Resonance Fluorescence in Transport through Quantum Dots: Noise Properties

Rafael Sánchez,[1] Gloria Platero,[1] and Tobias Brandes[2]
[1]*Instituto de Ciencia de Materiales, CSIC, Cantoblanco, Madrid, 28049, Spain*
[2]*Institut für Theoretische Physik, Technische Universität Berlin, D-10623 Berlin, Germany*
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We study a two-level quantum dot embedded in a phonon bath and irradiated by a time-dependent ac field, and develop a method that allows us to extract simultaneously the full counting statistics of the electronic tunneling and relaxation (by phononic emission) events as well as their correlation. We find that the quantum noise of both the transmitted electrons and the emitted phonons can be controlled by the manipulation of external parameters such as the driving field intensity or the bias voltage.

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The complete knowledge of the statistics and, in concrete, the properties of the fluctuations of the number of particles emitted from a quantum system has been a topic of intense studies in quantum optics [1,2] and, in more recent years, in quantum transport [3,4]. In particular, *purely quantum* features like an antibunching of photons emitted from a closed two-level atom under a resonant field [5], or a bunching of electrons tunneling through interacting two-level quantum dots (QD) [6] have been reported. The electronic case receives special interest since it has been recently addressed for many different physical systems such as single electron tunneling devices [7–12], molecules [13], charge shuttles [14], surface acoustic waves driven single electron pumps [15], or beam splitting configurations [16]. Here, we show that the *combined* statistics of fermions and bosons is a very sensitive tool for extracting information from time-dependent, driven systems. In particular, phonon emission has been measured by its influence on the electronic current in two-level systems [17]. We analyze the electron and phonon noises and find that they can be tuned back and forth between sub- and super-Poissonian character by using the strength of an ac driving field or the bias voltage. For this purpose, we develop a general method to simultaneously extract the full counting statistics of single electron tunneling and (phonon mediated) relaxation events.

Our system consists of a two level QD connected to two fermionic leads by tunnel barriers; cf. Fig. 1. The Coulomb repulsion inside the QD is assumed to be so large that only single occupation is allowed (*Coulomb blockade* regime). The lattice vibrations induce, at low temperatures, inelastic transitions from the upper to the lower state. In analogy to resonance fluorescence (RF) in quantum optics, a time-dependent ac field with a frequency ω drives the transition between the two levels ε_1, ε_2 close to resonance, $\Delta_\omega = \varepsilon_2 - \varepsilon_1 - \omega \approx 0$, which allows us to assume the rotating wave approximation. Thus, the electron in the QD is coherently delocalized between both levels performing *photon-assisted Rabi oscillations* [18]. For simplicity, we consider spinless electrons.

The components of the density matrix $\rho(t) = \sum_{n_e,n_{\rm ph}} \rho^{(n_e,n_{\rm ph})}(t)$ give the probability that, during a certain time interval t, n_e electrons have tunneled out of a given electron-phonon system and $n_{\rm ph}$ phonons have been emitted [19]. We define the generating function (GF) [20,21] $G(t, s_e, s_{\rm ph}) = \sum_{n_e,n_{\rm ph}} s_e^{n_e} s_{\rm ph}^{n_{\rm ph}} \rho^{(n_e,n_{\rm ph})}(t)$, where $s_{e({\rm ph})}$ are the electron (phonon) counting variables whose derivatives give us the correlations

$$\frac{\partial^{p+q}{\rm tr}G(t,1,1)}{\partial s_e^p \partial s_{\rm ph}^q} = \left\langle \prod_{i=1}^{p}\prod_{j=1}^{q}(n_e - i + 1)(n_{\rm ph} - j + 1)\right\rangle. \quad (1)$$

Thus, we are able to obtain the mean number $\langle n_\alpha\rangle$, the variance $\sigma_\alpha^2 = \langle n_\alpha^2\rangle - \langle n_\alpha\rangle^2$ (which give the $\alpha = e$, ph current and noise, respectively), or define the correlation between the electron and phonon counts, $\langle n_e n_{\rm ph}\rangle$.

