of a magnetic field is revealed in the shot noise. All these features are very characteristic of the d-wave symmetry and can be used as additional tests of this scenario in cuprates.

Our goal is to extend the theory of the shot noise to the case of superconducting cuprates. For this purpose, we consider a voltage biased contact, consisting of two $d_{x^2-y^2}$ superconductors separated by a single interface of arbitrary transparency. The order parameter on side i, $i = L, R$, is rotated by α_i with respect to the surface normal, and we denote junction type by the relative crystal orientations as d_{α_L}-d_{α_R}. There are several experimental realizations of this system, among which the bicrystal grain-boundary junctions are ideal examples [11]. To calculate the noise we use the formalism developed in Ref. [12]. In that work we introduced a formulation of boundary conditions that mimics interfaces for the quasiclassical theory of superconductivity and that are suitable for arbitrary transparency, and we established the machinery to determine the current fluctuations in unconventional junctions. Here we consider the case of point-contact-like geometry and assume that the voltage drop takes place at the interface. Thus, to compute the noise we first determine self-consistently the local electronic properties of the isolated electrodes. This includes effects on the order-parameter profile and on the local density of states (DOS) by pair breaking caused by quasiparticle scattering both off the interface and off homogeneously distributed impurities in the crystals [13–18]. Finally, the noise is calculated using the local surface Green's and solving the appropriate boundary conditions for a point contact, as detailed in Ref. [12].

The noise spectral density $S(\omega)$ is defined as

$$S(\omega) = \int d(t' - t) e^{i\omega(t'-t)} \langle \delta\hat{I}(t')\delta\hat{I}(t) + \delta\hat{I}(t)\delta\hat{I}(t')\rangle, \tag{1}$$

where $\delta\hat{I}(t) = \hat{I}(t) - \langle\hat{I}(t)\rangle$ are the fluctuations in the current. We consider only the zero-frequency limit at zero temperature. In the case of a constant bias voltage V, one can show (see Ref. [12]) that the noise oscillates in time with all the harmonics of the Josephson frequency, i.e., $S(t) = \sum_m S_m e^{im\phi(t)}$, where $\phi(t) = \phi_0 + (2eV/\hbar)t$ is the time-dependent superconducting phase difference. We consider only the dc noise, denoted from now on as S. Furthermore, we assume that the interface conserves the momentum of the quasiclassical trajectories, which allows us to write the noise as a sum over independent trajectory contributions: $S = \frac{1}{2}\int_{-\pi/2}^{\pi/2} d\hat{\boldsymbol{p}}_F S(\hat{\boldsymbol{p}}_F)\cos(\hat{\boldsymbol{p}}_F)$, where $\hat{\boldsymbol{p}}_F$ defines the Fermi surface position. For the angular dependence of the transmission coefficient, we use the expression $D(\hat{\boldsymbol{p}}_F) = D\cos^2(\hat{\boldsymbol{p}}_F)/[1 - D\sin^2(\hat{\boldsymbol{p}}_F)]$, resulting from a δ-like potential. Here D is the transmission for the trajectory perpendicular to the interface. In the tunneling regime one can easily demonstrate that the zero-frequency noise reaches the Poisson value, i.e., $S = 2eI$. Thus, in this limit the noise does not contain new information as compared with the current. For this reason, we investigate the case of not too low interface transparency, $D \geq 0.1$, in which the multiple Andreev reflections (MAR) play a fundamental role in the transport [19,20].

Let us start by analyzing the case of a symmetric d_0-d_0 junction in the clean limit. In this case, the order parameter is constant up to the surface, and there are no bound states for any trajectory. The noise-voltage characteristics for a single trajectory, $S(\hat{\boldsymbol{p}}_F)$, coincide with those of isotropic s-wave superconductors [3] and can be seen in Fig. 1(a). As a consequence of the occurrence of MARs, the trajectory-resolved shot noise exhibits the following remarkable features: (i) the presence of a pronounced subharmonic gap structure (SGS) at voltages $eV = 2\Delta(\hat{\boldsymbol{p}}_F)/n$, (ii) the noise greatly exceeds the Poisson value $2eI$ in the subgap region, as can be seen in Fig. 1(b), and (iii) in the tunneling regime the effective charge is quantized in units of the electron charge. This last feature, illustrated in the inset of Fig. 1(b), was used to suggest that the noise provides a way of measuring the charge of individual MARs in s-wave

