Magnetic Interference Patterns and Vortices in Diffusive SNS Junctions

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We study theoretically the electronic and transport properties of a diffusive superconductor-normal metal-superconductor junction in the presence of a perpendicular magnetic field. We show that the field dependence of the critical current crosses over from the well-known Fraunhofer pattern in wide junctions to a monotonic decay when the width of the normal wire is smaller than the magnetic length $\xi_H = \sqrt{\Phi_0/H}$, where H is the magnetic field and Φ_0 the flux quantum. We demonstrate that this behavior is a direct consequence of the magnetic vortex structure appearing in the normal region and predict how this structure is manifested in the local density of states.

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Introduction.—The study of the modification of the properties of a normal metal in contact with superconductors, known as the *proximity effect*, has a long history [1]. In recent years there has been a renewed interest in this subject because new experimental techniques have allowed resolving properties on smaller length scales and very low temperatures [2]. Although many electronic and transport properties of hybrid SN structures are now well understood, the situation is less satisfactory when dealing with the magnetic field dependence of those properties. A few years ago, Heida *et al.* [3] measured the critical current as a function of a perpendicular magnetic field in ballistic superconductor-normal metal-superconductor (SNS) junctions of comparable length and width and found a periodicity close to $2\Phi_0$, where $\Phi_0 = h/2e$ is the flux quantum, instead of the standard Φ_0 of the Fraunhofer pattern [4]. This was qualitatively explained in Refs. [5,6] in terms of the classical trajectories associated with current-carrying Andreev states in a normal clean wire. In the case of diffusive junctions, numerous experiments have shown that in wide junctions the critical current exhibits a Fraunhofer-like pattern [7,8]. However, very recent experiments in junctions where the width is comparable to the superconducting coherence length have shown a monotonic decay of the critical current with field, i.e., the absence of magnetic interference patterns [9]. The unified description of these two very different behaviors is a basic open problem.

In this Letter we show that the solution to the previous puzzle is closely related to the issue of the formation of a magnetic vortex structure in the normal conductor. Vortex matter in mesoscopic superconductors is also a very active field [10]. It has been shown that the basic properties such as critical fields [11] and the magnetization [12] depend crucially on the size and topology of the mesoscopic samples, which in turn determine the vortex structure. There is also a great interest in the study of nucleation of superconductivity and vortex matter in hybrid structures [13]. However, little attention has been paid to the formation of vortices inside nonsuperconducting materials. Our goal here is to answer the following fundamental questions: is it possible to induce a vortex structure in a normal wire by proximity to a superconductor? If so, what are the properties of such *proximity vortices* and their influence on the Josephson effect? For this purpose, we have studied a diffusive SNS junction in the presence of a perpendicular magnetic field. By solving the two-dimensional Usadel equations [14], we are able to describe the electronic properties for arbitrary length, L, and width, W, of the normal wire. We find that a magnetic vortex structure may develop in the normal metal with properties similar to those in the mixed state of a type II superconductor [15]. This vortex structure is reflected in the appearance of an interference pattern in the critical current that tends to the Fraunhofer pattern in the wide-junction limit ($W \gg \xi_H = \sqrt{\Phi_0/H}$). On the contrary, when W is comparable or smaller than ξ_H, the formation of vortices is not favorable and the field acts as a pair-breaking mechanism which suppresses monotonously the critical current. Our results not only solve the puzzle described above, but also illustrate the richness of the vortex physics in hybrid structures.

Quasiclassical formalism.—We consider a SNS junction, where N is a diffusive normal metal of length L and width W coupled to two superconductors with gap Δ. The junction is subjected to an uniform magnetic field $\mathbf{H} = H\hat{z}$ perpendicular to the normal film lying in the xy plane, where $x \in [0, L]$ and $y \in [-W/2, W/2]$. We assume that the thickness of the normal wire is smaller than the London penetration depth; i.e., the field penetrates completely in the normal region. To describe the electronic properties we use the quasiclassical theory of superconductivity in the diffusive limit [14,16], where the mean free path is much smaller than the coherence length, $\xi = \sqrt{\hbar D/\Delta}$, D being the diffusion constant of the normal metal. In equilibrium situations this theory is formulated in terms of retarded Green functions $\hat{G}^R(\mathbf{R}, \epsilon)$, which depend on position $\mathbf{R}$ and energy ϵ. This propagator is a 2×2 matrix in electron-hole space