We integrate the equations of motion for the GF

$$\dot{G}(t, s_e, s_{\rm ph}) = M(s_e, s_{\rm ph})G(t, s_e, s_{\rm ph}), \quad (2)$$

that generalizes the Master equation, $\dot{\rho}(t) = M(1,1)\rho(t)$, by introducing the counting variables in those terms corresponding to the tunneling of an electron to the collector lead and the emission of a phonon.

The longtime behavior is extracted from the pole near $z = 0$ in the Laplace transform of the GF, $\tilde{G}(z, s_e, s_{\rm ph}) = (z - M)^{-1}\rho(0)$. From the Taylor expansion of the pole

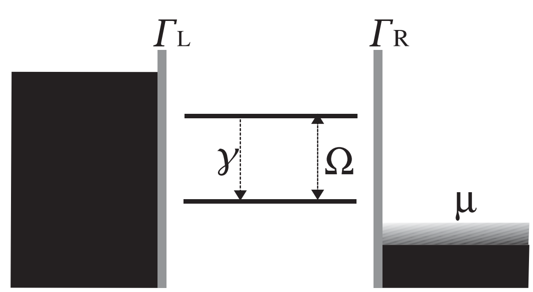

FIG. 1. A two-level QD coupled to two electronic leads. Electrons entering the QD from the left oscillate between both levels with Rabi frequency Ω, can relax with rate γ, or tunnel out if its energy is greater than μ.

$z_0 = \sum_{m,n>0} c_{mn}(s_e-1)^m(s_{\rm ph}-1)^n$, we obtain $G(t, s_e, s_{\rm ph}) \sim g(s_e, s_{\rm ph})e^{z_0 t}$ and the central moments

$$\langle n_{e({\rm ph})}\rangle = \frac{\partial g(1,1)}{\partial s_{e({\rm ph})}} + c_{10(01)}t, \tag{3a}$$

$$\sigma^2_{e({\rm ph})} = \frac{\partial^2 g(1,1)}{\partial s^2_{e({\rm ph})}} - \left(\frac{\partial g(1,1)}{\partial s_{e({\rm ph})}}\right)^2 + (c_{10(01)} + 2c_{20(02)})t, \tag{3b}$$

or the electron-phonon correlation (not discussed here), given by

$$\langle n_e n_{ph}\rangle - \langle n_e\rangle\langle n_{ph}\rangle = \frac{\partial^2 g(1,1)}{\partial s_e \partial s_{ph}} + c_{11}t. \tag{4}$$

Higher moments can be straightforwardly obtained by this formalism. In the large time asymptotic limit, all the information is included in the coefficents $c_{\rm mn}$. Then, the Fano factor is $F_{e({\rm ph})} = 1 + 2c_{20(02)}/c_{10(01)}$ so the sign of the second term in the right-hand side defines the sub-$(F<1)$ or super-$(F>1)$ Poissonian character of the noise.

We describe our system by the Hamiltonian $\hat{H}(t) = \sum_i \varepsilon_i \hat{d}_i^\dagger \hat{d}_i + \frac{\Omega}{2}(e^{-i\omega t}\hat{d}_2^\dagger \hat{d}_1 + {\rm H.c.}) + \sum_Q \omega_Q \hat{a}_Q^\dagger \hat{a}_Q + \sum_Q \lambda_Q(\hat{d}_2^\dagger \hat{d}_1 \hat{a}_Q + {\rm H.c.}) + \sum_{k\sigma}\varepsilon_{k\alpha}\hat{c}_{k\alpha}^\dagger \hat{c}_{k\alpha} + \sum_{k\alpha i} V_\alpha(\hat{c}_{k\alpha}^\dagger \hat{d}_i + {\rm H.c.})$, where $\hat{a}_Q$, $\hat{c}_{k\alpha}$, and $\hat{d}_i$ are annihilation operators of phonons and electrons in the leads and in the QD, respectively. Writing the density matrix as a vector, $\rho = (\rho_{00}, \rho_{11}, \rho_{12}, \rho_{21}, \rho_{22})^T$, the equation of motion of the GF (2) is described, in the Born-Markov approximation, by the matrix