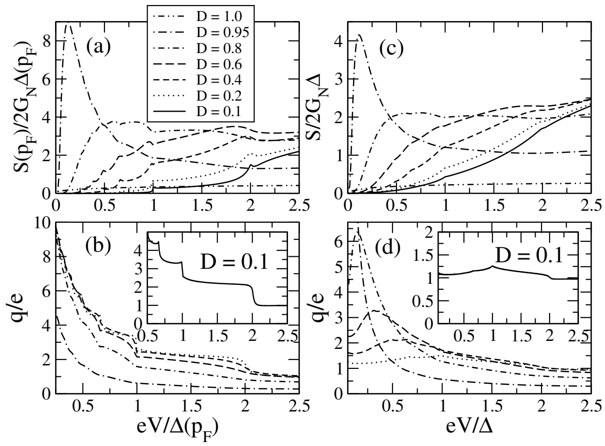

FIG. 1. d_0-d_0 contact in the clean case: (a) angle-resolved shot noise as a function of the voltage in units of the trajectory gap for different transmissions. G_N is the normal state conductance. (b) Angle-resolved effective charge, $q = S/2I$. (c) Angle-averaged shot noise. The voltage is normalized by the maximum gap Δ. (d) Angle-averaged effective charge.

superconductors [3]. Indeed, as mentioned in the introduction, these noise-voltage characteristics have been quantitatively confirmed in the context of superconducting atomic contacts [2]. The natural questions now are do these features survive after doing the average over the different directions in the Fermi surface? Can we still identify the charge of individual MARs in a d-wave junction? The answers to these questions can be seen in Figs. 1(c) and 1(d). First of all, notice that the SGS is still visible, but it is more rounded than in the s-wave case. It is worth remarking that it is the bulk maximum gap what is revealed in the SGS. Notice also that the effective charge does not show any sign of quantization even at low transparencies [see inset of Fig. 1(d)]. This is due to the fact that different MARs contribute simultaneously for different trajectories, and then the discreteness of q is washed out. Anyway, the dominant contribution of MAR at high transmission is still manifested as a huge enhancement of the effective charge at low bias ($q \gg e$).

Let us now consider the case of a $d_{\pi/4}$-$d_{-\pi/4}$ junction. In this case, assuming specular quasiparticle scattering at the interface, an Andreev bound state forms at zero energy for every trajectory [21]. This implies that the surface acts as a pair breaker [13,14] and the gap is depressed in the vicinity of the interface, vanishing exactly at the barrier. This order-parameter profile induces not only the appearance of bound states at zero energy, but also at the gap edges, as can be seen in the inset of Fig. 2(b). As a consequence of this local density of states, the noise exhibits a pronounced SGS due to resonant tunneling through the bound states [see Fig. 2(a)]. As in the case of the current, see Ref. [20], there is an even-odd effect in the SGS, in the sense that the even ($eV = \Delta/n$) structures are more pronounced. Its origin