$$\hat{G}^R = \begin{pmatrix} \mathcal{G}^R & \mathcal{F}^R \\ \tilde{\mathcal{F}}^R & \tilde{\mathcal{G}}^R \end{pmatrix}, \tag{1}$$

which satisfies the Usadel equation in the N region [16]

$$\frac{\hbar D}{\pi}\nabla(\hat{G}^R\check{\nabla}\hat{G}^R) + \epsilon[\hat{\tau}_3, \hat{G}^R] = \frac{ieD}{\pi}\mathbf{A}[\hat{\tau}_3, \hat{G}^R\check{\nabla}\hat{G}^R]. \quad (2)$$

Here, $\mathbf{A}$ is the vector potential, $\check{\nabla} = \nabla\hat{1} - (ie/\hbar)\mathbf{A}\hat{\tau}_3$, $\hat{\tau}_3$ is the Pauli matrix, and the Coulomb gauge ($\nabla\mathbf{A} = 0$) has been already used. Equation (2) is supplemented by the normalization condition $(\hat{G}^R)^2 = -\pi^2\hat{1}$ and proper boundary conditions. For the SN interfaces we use the boundary conditions introduced in Ref. [17], which allow us to describe the system for arbitrary transparency. For the metal-vacuum borders of the normal wire we impose that the current density in the y direction vanishes at $y = \pm W/2$ [18]. In general, the Usadel equation has to be solved together with the Maxwell equation $\nabla \times \mathbf{H} = \mu_0\mathbf{j}$ in a self-consistent manner. However, we are interested here in the case where the width W is smaller than the Josephson penetration length $\lambda_J = \sqrt{\hbar/2\mu_0 e j_c d}$, where j_c is the critical current per unit area and d is the effective length of the junction including the London penetration depths in the leads. In this case one can ignore the screening of the magnetic field by the Josephson currents and the field is equal to the external one [4].

The physical properties of interest can be expressed in terms of the Usadel-Green functions. Thus, for instance, the local density of states is given by $\rho(\mathbf{R}, \epsilon) = -\mathrm{Im}\, \mathcal{G}^R(\mathbf{R}, \epsilon)/\pi$. To quantify the superconducting correlations we use the pair correlation function defined as $F(\mathbf{R}) = (1/4\pi i)\int d\epsilon(\mathcal{F}^R - \mathcal{F}^A)\tanh(\beta\epsilon/2)$, where $\beta = 1/k_B T$. Finally, the supercurrent density in the junction can be written as $\mathbf{j}(\mathbf{R}) = (\sigma_N/4\pi^2 e)\int d\epsilon \tanh(\beta\epsilon/2) \times \mathrm{Re}\{\mathcal{F}^R\nabla\tilde{\mathcal{F}}^R - \tilde{\mathcal{F}}^R\nabla\mathcal{F}^R + (4ie/\hbar)\mathbf{A}\mathcal{F}^R\tilde{\mathcal{F}}^R\}$, where σ_N is the normal state conductivity. The net current is obtained integrating j_x across the y direction.

Equation (2) constitutes a set of coupled second-order nonlinear partial differential equations, whose resolution is a formidable task. In general, one has to resort to numerical methods [19]. However, one can get analytical insight in two limiting cases. By choosing the gauge $\mathbf{A} = -Hy\hat{x}$, one can identify in Eq. (2) the length $\xi_H = \sqrt{\Phi_0/H}$ as the characteristic variation scale of the Green functions in the transversal direction. In the narrow-junction limit, i.e., when $W < \xi_H$, the Green functions do not vary considerably in the y direction and after averaging Eq. (2) over this direction one obtains a one-dimensional equation analog to Eq. (2), but with the right-hand side replaced by $(\Gamma_H/2\pi)[\hat{\tau}_3\hat{G}^R\hat{\tau}_3, \hat{G}^R]$. Here, $\Gamma_H = De^2H^2W^2/(6\hbar)$ is a depairing energy, which in terms of the Thouless energy, $\epsilon_T = \hbar D/L^2$, can be written as $\Gamma_H = \epsilon_T(\pi\Phi/\sqrt{6}\Phi_0)^2$, where $\Phi = HLW$ is the flux enclosed in the junction. This equation can now be easily solved and describes the effect of a pair-breaking mechanism, such as magnetic impurities, that has been studied extensively in Ref. [20]. The other analytic case is the limit of a wide junction where $W \gg L, \xi_H$. A dimensional analysis shows that in this limit one can neglect the terms containing the derivatives with respect to the y coordinate. The field also disappears from the equation and its only effect is to change the superconducting phase difference ϕ into the gauge-invariant combination $\gamma = \phi - 2\pi(\Phi/\Phi_0)y/W$. Thus, it is easy to anticipate, in particular, that critical current exhibits a Fraunhofer-like pattern in this limit.