$$M(s_e, s_{\rm ph}) = \begin{pmatrix} -2\Gamma_{\rm L} - (\chi_1+\chi_2)\Gamma_R & s_e\bar{\chi}_1\Gamma_R & 0 & 0 & s_e\bar{\chi}_2\Gamma_R \\ \Gamma_{\rm L} + s_e^{-1}\chi_1\Gamma_R & -\bar{\chi}_1\Gamma_R & i\frac{\Omega}{2} & -i\frac{\Omega}{2} & s_{\rm ph}\gamma \\ 0 & i\frac{\Omega}{2} & -\frac{(\bar{\chi}_1+\bar{\chi}_2)\Gamma_R+\gamma}{2} + i\Delta_\omega & 0 & -i\frac{\Omega}{2} \\ 0 & -i\frac{\Omega}{2} & 0 & -\frac{(\bar{\chi}_1+\bar{\chi}_2)\Gamma_R+\gamma}{2} - i\Delta_\omega & i\frac{\Omega}{2} \\ \Gamma_{\rm L} + s_e^{-1}\chi_2\Gamma_R & 0 & -i\frac{\Omega}{2} & i\frac{\Omega}{2} & -\gamma - \bar{\chi}_2\Gamma_R \end{pmatrix}, \tag{5}$$

where $\gamma = 2\pi|\lambda_{\varepsilon_2-\varepsilon_1}|^2$ is the spontaneous phonon emission rate, $\Gamma_\alpha = 2\pi|V_\alpha|^2$ is the tunneling rate through the contact α, Ω is the Rabi frequency, which is proportional to the intensity of the ac field, and $\chi_i = f(\varepsilon_i - \mu) = (1 + e^{(\varepsilon_i-\mu)\beta})^{-1}$ and $\bar{\chi}_i = 1 - \chi_i$ weight the tunneling of electrons between the right lead (with a chemical potential μ) and the state i in the QD. The Fermi energy of the left lead is considered infinite, so no electrons can tunnel from the QD to the left lead. All the parameters in these equations, except the sample-depending coupling to the phonon bath, can be externally manipulated. In the limit $\Gamma_{L(R)} \to 0$, we recover the pure RF case for the statistics of the emitted phonons [20],

$$F_{\rm ph}(\Gamma_i = 0) = 1 - \frac{2\Omega^2(3\gamma^2 - 4\Delta_\omega^2)}{(\gamma^2 + 2\Omega^2 + 4\Delta_\omega^2)^2}, \tag{6}$$

yielding the famous sub-Poissonian noise result at resonance ($\Delta_\omega = 0$). In the following, we will restrict ourselves to the resonant case.

Electron noise.—By tuning μ, we control the tunneling of electrons from the two levels. Let us first consider the *nondriven* case ($\Omega = 0$). If $\mu < \varepsilon_2$ (i.e., $\chi_2 \approx 0$), the Fano factor becomes

$$F_e = 1 + \frac{2\Gamma_L\Gamma_R(-2(\gamma+\Gamma_R)^2 + \Gamma_L\Gamma_R\chi_1^2 + [2\gamma\Gamma_L + (\gamma+\Gamma_R)(2\gamma+3\Gamma_R)]\chi_1)}{((\gamma+\Gamma_R)(2\Gamma_L+\Gamma_R) - \Gamma_L\Gamma_R\chi_1)^2}. \tag{7}$$

The *dynamical channel blockade* case is of special interest, as for $\varepsilon_1 < \mu < \varepsilon_2$ ($\chi_1 \approx 1$) the occupation of the lower level blocks the transport through the upper level, leading to super-Poissonian noise [6]:

$$F_e = 1 + \frac{2\Gamma_L\Gamma_R}{\Gamma_R(\gamma+\Gamma_R) + \Gamma_L(2\gamma+\Gamma_R)}, \tag{8}$$

which has a finite well-defined value in spite of the strong current suppression. In contrast, the large-bias case, $\mu \ll \varepsilon_1$ ($\chi_1 \approx \chi_2 \approx 0$), where the conduction through both channels is allowed, results in sub-Poissonian noise [10]:

$$F_e = 1 - \frac{4\Gamma_L\Gamma_R}{(2\Gamma_L+\Gamma_R)^2}, \tag{9}$$

independent of the coupling to the phonon bath. Note that by comparing with the single resonant level case [22], the inclusion of an additional level in the bias window increases the noise. In the opposite limiting case, $\mu \gtrsim \varepsilon_2$ ($\chi_2 \lesssim 1$), where the transport is still possible due to the thermal smearing of the Fermi function, the noise tends to be Poissonian before the current is completely suppressed for higher μ (cf. Fig. 2). The phonon bath mainly affects the current noise through the nondriven device in the region $\varepsilon_1 \lesssim \mu \lesssim \varepsilon_2$ by reducing it as γ increases (''noise reduction by noise'') without modifying its super-Poissonian character cf. Eq. (8), and in a extremely sensible way in the regime $\mu \gtrsim \varepsilon_2$, where the small rate for electrons tunneling from the upper level to the right lead is not able to compete with the phonon-induced relaxation that blocks the current and suppresses the super-Poissonian noise (Fig. 2).

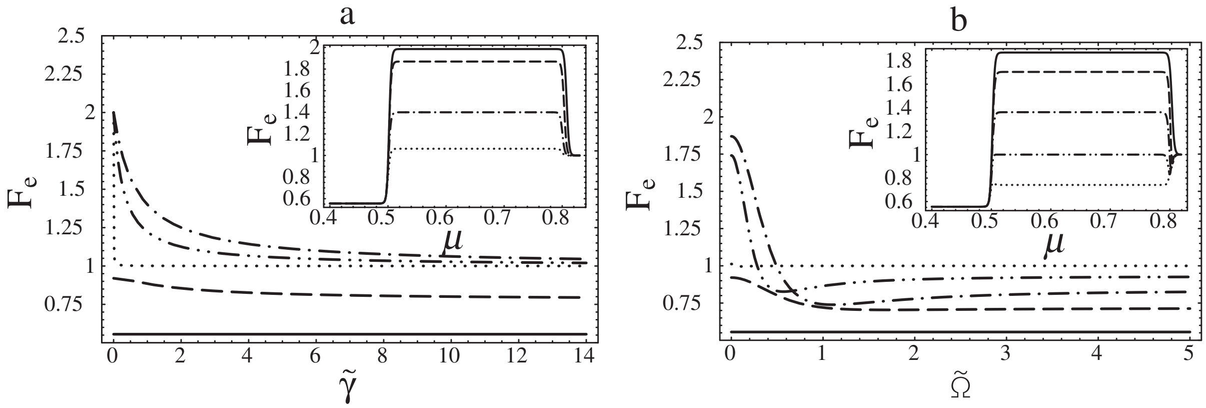

FIG. 2. Electron Fano factor as a function of (a) $\tilde{\gamma} = \gamma/\Gamma$ for $\tilde{\Omega} = \Omega/\Gamma = 0$ ($\Gamma = \Gamma_{L/R}$) and (b) $\tilde{\Omega}$ for $\tilde{\gamma} = 0.1$ for different chemical potentials: $\mu < \varepsilon_1$ (solid), $\mu = \varepsilon_1$ (dashed), $\varepsilon_1 < \mu < \varepsilon_2$ (dash-dotted), $\mu = \varepsilon_2$ (dash-dot-dotted) and $\mu \gtrsim \varepsilon_2$ (dotted). Note that, in the nondriven case (a), the phonon bath does not affect the sub or super-Poissonian character of the electron noise which can be manipulated introducing the resonant driving field (b). Insets: (a) F_e as a function of μ for $\Omega = 0$ and different phonon emission rates: $\tilde{\gamma} = 0.01$ (solid line), $\tilde{\gamma} = 0.1$ (dashed), $\tilde{\gamma} = 1$ (dash-dotted line) and $\tilde{\gamma} = 10$ (dotted line) and (b) for $\tilde{\gamma} = 0.1$ and different driving intensities: $\tilde{\Omega} = 0$ (solid line), $\tilde{\Omega} = 0.15$ (dashed line), $\tilde{\Omega} = 0.3$ (dash-dotted line), $\tilde{\Omega} = \tilde{\Omega}_P = 0.492$ (dash-dot-dotted line), $\tilde{\Omega} = 1$ (dotted line). Parameters (holding for all figures, in meV): $\varepsilon_1 = 0.5$, $\varepsilon_2 = 0.8$, $\kappa_B T = \beta^{-1} = 2 \times 10^{-3}$.