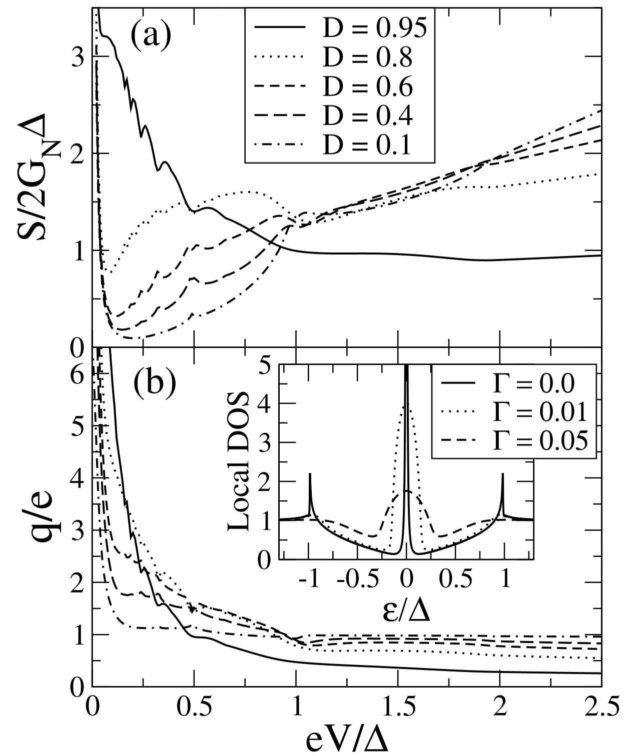

FIG. 2. $d_{\pi/4}$-$d_{-\pi/4}$ contact in the clean case: (a) shot noise as a function of voltage for different transmissions. (b) Effective charge vs voltage. The curves were computed using a small inelastic broadening ($\sim 0.003\Delta$). Inset: local DOS at the interface for a 45° misorientation for different values of the bulk-impurity scattering rate Γ (Born scatterers), measured in units of $2\pi T_C$, where T_C is the critical temperature in the clean case. Δ is the maximum bulk gap for the clean superconductor.

can be understood as follows. In this geometry there are two types of MARs which dominate the transport: (a) those which connect the bound states with the gap edges and (b) the usual ones connecting the gap edges. The first ones give rise to the SGS at $eV = \Delta/n$, while the second ones contribute to the whole series $eV = 2\Delta/n$. However, the bound states at the gap edges do not appear for every trajectory, which weakens the SGS due to these processes. On the other hand, as we show in Fig. 2(b) the effective charge is not even quantized at low transparencies, again due to the average over the different trajectories. However, at high transparencies the dominant contribution of the MARs gives rise to a huge enhancement of the effective charge at low bias. This is a robust feature which survives for all crystal misorientations, and it is an unambiguous signature of the fact that the MARs control the low voltage transport. Indeed, this pronounced increase of the noise-current ratio at low bias has been recently observed in symmetric bicrystal YBaCuO Josephson junction [22] in, to our knowledge, the first experimental analysis of the shot noise in cuprate Josephson junctions. In this experiment a mean transparency of $D \approx 0.01$ was estimated, but in our opinion this enhancement is due to MARs in high transparent conduction channels, probably due to the presence of pinholes such as in the conventional *SIS* tunnel junctions of Dieleman *et al.* [23].

In d-wave superconductors the order parameter is very sensitive to scattering from nonmagnetic impurities and surface roughness. In particular, it is known that these elastic scattering mechanisms provide an intrinsic broadening for the zero-energy bound states [17,18]. For the case of Born scatterers this broadening is $\propto \sqrt{\Gamma\Delta}$, where $\Gamma = 1/2\tau$ is the effective pair-breaking parameter locally at the surface. This is illustrated in the inset of Fig. 2(b) for the case of bulk impurities. The interesting question now is what is the signature of impurities in the shot noise of a d-wave junction? In Fig. 3 we show the shot noise and effective charge for a $d_{\pi/4}$-$d_{-\pi/4}$ junction for different values of the bulk-impurity scattering rate. As the scattering rate increases, there are two major effects that one should notice: (i) the disappearance of the SGS in the noise, and (ii) a reduction of the effective charge, specially pronounced at low voltages. Both features can be understood as follows: the increase of density of states in the gap region enhances the probability of single-quasiparticle processes, producing the subsequent reduction of the probability of the Andreev processes, which in turn leads to both the suppression of the SGS and the reduction of the effective charge.

