Transport through superconductor/magnetic dot/superconductor structures

M. Andersson [a], J.C. Cuevas [b], M. Fogelström [a],*

[a] *Institute of Theoretical Physics, Chalmers University of Technology and Göteborg University, S-412 96 Gothenburg, Sweden*
[b] *Institut für Theoretische Festkörperphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany*

Abstract

The coupling of two s-wave superconductors through a small magnetic dot is discussed. Assuming that the dot charging energy is small compared to the superconducting gap, $E_c \ll \Delta$, and that the moment of the dot is classical, we develop a simple theory of transport through the dot. The presence of the magnetic dot will position Andreev bound states within the superconducting gap at energies tunable with the magnetic properties of the dot. Studying the Josephson coupling it is shown that the constructed junction can be tuned from a "0" to a "π"-junction via a degenerate two-level state either by changing the magnetic moment of the dot or by changing temperature. Furthermore, it is shown that details of the magnetic dot can be extracted from the sub-harmonic structure in the current–voltage characteristics of the junction. © 2002 Elsevier Science B.V. All rights reserved.

PACS: 74.50.+r; 74.25.Ha; 74.80.−g; 74.80.Fp
Keywords: Magnetic dot; Superconducting gap; Josephson junction; Andreev reflection

1. Introduction

If a superconductor is exposed to magnetically active impurities [1,2] or materials [3] the superconducting state is modified. Coupling two superconductors through a magnetically active barrier may lead to what is known as a "π"-junction [2], a junction for which the ground state has an internal phase shift of π between the superconductors across the barrier. If the barrier is extended to an S/F/S structure, a ferromagnetic (F) layer sandwiched between two superconductors (S), the critical current will oscillate as the thickness of F is varied [3]. Recently, this behavior was reported in a mesoscopic S/F/S structure and a systematic change from a "0" to "π" Josephson junction as a function of the F-layer thickness was seen [4]. In this paper we present a different scenario which gives rise to qualitatively the same physics, namely, two s-wave superconductors connected through a small magnetic dot. In this case the effect of the magnetic dot arises from the spin-active contact at the interface between the dot and the superconductor in contrast to the internal exchange field of the ferromagnet as in Refs. [3,4].

In Fig. 1 we show a cartoon of our system: two *conventional* superconductors brought in electrical contact through a magnetic grain. The size of the grain, d, is small compared to the superconducting coherence length, ξ_0, so that $d/\xi_0 \ll 1$ holds. The dot is characterized by a large classical magnetic

---
* Corresponding author. Tel.: +46-31-772-3196; fax: +46-31-772-3204.
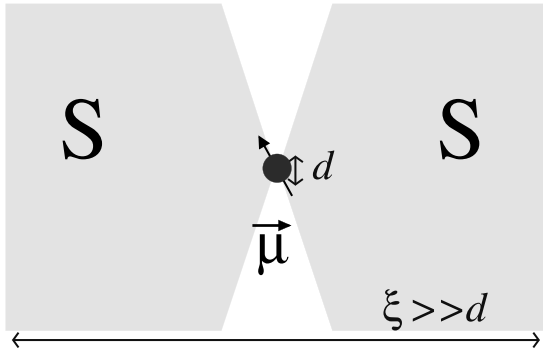*E-mail address:* fogelstr@fy.chalmers.se (M. Fogelström).

Fig. 1. A schematic illustration of the superconductor/magnetic dot/superconductor contact considered. The size of the dot, d is small compared to the superconducting coherence length ξ_0. It is also assumed that the moment, $\vec{\mu}$, is large, i.e. in the classical limit.

