SUPPLEMENTARY MATERIAL: EXPERIMENTAL REALISATION OF A QUANTUM DOT ENERGY HARVESTER

Estimating temperature differences

In this section, we discuss how the temperatures in Fig. 1 (a) were estimated. The cavity temperature was estimated as a function of the current applied to the heating channel, whose measurement set-up is similar to the one shown in Fig. 1 (b). A DC current was applied to the heating channel using an *NicDAQ-9178*, and the conductance was measured through the left dot between $A-B$ with a lock-in amplifier (no R_{Load} was present i.e. $R_{\mathrm{Load}}=\infty$), while sweeping the voltage V_{LD} on the left plunger gate with fixed voltages on the other gates. To extract the temperature of the cavity at each DC heating current, the conductance, G, of a Coulomb Peak is fitted to a thermally broadened Lorentzian, parameterised by a full width at half-maximum Γ [54],

$$G=\frac{e^2}{h}\frac{1}{4k_BT_C}B\int_{-\infty}^{+\infty}\cosh^{-2}\left(\frac{E}{2k_BT_C}\right)\times \frac{(\Gamma/2)\pi}{(\Gamma/2)^2+[(e\alpha V_g-E_{res})-E]^2}dE. \tag{6}$$

Where e is the electron charge, k_B is the Boltzmann Constant, T_{C} is the electron temperature of the cavity, B is the temperature-independent energy-integrated strength of the resonance, E and E_{res} are the energy of the dot level and the energy of resonance respectively. Here $E=e\alpha V_{LD}$, with $\alpha=0.025$ the lever arm of the plunger gate, ascertained via measurement of the Coulomb Diamonds for each dot, which is discussed in the next section. For DC currents from 0 to 100 nA, Eq. (6) gives a temperature range from 75 mK to 150 mK, as shown in Fig. 6(b).

Our data shows the power dissipated per electron[50] is best fit with $P=I^2R/(n_eC)=\beta(T_C^5-T_0^5)$ with $R=500\,\Omega$, $C=350\,\mu\mathrm{m}^2$ and $\beta=1.8\times10^{-15}\,\mathrm{WK}^{-5}$, shown as the orange line in Fig. 6(b). Here, I is the heating current, R is the resistance of the heating channel and C is the area of the heating channel. This T^5 behavior was attributed to acoustic phonon scattering in the Grüneisen-Bloch regime in the heating channel, with coupling via a screened piezoelectric potential [55]. The $\beta=1.8\times10^{-15}\,\mathrm{WK}^{-5}$ is larger than the theoretical prediction of $9.6\times10^{-18}\,\mathrm{WK}^{-5}$ ($60\,\mathrm{eV/sK}^5$) [50]. This can be because some heat is leaking through ohmic contacts at each end of the heating channel. For $T\sim500\,\mathrm{mK}$, the heated electrons relax to the lattice temperature over a distance $l_{\text{e-ph}}\sim200\,\mu\mathrm{m}$, and as T is lowered further, $l_{\text{e-ph}}$ can significantly exceed the size of the cavity of the device [13]. In this regime, the energy redistribution is achieved via electron diffusion to the cold reservoirs [56].

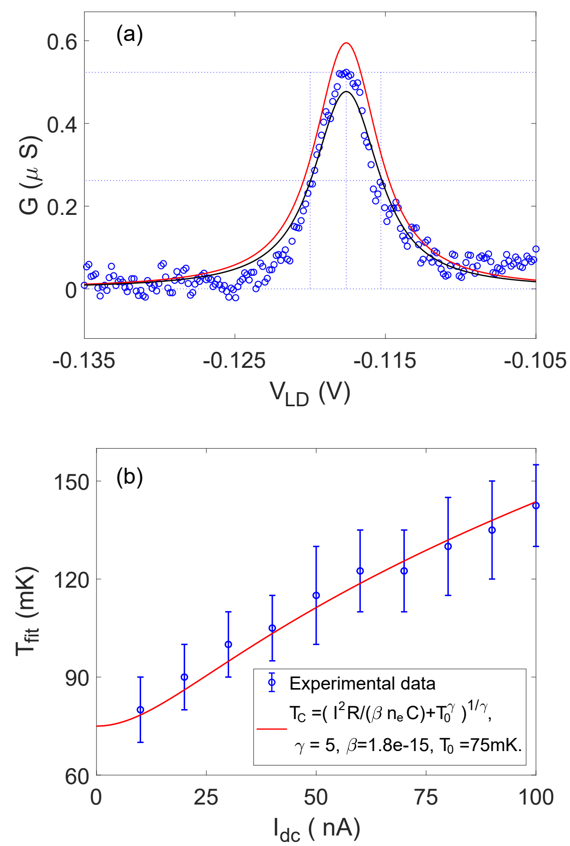

FIG. 6. (a) Conductance G plotted versus plunger voltage V_{LD}. The plot is a representative Coulomb resonance peak, measured at a lattice temperature of 75 mK and with a DC heating current of 30 nA applied to the heating channel. The circles are the experimental data points and the red and the black lines are the minimum 90 mK and maximum 110 mK theoretical fits by using Eq. (6). (b) The blue dots are the estimated electrical temperatures T_C using Eq. (6), with the fitting method shown in (a), and the red line is the relative general relationship between the energy loss rate and the temperature.