Fogelström *et al.* [15] have shown that the Andreev bound states should split in the presence of a magnetic field perpendicular to the ab plane. This splitting results in a splitting of the zero bias conductance peak observed in tunnel junctions [24]. It is then interesting to analyze what the signature is of this time reversal symmetry breaking in the shot noise. Let us consider a magnetic field perpendicular to the ab plane, $\mathbf{H} = H\hat{\mathbf{z}}$. As mentioned above, this field leads to a Doppler shift in the continuum excitations given by $\mathbf{v}_f \cdot \mathbf{p}_s$, where the condensate momentum is $\mathbf{p}_s = -(e/c)A(x)\hat{\mathbf{y}}$, with A the self-consistently determined vector potential [15]. This means

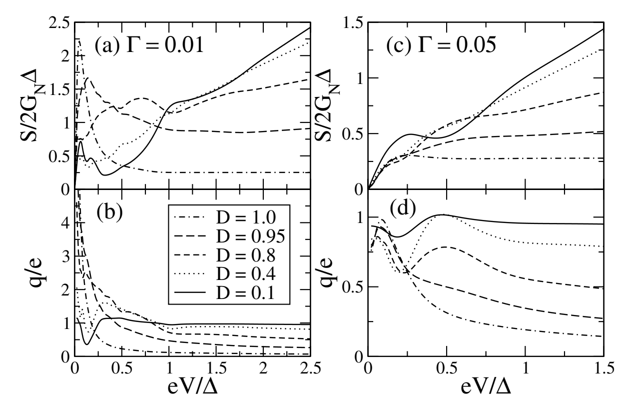

FIG. 3. Shot noise and effective charge as a function of voltage for a $d_{\pi/4}$-$d_{-\pi/4}$ contact for two values of the bulk-impurity scattering rate Γ.

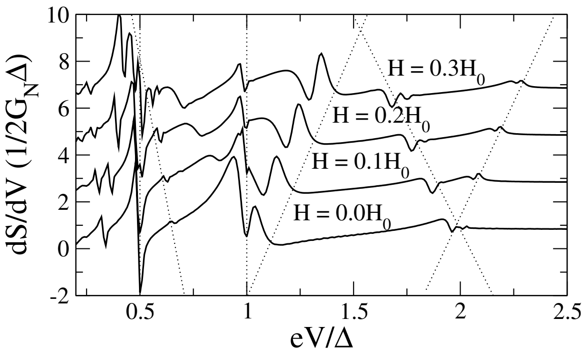

FIG. 4. Differential shot noise of a clean $d_{\pi/4}$-$d_{-\pi/4}$ junction with $D = 0.2$ and different values of the magnetic field. The curves have been vertically displaced for clarity and dotted lines have been added to guide the eye.

that the Andreev bound states are shifted to an energy which, in the limit of a large ratio λ/ξ_0, can be estimated to be $\epsilon_b(\hat{\boldsymbol{p}}_F) = (e/c)v_f H\lambda \sin\hat{\boldsymbol{p}}_F$, λ being the ab-plane penetration depth. We use a natural field scale set by a screening current of order the bulk critical current, $H_0 = c\Delta/ev_f\lambda$, which is of the order of a Tesla [15]. The screening currents flow parallel to the interface and in opposite directions in both electrodes, which means that the trajectory-resolved DOS of the left and right superconductors are shifted by $2\epsilon_b(\hat{\boldsymbol{p}}_F)$ relative to each other. As explained in Ref. [20], this shift modifies the threshold voltages of MARs starting and ending in different electrodes, leading to the splitting of the peaks with an odd order n in the SGS. On the contrary, since the magnetic field produces a rigid shift of the spectrum, the threshold voltages of those MARs starting and ending in the same electrode are not modified. This means that the positions of the structures with an even order n in the SGS remain unchanged. This is illustrated in Fig. 4 where we show the differential shot noise dS/dV for a $d_{\pi/4}$-$d_{-\pi/4}$ junction with transmission $D = 0.2$ for different values of the magnetic field. Starting at large voltages, the structure at 2Δ splits with applied field. Around $eV = \Delta$ there is a maximum at $eV = \Delta$, unaffected by the applied field, as well as a field shift of the peak just above Δ. The field dependence of the differential noise is most clearly resolved at larger biases, $eV \geq \Delta/2$, as the marks of the various processes begin to overlap at small bias. The effect of the Doppler shift on the SGS of the noise is prominent only in junctions with a sizable misorientation. For junctions close to the d_0-d_0 case, the main contribution to the SGS comes from trajectories close to perpendicular incidence, i.e., with $\sin\hat{\boldsymbol{p}}_F \sim 0$ and thus having a vanishing Doppler shift.