moment, $\vec{\mu}$, and despite the small size of the grain we shall assume that charging effects on the dot can be neglected, i.e. $E_c \ll \Delta$. In our approach the effect of the magnetic dot enters via boundary conditions for the quasiclassical Green's function. In this spirit the contact over the magnetic dot is described by the following $\hat{S}$-matrix

$$\hat{S} = \begin{pmatrix} r & t \\ t & -r \end{pmatrix} \exp\left(i\frac{\Theta}{2}\hat{\sigma}_\mu\right) \qquad (1)$$

where r and t are the usual reflection and transmission coefficients. The spin-dependence enters via a rotation of the quasiparticle spin by an angle Θ around the moment axis, $\hat{\sigma}_\mu$. The angle Θ is a phenomenological parameter that quantifies the degree of "spin-mixing" induced by the moment $\vec{\mu}$ and varies between 0 (no spin mixing) and π (strong spin mixing) [5,6]. The calculations presented here are done within the quasiclassical theory as presented in Refs. [6,7] and references therein.

2. Local quasiparticle density of states

One of the most pronounced effects of the moment, $\vec{\mu}$, on the superconducting density of states (DOS) is the formation of Andreev states at $\varepsilon_B(\Theta) = \pm\Delta\cos(\Theta/2)$. The two signs refer to the two "spin bands" on which the pairing amplitudes

are $\sim\langle\psi_\uparrow\psi_\downarrow\rangle(+)$ and $\sim\langle\psi_\downarrow\psi_\uparrow\rangle(-)$ respectively. The formation of these Andreev states is quite easily understood since the moment will give reflected quasiparticles a phase shift of $\exp(\pm i\Theta)$. The sign of the phase shift depends on the spin of the quasiparticle: $(+)$ for spin up and $(-)$ for spin down. Introducing the moment axis, $\hat{\sigma}_\mu$, breaks the rotational symmetry in spin space of the superconductors and pins down the direction of the spin-quantization axis to be parallel to $\hat{\sigma}_\mu$.

Formation of Andreev states is a signature of a reduced superconducting order by pair breaking. The Andreev states are spatially localized at the magnetic moment and their spectral weight decays exponentially in to the superconductors over a distance given by $\xi(\varepsilon_B) = v_F/2(\Delta^2 - \varepsilon_B^2)^{1/2}$, where v_F is the Fermi velocity in the superconductor. This gives that superconductivity heals to its bulk value over a distance $\sim\xi(\varepsilon_B)$ into the superconductor. However, in the limit $d/\xi_0 \ll 1$ the pair-breaking effect of the dot on the superconducting order parameter is small and we can, to good approximation, assume a constant order parameter up to the contact. Another consequence of the relative smallness $(d/\xi_0 \ll 1)$ of the dot is that the broadening of the Andreev state by elastic impurity scattering is suppressed.

It is important to note that the positions of the bound states do not disperse with momentum $\hat{p}$ but do only depend on the angle Θ. This makes the position of states ε_B robust and the DOS shown in Fig. 2(a) are angle averaged ones at the dot. Also important to note is that the DOS in both superconductors are symmetric and ε_B coincide in both sides. This makes the DOS independent of the transmission probability, D, as long as there is no phase difference χ applied across the junction.

3. Tunable Josephson-junction properties

It is well known that the Josephson current is carried by junction Andreev states [8–10]. In case of a magnetic dot the junction Andreev states split into four states, labeled by their momentum directions $\pm\hat{p}$ and by spin band, as a phase difference χ is applied across the junction [7]. We have
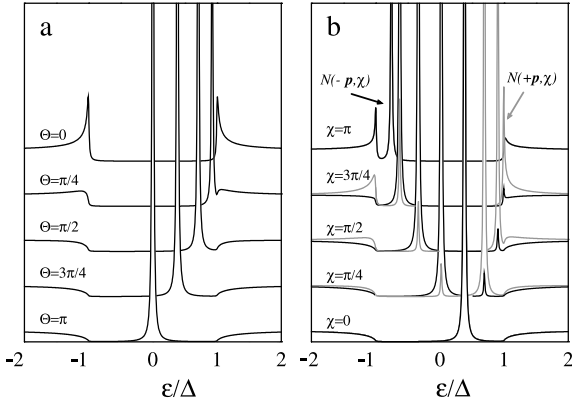
Fig. 2. The DOS at a magnetic dot for different values of Θ in (a). The bound states appear at $\varepsilon_B = \pm\Delta\cos(\Theta/2)$, the different signs refer to two the spin bands (see text). Here we plot $\varepsilon_B = +\Delta\cos(\Theta/2)$. In panel (b) we display the DOS for the momenta $\pm\hat{\boldsymbol{p}}$, on the same spin band as in (a), with $\Theta = 3\pi/4$ for various values of phase difference χ over the junction, $D = 0.9$.