The introduction of a finite ac field affects the character of the electron noise only in the region where μ is between the energies of the two levels (see Fig. 2). The possibility of pumping electrons from the lower to the upper level by means of the absorption of one photon suppresses the dynamical channel blockade and, consequently (at sufficiently large Rabi frequencies), the electronic noise turns out to be sub-Poissonian:

$$F_e = 1 + \frac{2\Gamma_L\Gamma_R[\Gamma_L(2\gamma+\Gamma_R)(\gamma+\Gamma_R)^2 + \Gamma_R(\gamma+\Gamma_R)^3 - \Omega^2(2(\gamma^2+2\Omega^2) + 4\Gamma_L(2\gamma+\Gamma_R) + \Gamma_R(7\gamma+5\Gamma_R))]}{(\Gamma_R[(\gamma+\Gamma_R)^2 + 3\Omega^2] + \Gamma_L[(2\gamma+\Gamma_R)(\gamma+\Gamma_R) + 4\Omega^2])^2}. \quad (10)$$

For $\mu \ll \varepsilon_1$, the Fano factor does not depend on the ac field and expression (9) is recovered.

Phonon noise—We now turn to the phonon statistics cf. Fig. 3. In the *nondriven* case, a phonon can be emitted only when an electron enters the upper level with the lower level being already empty by an electron having tunneled out into the right contact. Therefore, the emission of phonons is antibunched and suppressed in the region where the transport of electrons is blocked by the occupation of the lower level (i.e., when $\mu > \varepsilon_1$):

$$F_{\rm ph} = 1 - \frac{2\gamma\Gamma_L\Gamma_R(\gamma + 2\Gamma_L + 2\Gamma_R)(1-\chi_1)}{(\Gamma_L[2\gamma + \Gamma_R(2-\chi_1)] + \Gamma_R(\gamma+\Gamma_R))^2}. \quad (11)$$

However, the driving field gives an additional way of populating the higher level, increasing the noise and leading, for high enough field intensities, to the bunching of phonons (see Fig. 3). A crossover from this *pure transport* infinite bias regime ($\mu \ll \varepsilon_1$), where the phonon noise increases with the Rabi frequency, to the strictly sub-Poissonian noise where the electron transport is suppressed [and the RF limit (6) is recovered] can be observed by increasing μ and successively blocking the transport through the lower (for $\mu > \varepsilon_1$) and upper level (for $\mu > \varepsilon_2$) cf. Fig. 3. It can be easily seen that the noise suppression for this last case is largest when $\Omega = \gamma/\sqrt{2}$. Thus, in the more interesting intermediate regime, $\varepsilon_1 < \mu < \varepsilon_2$, as in the RF case, a minimum at low intensities is observed which is shifted and modified by the electronic transport, and also the super-Poissonian noise characteristic of the