In summary, we have presented a theory of shot noise in d-wave/d-wave contacts with arbitrary transparency. We have shown that in the MAR regime these nonequilibrium current fluctuations exhibit very peculiar features such as subharmonic gap structure, super-Poissonian noise ($S \gg 2eI$), reduction of the effective charge q by impurities, and the splitting of the SGS in a magnetic field. All these features are unique fingerprints of the d-wave scenario, and we hope that our analysis will trigger experimental study of shot noise in cuprate junctions.

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[1] Ya. M. Blanter and M. Büttiker, Phys. Rep. **336**, 1 (2000).
[2] R. Cron *et al.*, Phys. Rev. Lett. **86**, 4104 (2001).
[3] J. C. Cuevas, A. Martín-Rodero, and A. Levy Yeyati, Phys. Rev. Lett. **82**, 4086 (1999); Y. Naveh and D. V. Averin, Phys. Rev. Lett. **82**, 4090 (1999).
[4] J. X. Zhu and C. S. Ting, Phys. Rev. B **59**, 14 165 (1999); Y. Tanaka *et al.*, Phys. Rev. B **61**, 11 902 (2000); T. Löfwander, V. S. Shumeiko, and G. Wendin, Physica (Amsterdam) **367C**, 86 (2002).
[5] D. J. Van Harlingen, Rev. Mod. Phys. **67**, 515 (1995).
[6] C. C. Tsuei and J. R. Kirtley, Rev. Mod. Phys. **72**, 969 (2000).
[7] M. Covington *et al.*, Phys. Rev. Lett. **79**, 277 (1997).
[8] W. K. Neils and D. J. Van Harlingen, Phys. Rev. Lett. **88**, 047001 (2002).
[9] L. Saminadayar *et al.*, Phys. Rev. Lett. **79**, 2526 (1997); R. de-Picciotto *et al.*, Nature (London) **389**, 162 (1997).
[10] For a recent review on tunneling effect in unconventional superconductors, see S. Kashiwaya and Y. Tanaka, Rep. Prog. Phys. **63**, 1641 (2000).
[11] L. Alff *et al.*, Phys. Rev. B **58**, 11 197 (1998); L. Alff *et al.*, Eur. Phys. J. B **5**, 423 (1998).
[12] J. C. Cuevas and M. Fogelström, Phys. Rev. B **64**, 104502 (2001).
[13] M. Matsumoto and H. Shiba, J. Phys. Soc. Jpn. **64**, 3384 (1995); **64**, 4867 (1995).
[14] L. J. Buchholtz *et al.*, J. Low Temp. Phys. **101**, 1079 (1995); **101**, 1099 (1995).
[15] M. Fogelström, D. Rainer, and J. A. Sauls, Phys. Rev. Lett. **79**, 281 (1997).
[16] Yu. S. Barash, A. A. Svidzinsky, and H. Burkhardt, Phys. Rev. B **55**, 15 282 (1997).
[17] A. Poenicke *et al.*, Phys. Rev. B **59**, 7102 (1999).
[18] A. Poenicke, M. Fogelström, and J. A. Sauls, Physica (Amsterdam) **284B–288B**, 589 (2000).
[19] T. Löfwander, V. S. Shumeiko, and G. Wendin, Supercond. Sci. Technol. **14**, R53 (2001), and references therein.
[20] A. Poenicke, J. C. Cuevas, and M. Fogelström, Phys. Rev. B **65**, 220510 (2002).
[21] C. R. Hu, Phys. Rev. Lett. **72**, 1526 (1994).
[22] K. Y. Constantinian *et al.*, Physica (Amsterdam) **367C**, 276 (2002).
[23] P. Dieleman *et al.*, Phys. Rev. Lett. **79**, 3486 (1997).
[24] M. Aprili, E. Badica, and L. H. Greene, Phys. Rev. Lett. **83**, 4630 (1999); R. Krupke and G. Deutscher, Phys. Rev. Lett. **83**, 4634 (1999).