$$\varepsilon_B(\chi) = \pm\Delta\left[\cos^2\left(\frac{\Theta}{2}\right) - D\cos\Theta\sin^2\left(\frac{\chi}{2}\right)\right.$$
$$\left. \pm\sqrt{D}\sin\Theta\sin\left(\frac{\chi}{2}\right)\sqrt{1 - D\sin^2\left(\frac{\chi}{2}\right)}\right]^{1/2} \tag{2}$$

which now not only depends on Θ but also on χ and D. In Fig. 2(b) we show the DOS for the two opposite momentum directions, $+\hat{\boldsymbol{p}}$ and $-\hat{\boldsymbol{p}}$, on the positive spin band with $\Theta = 3\pi/4$, for various χ at $D = 0.9$. As seen in Fig. 2(b) the Andreev states in the DOS splits equally for *both* $+\hat{\boldsymbol{p}}$ and $-\hat{\boldsymbol{p}}$ with applied phase difference χ as given by Eq. (2). The spectral weight of the bound states, $w(\varepsilon_B(\chi, \hat{\boldsymbol{p}}))$, however, is not symmetric in momenta, i.e. $w(\varepsilon_B(\chi, -\hat{\boldsymbol{p}})) \neq w(\varepsilon_B(\chi, +\hat{\boldsymbol{p}}))$. This leads to, after summing up all contributions, a net Josephson current that may have additional zeros in the phase interval 0 and π.

The sensitivity of the quasiparticle spectra in Eq. (2) to the degree of spin mixing, Θ, and to the transmission probability D at finite phase difference, makes the magnetic dot a very tunable Josephson contact between the two superconductors it connects. In d-wave superconductors, as the high-T_c cuprates, Andreev states at zero-energy

lead to anomalous Josephson properties [11,12]. In an ideal junction between two d-wave superconductors the zero-energy states will cause the critical current to diverge $\sim 1/T$ at low temperature [11]. In addition, the energetics of the junction may change from being a 0-junction at temperatures just below T_c to a π-junction at low temperatures $T \ll T_c$ [11,12].

As demonstrated in Refs. [6,7] the Josephson coupling of two s-wave superconductors through a magnetically active barrier shows much of the same anomalies as seen in the d-wave/d-wave contact. In Fig. 3(a) we show the critical current as a function of temperature for several values of Θ ranging for 0 to π. At low temperatures and for Θ close to π the critical current is roughly an order of magnitude larger than the value of a non-magnetic dot, i.e. at $\Theta = 0$. In fact, it is a finite transparency, in Fig. 3(a) put to $D = 0.1$, or an inelastic or phase-breaking life time, $\tau_{\text{inel,pb}}$, depending on which gives the largest broadening of the junction
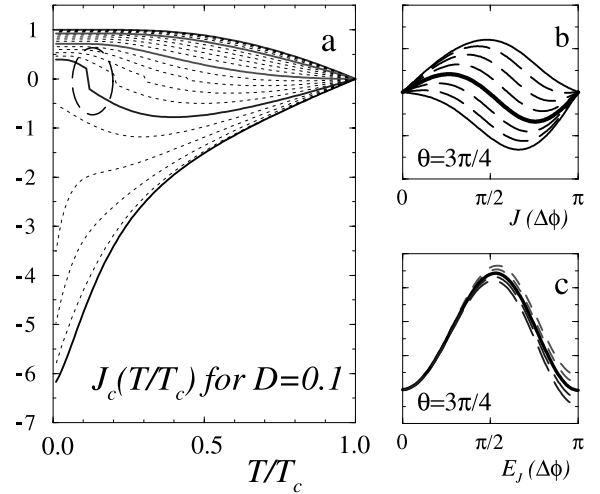