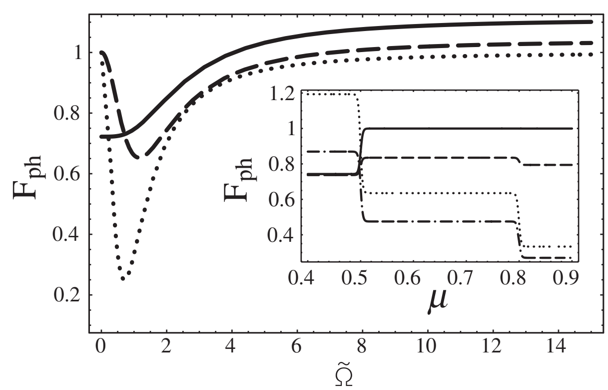

FIG. 3. $F_{\rm ph}$ as a function of $\tilde{\Omega}$ for $\tilde{\gamma} = 1$ and different chemical potentials: $\mu < \varepsilon_1$ (solid line), $\varepsilon_1 < \mu < \varepsilon_2$ (dashed line) and $\mu > \varepsilon_2$ (dotted line). In the large-bias case, the noise always increases with $\tilde{\Omega}$ and becomes super-Poissonian (in this concrete configuration) for $\tilde{\Omega} > 4.046$. In the case $\mu > \varepsilon_2$, there is a pronounced minimum at $\tilde{\Omega} = 1/\sqrt{2}$ typical of the RF. The case $\varepsilon_1 < \mu < \varepsilon_2$ follows an intermediate behavior showing the RF-like minimum and the super-Poissonian noise (typical of the large-bias case) for $\tilde{\Omega} > 5\sqrt{2}$ (see text). Inset: $F_{\rm ph}$ as a function of μ for $\tilde{\gamma} = 10$ and different Rabi frequencies: $\tilde{\Omega} = 0.1$ (solid line), $\tilde{\Omega} = 2$ (dashed line), $\tilde{\Omega} = 6$ (dash-dotted line), $\tilde{\Omega} = 10$ (dotted line). For high $\tilde{\Omega}$, μ changes the character of the noise.

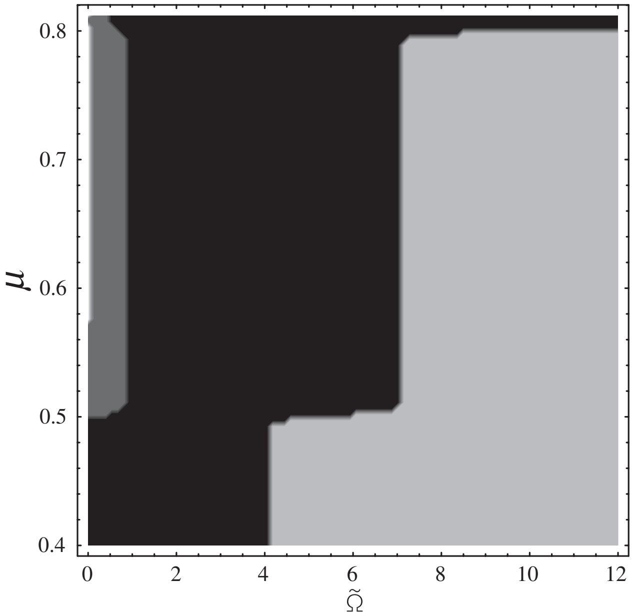

FIG. 4. Color plot showing the different regions where one can find F_e, $F_{ph} \geq 1$ (white, appearing only for $\tilde{\Omega} = 0$), $F_e < 1$, $F_{ph} \geq 1$ (light gray), $F_e \geq 1$, $F_{ph} < 1$ (dark gray), and F_e, $F_{ph} < 1$ (black) by tuning μ and $\tilde{\Omega}$, for $\tilde{\gamma} = 1$.

large-bias regime for $\Omega > [4(3\gamma + 2\Gamma_R)\Gamma_L^2/\Gamma_R + (13\gamma + 11\Gamma_R)\Gamma_L + 3\Gamma_R(\gamma + \Gamma_R)]^{1/2}$ (see Fig. 3).

Joining both studies, it is possible to find different regions where the electron and phonon noise show all the sub and super-Poissonian combinations except the bunching of both electrons and phonons, since in the low intensities regime, where the electron noise tends to be super-Poissonian, the phonon noise is sub-Poissonian (strictly Poissonian for $\Omega = 0$), see Fig. 4.

In conclusion, we propose a solid-state, transport version of resonance fluorescence (driven two-level quantum dot with phonon emission), where the noise for both the transferred electronic current and the emitted phonons can be experimentally controlled by tuning the transport (chemical potential [7]) and optics (ac field intensity) parameters. We found various regimes showing different combinations of sub and super-Poissonian electron and phonon Fano factors. Although not discussed here, our technique can also be applied to driven electron-phonon systems with higher complexity such as double quantum dots [23], and to obtain higher moments and correlations between electron and phonon distributions, which will be the scope of a future work.

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