Discussion of the results.—We start by analyzing the local density of states (DOS) in the normal wire. In the absence of field the main feature is the presence of a minigap, Δ_g [21–23]. This minigap is the same throughout the normal wire and for perfect transparency scales as $\Delta_g \sim 3.1\epsilon_T$ in the limit $L \gg \xi$. In Fig. 1 we show the local DOS in the middle ($x = L/2$) of a wire of length $L = 2\xi$ for two different values of the width and the magnetic flux. Notice that for $W = \xi$ (see upper panels), the local DOS is practically independent of the y coordinate. Moreover, when the field is not very high, there is a clear minigap (see upper left panel), which closes at higher fields (see upper right panel). As one can see in the lower panels, when $W \gg L$, the local DOS is strongly modulated along the y direction. For low fields ($\Phi < \Phi_0$), the minigap is still open throughout the wire, but for higher fields the minigap changes in a periodic fashion from its maximum value (equal to the value in the absence of field) to exactly zero at well-defined positions where the DOS is the normal state one.

These results are in agreement with the limiting cases discussed above. If the wire is narrow the field acts as a pair-breaking mechanism. It is well known that the minigap is reduced by such mechanisms [20,22] and, in particular, it closes at a critical value of the depairing energy $\Gamma_H^C = \pi^2\epsilon_T/2$ [24], i.e., in our case at a critical flux $\Phi^C = \sqrt{3}\Phi_0$. This explains the results for $W = \xi$. To understand the results for $W = 50\xi$, we remind that in the wide limit the magnetic field only enters in the gauge-invariant phase difference γ. It has been shown that in the absence of field

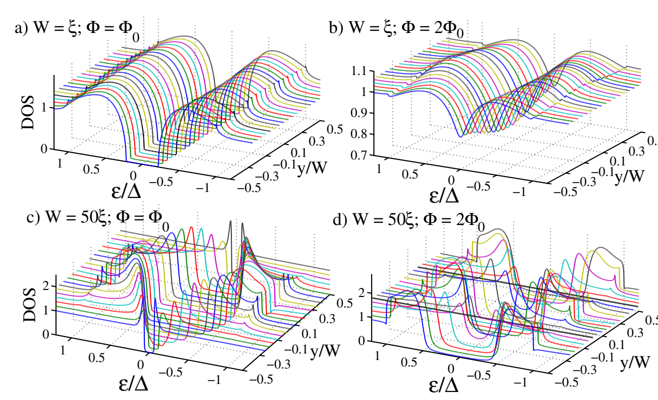

FIG. 1 (color online). Local density of states as a function of the energy in the middle of a wire of length $L = 2\xi$ for two values of the magnetic flux. The different curves correspond to different values of the y coordinate. We have assumed perfect transparency for the interfaces and a phase difference $\phi = 0$. In panel (d) we have used thicker lines to highlight the curves where the DOS is equal to the normal state one.

the minigap decreases monotonously as the phase difference increases and it closes when the phase is equal to π [23]. Bearing this in mind, one can easily understand the results of Fig. 1(c) and 1(d). When $\gamma = 0$ the minigap is completely open reaching the value in the absence of field. However, when $\gamma = \pi$ the minigap closes. For $\Phi = 2\Phi_0$ and $\phi = 0$, the phase γ takes the values $\mp\pi$ at $y/W = \pm 1/4$, which explains why the two thick curves in Fig. 1(d) correspond to normal state DOS.