Fig. 3. The critical current vs. temperature of a magnetic dot Josephson contact is shown in (a). The spin-mixing angle, Θ is varied from 0 to π (top to bottom) in steps of $\pi/20$. The transmission probability D is 0.1 ($D = |t|^2$). In panels (b) and (c) we show the current–phase relation (b) and the junction energy vs. phase (c) for $\Theta = 3\pi/4$ at temperatures just around the switching temperature $T_{\text{sw}} \sim 0.12 T_c$, highlighted by the dashed circle. At T_{sw} the junction changes state from a π to 0-junction. As seen, the junction goes through a degenerate π-periodic state at the switching temperature.

states, that cuts of the growing critical current as $T \to 0$ [6,7].

In Fig. 3(b) and (c) we show the current–phase and the energy–phase relations of a junction with $\Theta = 3\pi/4$ and $D = 0.1$. The temperature is swept over a small interval including T_{sw}. At $T = T_{sw}$ the junction changes its energy state from being a 0-junction at low temperatures to being a π-junction at $T > T_{sw}$. This transition goes over π-periodic current–phase relation at $T = T_{sw}$ and here the 0 and π states of the junction are degenerate. A similar transition may be achieved at fixed T and instead sweeping Θ.

4. Current–voltage characteristics

As introduced, the spin-mixing angle is a phenomenological parameter in the same spirit as the junction transmission probability is. In principle one should be able to give a microscopic justification of the value of Θ by calculation along the lines in Ref. [5]. In a practical realization of a magnetic dot it is more likely that an effective Θ is measured and it is important when characterizing the junction to extract the value of Θ by comparing measurements with calculations. Pinning down Θ and D can be done by measuring several junction specific properties such as the Josephson current, the current–voltage characteristics and the noise spectrum of the junction as was recently done in Al point contacts [13]. Here we compute one of these properties, the dc current–voltage curves, for a single-channel contact as a function of transparency, D, and spin mixing, Θ, using the methods described in Ref. [7].

At a constant voltage bias, $|eV| < 2\Delta$, applied over a junction between two superconductors the only way a dc current can flow through the contact is via multiple Andreev reflection (MAR) [14]. This simply means that in order to transfer a quasiparticle at energy $\varepsilon = -\Delta$ in the superconductor at bias $-|eV|/2$ to energy $\varepsilon = \Delta$ in the superconductor at bias $+|eV|/2$ the quasiparticle must undergo $2\Delta/|eV|$ Andreev reflections. At each reflection event the quasiparticle is accelerated and shifted up in energy by an amount $|eV|$. In the current–voltage characteristics this shows up in an abrupt

increase in the current each time the voltage is swept over a value where $\mathrm{mod}(2\Delta, |eV|)$ changes. This holds true for the conventional superconductor for which there are no states within the superconducting gap [15,16]. The resulting sub-gap structure in the current–voltage curve for $\Theta = 0$ is shown in Fig. 4(a) for several values of D.

Introducing Andreev states within the gap, by a finite value of Θ, we modify the MAR picture above. Now, having the bound state $\varepsilon_B(\Theta) = \pm\Delta\cos(\Theta/2)$ and starting at a large voltage bias $|eV| \lesssim 2\Delta$, the first voltage below 2Δ to connect two points in the quasiparticle spectra, on either side of the junction, with a finite DOS, and thus to give a contribution to the current, is at $|eV|_1 = (1 + \cos(\Theta/2))\Delta$. There are two possible processes giving current at $|eV|_1$ per spin band. For the band with $\varepsilon_B(\Theta) = +\Delta\cos(\Theta/2)$ we have: (i) connecting an energy at $-\Delta$ with $\varepsilon_B(\Theta)$, i.e. an electron-like quasiparticle tunneling into the bound state from the continuum states below $-\Delta$, (ii) connecting $\varepsilon_B(\Theta)$ with the continuum states at $-\Delta$, i.e. a hole-like quasiparticle tunneling from the bound state into the continuum states below $-\Delta$. For the spin band with $\varepsilon_B(\Theta) = -\Delta\cos(\Theta/2)$ the same processes occur with the modification that the quasiparticle (hole) start in the bound state and ends up in the continuum at $+\Delta$. The next "spectral current channel" to open is at voltages $|eV|_2 = (1 + \cos(\Theta/2))(\Delta/2)$ which, via a single Andreev reflection off the opposing superconductor, connects finite DOS in the same electrode. Adding more and more spectral current channels, $|eV|_n$, we arrive at