Therefore, in this device, hot electrons diffuse out from the heating channel to the central cavity and replace cold electrons, serving to redistribute energy and leading to a well-defined electron temperature profile.

We assume the central cavity and the reservoir of the heating channel share the same temperature T_C, and the left and the right reservoirs have the same temperature with the base temperature, $T_L=T_R=T_0$. The base temperature estimated by this analysis is 75 mK. The quantum dot was used as the thermometer in this experiment, because it has greater accuracy than the thermometer in the mixing chamber of this dilution refrigerator, which gave a base temperature of 50 mK.

Coulomb blockade analysis

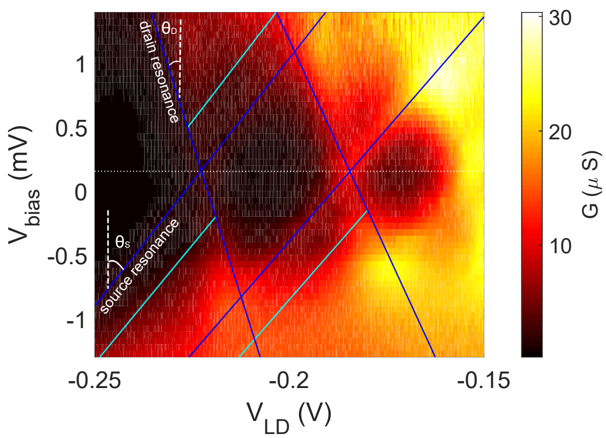

FIG. 7. Coulour scale plot of measured transport through the left quantum dot in Fig. 1, plotted as a function of the voltage on the plunger gate $\mathrm{V_{LD}}$, and of the source drain bias $\mathrm{V_{bias}}$. The 'Coulomb Diamond' in which transport through the dot is blockaded is depicted by the blue solid lines. Increasing the source drain bias increases the range of plunger voltage over which the dot is unblocked. At a sufficiently large source drain bias, the first excited level enters the transport window, and conduction through the dot increases, depicted as the light blue lines in the figure. The horizontal dashed line marks the actual zero bias point of Coulomb Diamonds, and the $\mathrm{V_{bias}}$=0.155 mV is the offset from the measurement.

When an isolated island of charges have a sufficiently small capacitance, the energy required to change its charge by even one electron may be large. Until this energy is supplied, no charge may move onto or off the island. This is called *Coulomb blockade*. During our experiment, the two quantum dots have firstly been tuned to have energy levels lying between the potentials of source and drain reservoirs. In this situation, the Coulomb blockade has been lifted and electrons with energies that match the energy level lying within the bias window can tunnel through the dots. When the potential of the dots is changed by a gate electrode, the plunger gate in Fig. 1, the current through a quantum dot shows periodic oscillations in plunger-gate voltage, which is known as the *Coulomb Peak*. Increasing the source-drain bias across a quantum dot will increase the window of energies over which states in the source are full and states in the drain are empty. With a large enough bias, the widened conductance peaks from adjacent charge states of dots overlap leaving the diamond shape regions, which are commonly referred to as *Coulomb Diamonds* [49]. In reality, the excited states start contributing to transport before the transport window is wide enough to include the next charge state of the dot, shown as light blue lines in Fig. 7. The boundaries of the Coulomb diamonds are labeled as 'source resonance' and 'drain source' in Fig. 7. This is because they correspond to the dot level being aligned with the source and drain potentials respectively. The gradient of these resonances can be used to calibrate the conversion factor between ΔV_{g} and the electrochemical potential of the dot. The gradient are defined as [49]:

$$m_{\mathrm{S}} = \tan(\theta_{\mathrm{S}}) = \left(\frac{\Delta V_{\mathrm{g}}}{\Delta V_{\mathrm{SD}}}\right)^{(S)}, \tag{7}$$

$$m_{\mathrm{D}} = \tan(\theta_{\mathrm{D}}) = \left(\frac{\Delta V_{\mathrm{g}}}{\Delta V_{\mathrm{SD}}}\right)^{(D)}. \tag{8}$$

The superscript in the right hand expression denotes whether the gradient is of the source or drain resonance line. The gate electrode *lever arm* is given by the value as:

$$\alpha_{\mathrm{G}} = \frac{1}{m_{\mathrm{S}} - m_{\mathrm{D}}}. \tag{9}$$

The *lever arm* for the gating effect on the dot energy from the biased reservoir (the drain) can also be found:

$$\alpha_{\mathrm{D}} = \frac{1}{1 - m_{\mathrm{S}}/m_{\mathrm{D}}}. \tag{10}$$

With the bias applied to only the drain reservoir, α_{S}, the *lever arm* of the source cannot be found.

The lever arm of a gate (or lead) quantifies the effect of the gate on the potential of the dot. It is a useful parameter to convert the experimentally measured voltage to energy. During this experiment, Coulomb diamonds from the left dot gives 0.025 lever arm, which is crucial to the analysis of electrical temperatures.