The peculiar DOS suggests the presence of vortices in the normal wire. To confirm this idea, we have analyzed the pair correlation function, $F(\mathbf{R})$. In Fig. 2 we show a map of the modulus of this function throughout the normal wire for $L = 2\xi$ and $W/\xi = 1, 3, 50$. All the panels show that F diminishes towards the center of the wire, which simply reflects the decay of the superconducting correlations inside the normal wire. The main difference is the modulation along the y direction. In the case $W = \xi$, at low fields (see panel for $\Phi = \Phi_0$) F is still finite everywhere, while for higher fields it can be very small in the center of the wire, but with practically no modulation. The situation changes by increasing the width of the junctions. Already for $W = 3\xi$ [see Fig. 2(b)] one can clearly see that a linear array of vortices located on $x = L/2$ appears. Finally, for a very wide junction [see Fig. 2(c)] the vortex array becomes completely regular.

It is possible to get a deeper insight into the vortex structure by linearizing Eq. (2). This can be done assuming that the proximity effect is weak. In this case, using the gauge $\mathbf{A} = -Hy\hat{x}$ one obtains the following equation for the anomalous Green function

$$\partial^2_{\tilde{x}}\mathcal{F}^R + \left(\frac{L}{W}\right)^2 \partial^2_{\tilde{y}}\mathcal{F}^R + 4is\tilde{y}\partial_{\tilde{x}}\mathcal{F}^R - 4s^2\tilde{y}^2\mathcal{F}^R = -2i\frac{\epsilon}{\epsilon_T}\mathcal{F}^R,$$

where $s = \pi\Phi/\Phi_0$, $\tilde{x} = x/L$, and $\tilde{y} = y/W$. In the wide limit $W \gg L$, it is easy to find the solution of this equation, from which one can deduce that the zeros of the pair correlation function are given by: $x = L/2$ and $\phi - 2\pi(\Phi/\Phi_0)y/W = (2m+1)\pi$, where $m = 0, \pm 1, \ldots$ and $y \in [-W/2, W/2]$. This means that the vortex cores are located exactly on the middle of the wire forming a regular linear array along the y direction and they are separated by a distance Φ_0/HL. Thus, for the case $W = 50\xi$ in Fig. 2 this condition tells us that for $\Phi = 4\Phi_0$ there are four vortex cores located on $y/W = \pm 1/8$, $\pm 3/8$, which are the positions that one can read off from Fig. 2(c). Notice also that the phase ϕ simply shifts rigidly the line of vortices along the y direction. Thus, measurements of the local DOS at the outer interfaces ($y = \pm W/2$) of a wide junction changing the supercurrent through the structure should show an oscillatory behavior. Moreover, from the analytical solution of the linearized Usadel equation in the wide-junction limit and from the numerical results for arbitrary cases, one can show that the phase of the pair correlation changes in 2π around the cores; i.e., each vortex has a unit topological charge. These vortices are reminiscent of the those known in the literature as Josephson vortices, which appear in tunnel junctions where $W < \lambda_J$ [4]. However, there is a crucial difference. The vortices found in this work do have a normal core, which is absent in the standard Josephson vortices. This property is essential to have a complete analogy with the Abrikosov vortices. The only difference of the vortices presented here with those in a bulk superconductor of type II is that they are arranged in one-dimensional array instead of forming a two-dimensional lattice and due to the confining geometry they do not possess a rotational symmetry [15].

We discuss finally the magnetic field dependence of the critical current. In Fig. 3 we show an example for $L = 8\xi$, which is a typical value in the experiments [9], and different values of W. Notice that for small values of W, the critical current decays monotonously. This is simply due to