$$|eV|_n = \left(1 + \cos\left(\frac{\Theta}{2}\right)\right)\frac{\Delta}{n} \qquad (3)$$

which gives the location of the strong features in the current–voltage curves at different spin-mixing angles, Θ. This is clearly demonstrated in Fig. 4 where the position of the sub-gap structures described by Eq. (3) are marked.

Why do you not see structure in $|eV|_n = 2\Delta/n$ as well? In principle, one could think that the usual sub-harmonic gap structure should be superimposed on this new one. The answer is that you still have the usual processes between $-\Delta$ and $+\Delta$, but
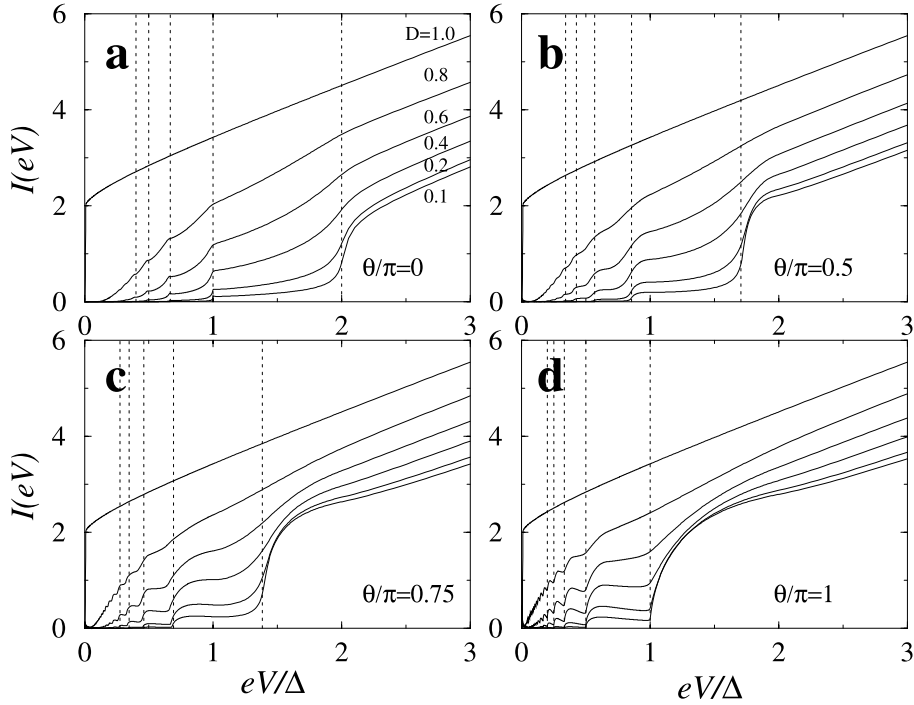
Fig. 4. The current–voltage characteristics for the magnetic Josephson contact shown for $\Theta = 0, \pi/2, 3\pi/4$, and $\Theta = \pi$ in panels (a) through (d). In each panel we show the *IV* curve for different transparencies as labeled in panel (a). The *IV* curves in panel (a) correspond to an ordinary single-channel point contact between two s-wave superconductors. The appearance of Andreev states at $\varepsilon_B = \pm\Delta\cos(\Theta/2)$ changes the sub-gap structure of the magnetic dot (b–d) as described in Eq. (3) and here marked by the vertical dashed lines. In each case the current is given in units $(G_N\Delta/e)$ where G_N is the normal conductance.

now the DOS at the gap edges is rather small. Thus, opening the possibility for one of these processes, its probability is small (although its contribution will increase with voltage).

Finally we should mention that the *IV* curves in Fig. 4 are calculated with a very small intrinsic broadening, $1/2\tau_{inel,pb} = 10^{-4}\Delta$. Increasing $1/2\tau_{inel,pb}$ smears out the sharp features at low voltages in to a large current peak. This is particularly strong when $\varepsilon_B \approx 0$, i.e. for Θ close to π. At $\Theta = \pi$ the *IV* curves are similar to those of a symmetric, 45° rotated, d-wave/d-wave junction [7,17].

limit where charging effects may be neglected. We show that the Andreev bound state spectra is tunable with the properties of the dot. This allows realizations of Josephson junctions that are either 0 or π-junctions. Furthermore, we look at ways to extract information of the dot from experimental data and in detail study the current–voltage characteristics. The presence of Andreev states in the DOS allows for current through resonant tunneling [18] and modifies the sub-gap structure in the *IV* curves compared to the case of a conventional SIS contact.

5. Conclusions

In conclusion we have presented a simple theory to handle transport through magnetic dots connecting two conventional superconductors in the

Acknowledgements

References

[1] H. Shiba, Prog. Theor. Phys. 40 (1968) 435;
H. Shiba, T. Soda, Prog. Theor. Phys. 41 (1969) 25.

[2] L.N. Bulaevskiǐ, V.V. Kuziǐ, A.A. Sobyanin, JETP Lett. 25 (1977) 290.

[3] A.I. Buzdin, L.N. Bulaevskiǐ, S.V. Panyukov, JETP Lett. 35 (1982) 178.

[4] V.V. Ryazanov et al., Phys. Rev. Lett. 86 (2001) 2427.

[5] T. Tokyyasu, J.A. Sauls, D. Rainer, Phys. Rev. B 38 (1988) 8823.

[6] M. Fogelström, Phys. Rev. B 62 (2000) 11812.

[7] J.C. Cuevas, M. Fogelström, Phys. Rev. B 64 (2001) 104502.

[8] A. Furusaki, M. Tsukada, Physica B 165–166 (1990) 967;
C.W.J. Beenakker, Phys. Rev. Lett. 67 (1991) 3836.

[9] R.A. Riedel, P.F. Bagwell, Phys. Rev. B 57 (1998) 6084.

[10] Yu.S. Barash, Phys. Rev. B 61 (2000) 678.

[11] Y. Tanaka, S. Kashiwaya, Phys. Rev. B 53 (1996) 11957;
Yu.S. Barash, H. Burkhardt, D. Rainer, Phys. Rev. Lett. 77 (1996) 4070.

[12] S.-K. Yip, Phys. Rev. B 52 (1995) 3087;
M. Fogelström, S.-K. Yip, Phys. Rev. B 57 (1998) R14060.

[13] R. Cron et al., Phys. Rev. Lett. 86 (2001) 4104.

[14] T.M. Klapwijk, G.E. Blonder, M. Tinkham, Physica B 109–110 (1982) 1657.

[15] E.N. Bratus, V.S. Shumeiko, G. Wendin, Phys. Rev. Lett. 74 (1995) 2110;
D. Averin, A. Bardas, Phys. Rev. Lett. 75 (1995) 1831;
J.C. Cuevas, A. Martín-Rodero, A. Levy Yeyati, Phys. Rev. B 54 (1996) 7366.

[16] E. Scheer et al., Phys. Rev. Lett. 78 (1997) 3535;
E. Scheer et al., Nature (London) 394 (1998) 154.

[17] T. Löfwander, G. Johansson, G. Wendin, J. Low Temp. Phys. 117 (1999) 593.

[18] A. Levy Yeyati et al., Phys. Rev. B 55 (1997) R6137;
G. Johansson et al., Phys. Rev. B 60 (1999) 1382.