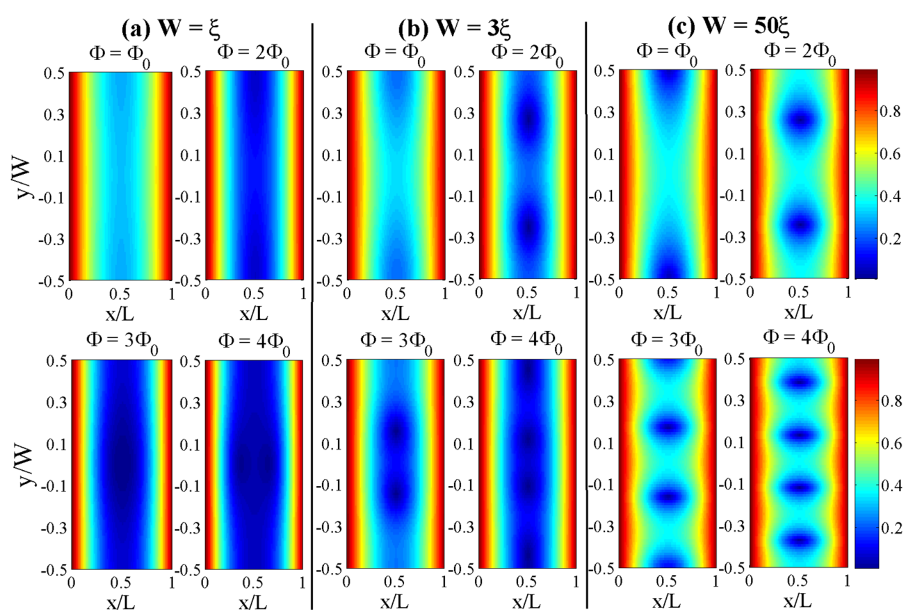

FIG. 2 (color online). Spatial map of the modulus of the pair correlations, $|F(\mathbf{R})|$, for $L = 2\xi$ and $\phi = 0$. The different panels correspond to different values of the width W and the magnetic flux Φ, as indicated in the graphs. $|F(\mathbf{R})|$ has been normalized to its value inside the electrodes, the temperature is $k_BT = 0.01\Delta$, and perfect transparency was assumed.

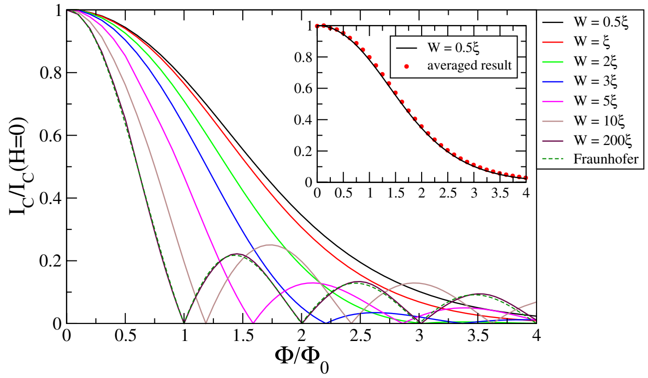

FIG. 3 (color online). Critical current normalized by the zero-field value vs magnetic flux for a wire length $L = 8\xi$, perfect transparent interfaces, $k_B T = 0.01\Delta$, and different values of W. The dashed line shows the standard Fraunhofer pattern given by $\sin(\pi\Phi/\Phi_0)/(\pi\Phi/\Phi_0)$. The inset shows for $W = 0.5\xi$ the comparison between the exact result and the approximation used for the narrow-junction limit.

the fact that in this limit no vortices appear and the field suppresses progressively the superconductivity in the normal wire [20]. Indeed, as we show in the inset of Fig. 3, the analytical result of the narrow-junction limit describes quantitatively the field dependence in this limit. As the width increases the vortex structure appears and as a consequence one observes an interference pattern where the critical current vanishes at certain values of the magnetic flux. Notice that these values are clearly larger than Φ_0 for intermediate widths and the patterns are not ‘‘periodic’’. Only in the limit $W \gg \xi_H, L$ one obtains a regular pattern with zeros at multiples of Φ_0, recovering the Fraunhofer pattern [4]. These results explain in a unified manner the different behaviors observed experimentally [7–9], which at first glance seemed to be contradictory.

Finally, we have studied systematically the role of the length L in the crossover from the narrow-junction to the wide-junction behavior. We have found that as L increases this transition occurs at larger values of W. This confirms the fact that the condition for the appearance of an interference pattern, i.e., zeros in the critical current, is given, roughly speaking, by $W > \xi_H$, which is equivalent to $W/L > \Phi_0/\Phi$. The standard Fraunhofer pattern is approached when $W \gtrsim L$.

Conclusions.—We have shown that the appearance of magnetic interference patterns in the critical current of diffusive SNS junctions is intimately linked to the formation of a vortex array in the normal wire. Our results provide a unified description of the critical current for arbitrary width of the junctions and solve the puzzle put forward by recent experiments [9]. Our work also paves the way to study the vortex matter in a great variety of hybrid structures like the recently introduced superconducting graphene junctions, where a standard Fraunhofer pattern has been observed [25].

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